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FINAL REPORT

ON

STUDY OF THE VIBRATION CHARACTERISTICS OF BEARINGS

December 6, 1963

Contributors:

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Submitted to:

U. S. DEPARTMENT OF THE NAVY
CHIEF, BUREAU OF SHIPS
WASHINGTON 25, D.C.

MAILED 1964

RESEARCH LABORATORY
SKF INDUSTRIES, INC.
ENGINEERING AND RESEARCH CENTER
KING OF PRUSSIA, PA.

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FINAL REPORT

ON

STUDY OF THE VIBRATION CHARACTERISTICS OF BEARINGS

INTRODUCTION

This study covers work performed by SKF Industries, Inc. under U. S. Navy Contract No. NObS-78552, between April 1960 and February 1963.

The purpose of the study was to investigate the vibration and noise producing characteristics of large roller bearings for propulsion machinery of ships as well as smaller rolling bearings used in auxiliary drives, with the ultimate aim of devising means to reduce vibration and noise caused by these bearings for all audio and subsonic frequencies.

A similar contract (NObS-77184) was awarded to the Franklin Institute in July 1959, and has also recently been completed. That contract covered only the vibration of small bearings for auxiliary drives.

Work performed by SKF Industries under Contract NObS-78552 has been covered by the following reports:

Progress Reports No. 1-17, issued bi-monthly during the period between April 1960 and January 1963.

The following "Special Reports".

Relative Axis Motion Induced by Variable Elastic Compliance in Ball Bearings (SKF Report L60L023)

Harmonic Analysis of the Relative Axis Motion Induced by Variable Elastic Compliance in Ball Bearings (SKF Report AL61L009)

A Study of Elastic Vibrations of the Outer Race of a Rolling Element Bearing (SKF Report AL61L027)

Analytical Study of the Vibration of a Bearing with Flexurally Rigid Races (SKF Report AL61L032)

Flexural Vibrations of a Ball Bearing Outer Ring due to Ball Loads (SKF Report AL61L037)

Analytical Study of the Radial, Axial and Angular Vibration of a Bearing with Flexurally Rigid Races (SKF Report AL62L005)

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This report is the Summary Report of the study. It consists of three parts:

Part I, covering Phase III of the Contract (Study of bearings with bore size range of 25-100 mm).

Part II, covering Phases I and II of the Contract (Study of large cylindrical, spherical and tapered roller bearings).

Part III, Design Guidelines, in which the effects of various parameters on the vibration of both small deep groove ball bearings and large roller bearings are summarized, and a procedure, is given for use in designing bearings for quiet running applications.

SUMMARY

The main results of the study are listed below:

1. Equipment and techniques for measuring vibration characteristics of small ball bearings and large roller bearings is now available. It includes equipment developed especially for this contract and equipment in general use at SKF Industries, which was applied to contract work.
 - a. Vibration test equipment to measure radial, axial and angular vibrations of the bearing outer ring or of a housing in which the bearing is mounted, in the frequency range between 3 and 10000 cps, both at discrete frequencies and in finite frequency bands. The equipment permits measurement at various radial and axial loads and rotational speeds.
 - b. Equipment for measurement of the airborne noise emitted by the bearing.
 - c. Equipment for study of vibration damping characteristics of bearing housings.
 - d. Equipment for measurement of the micro-geometry of bearing parts.
2. A mathematical theory of bearing vibration has been developed and presented. This theory relates the amplitudes and frequencies of bearing vibration to its sources. Natural frequencies of the bearing have been computed and the effect of housing elasticity on the bearing vibration has been analyzed.

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3. Experiments have been conducted for the following purposes:

- a. To verify the mathematical theory relating vibration to its sources, with special emphasis on the correlation between vibration and waviness, measured at discrete frequencies and in finite bands.
- b. To experimentally determine natural frequencies of the bearing, to compare these to the computed frequencies.
- c. To study the effect of damping, both within the bearing and in the housing, on bearing vibrations, and to test the feasibility of using special vibration attenuating bearing mounts.
- d. To study the effect of rotational speed, and axial and radial load on bearing vibration.
- e. To study the airborne noise of bearings under various loads and rotational speeds and correlate airborne noise with structureborne vibrations.
- f. To construct, and study the vibration characteristics of, bearings with improved micro-geometry of parts and of improved design.

4. The effects of various bearing parameters (both micro-geometry and design parameters), as well as operating and mounting parameters on the vibration characteristics of bearings have been examined. Guide lines for the selection of parameters have been given.

CONCLUSIONS

1. The principal accomplishments of this research are:

- a. Design principles to make quieter rolling bearings.
- b. Theory and experimental evidence identifying the micro-geometrical causes of bearing vibration.
- c. Methods to dampen vibration emanating from a rolling bearing.
- d. Instrumentation to measure vibration and noise of large and small rolling bearings.

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2. The instrumentation and experimental techniques developed under this contract have been found to be fully adequate for study of vibration characteristics of bearings in the size range and under the conditions covered by the contract. They can without modification be used in future studies of bearing vibration.

3. On the basis of theoretical and experimental findings of this study it is possible to make reasonably accurate predictions regarding the vibration spectrum of a bearing of a given design, if the detailed micro-geometry of the bearing parts is known.

4. If the bearing vibration level in any given frequency range under given load and speed conditions is known, order a magnitude estimates of the vibration level under other conditions of speed and load can be obtained.

5. The vibration level of a bearing in any given frequency range may be reduced by

- a. Improving the micro-geometry of the rolling surfaces, or
- b. by changing the bearing design.

Procedures are given for most efficient selection of micro-geometrical and design parameters to obtain a desired reduction in vibration level.

6. Large spherical and cylindrical roller bearings of improved vibration quality due to improved micro-geometry have been manufactured and tested. The reduction in vibration level is approximately as expected from the improvements in micro-geometry.

7. Ball bearings of improved vibration quality in the low frequency range due to design changes, have been manufactured and tested. The new design has smaller balls, a larger number of balls and a thicker outer ring than bearings of conventional design.

8. Large spherical roller bearings with two rows of rollers were found to have lower vibration level than single row cylindrical and tapered roller bearings, for the same micro-geometry of parts. This is partly explainable by the reduction due to the double-row design and partly attributed to other causes.

9. The vibration level of a bearing, except for very low frequencies, can be reduced effectively by mounting the bearing in a housing supported by laminated elastic members.

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10. The vibration level of a bearing in the radial direction can be reduced by mounting the bearing in a housing which is comparatively flexible angularly and comparatively stiff radially. The vibration level of the angular vibrations can be reduced by mounting the bearing in a housing which is radially flexible and angularly stiff.

RECOMMENDATIONS FOR FUTURE RESEARCH

1. Study of design and mounting parameters influencing the vibration of large roller bearings. The effects of roller skewing, flange geometry, misalignment between shaft and housing and effects peculiar to double-row bearings should be studied. Various housing designs should be examined analytically and experimentally and the optimum design procedure previously used for small ball bearings applied to large bearings. Bearings of improved vibration quality should be manufactured and tested.
2. Development, manufacture and testing of deep groove ball bearings of improved design and extremely quiet running quality.
3. Study of a vibratory system simulating a piece of rotating machinery and consisting of a rotating mass supported by two bearings mounted in elastic housings in turn attached to large masses, to find interrelationships between bearing and system vibration.
4. Study of externally forced vibrations of a rolling bearing under time-variable load.
5. Study of cage noises in large bearings.
6. Study of vibration influencing effects of lubricants and development of quiet running bearing lubricants.
7. A more detailed study of the subsonic vibrations generated by large rolling bearings.
8. A study of the bearing vibrations in the frequency range above 10000 cps, including the near ultrasonic range.

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PART I

STUDY OF VIBRATION AND NOISE CHARACTERISTICS OF ROLLING BEARINGS WITH BORE SIZE IN THE RANGE OF 25-100 MM.

1.1 INTRODUCTION

A rolling bearing is a complex vibratory system. Both the outer and inner ring, the rolling elements and the cage of a rotating bearing vibrate. The vibrations are transmitted to masses attached to the bearing (housing and shaft) and to the surrounding air and structures.

The amplitudes and frequencies of the vibrations emitted by the bearing depend on:

1. The source of vibration which may be within or outside the bearing. Only vibrations generated by the bearing itself are discussed in this report.
2. The response of the system. The various vibration sources act as inputs to a system consisting of the bearing races (with possible masses attached), and the output is the vibration measured at the outer ring, on a housing attached to the outer ring or at some other part of the system.

Part I of this report covers bearings in the size range used in noise critical applications of auxiliary naval equipment. In the majority of such applications the outer ring is stationary and fitted into a housing while the inner ring supports the rotating shaft. This condition was simulated in most of the tests performed as part of this study and used as a basis for mathematical analysis. Since the vibrations of a bearing mounted in this manner are transmitted mainly through the outer race, the main emphasis was put on the study of outer ring vibrations.

This part of the study covers deep groove ball bearings with bore sizes in the range between 25 and 100 mm. The rotational speed range is from 1000 to 3600 RPM. The findings apply to stationary radial loads up to approximately 15% and axial loads up to approximately 30% of the basic static load rating of the bearings.

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The test equipment used in the experimental study of bearings with 25-100 mm bore is discussed in Section 1.2.

Various sources of bearing vibrations are reviewed in Sections 1.3.1 - 1.3.6.

A brief discussion of contact deformations in ball bearings is included in Section 1.3.7, since they influence the vibration response characteristics of the bearings.

The frequencies generated by various vibration sources are determined in Sections 1.3.8 - 1.3.11.

Factors influencing the frequency response of the bearing as a vibratory system are discussed in Sections 1.3.12 - 1.3.18. This includes a brief discussion on natural frequencies and output amplitudes generated by various input sources.

The effect of load and speed is covered in Sections 1.3.19 and 1.3.20.

A comparison of vibration levels in different measuring directions and generated by various sources is given in Sections 1.3.21 - 1.3.23.

The effect of damping and means of attenuating the bearing vibrations are discussed in Sections 1.3.24 - 1.3.26.

The airborne noise characteristics of ball bearings are examined in Section 1.3.27.

Additional parameters influencing bearing vibration are briefly enumerated in Section 1.3.28.

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1.2 TEST EQUIPMENT

Equipment used in the experimental study, in data processing and analysis of the vibration characteristics of bearings with 25-100 mm bore, is listed below:

1.2.1 Vibration Tester for Measurement of Bearing Vibration under Radial and Axial Load

This tester was designed and built by SKF Industries for the purpose of measuring the bearing vibration under various load and speed conditions. A photograph of the tester is shown on Enclosure 1.

The tester accommodates two test bearings and two support bearings mounted on a common shaft. The support bearings are mounted in pillow blocks and are ball bearings of the same bore size as the test bearings. The test bearings are located between the support bearings and can be tested either without a housing or with the outer ring mounted in a cylindrical housing. Test bearing housings of various thicknesses were provided for each bearing size tested. A separate shaft is needed for each test bearing size. Support bearings of various O.D. sizes are accommodated by interchangeable inserts in the pillow block bores.

Radial load is applied to the test bearings through flexible steel straps slung around the outer rings of the test bearing housings (or around the O.D. of the test bearings), by elastic elements (springs or rubber pads) compressed by nuts on top of the frame. To avoid flexural deformation of the outer ring of the test bearings due to the steel straps, most of the tests were performed with the bearings mounted in housings. Strain gages on the tiebars are used to measure the magnitude of the radial load. The radial load range is 0-600 lbs.

Axial load is applied by nuts on both threaded ends of the shaft, over elastic thrust washers. The axial load range is 0-200 lbs. The method of applying the loads to the test bearings is illustrated by Enclosure 2. Calibration of axial loads was accomplished by measurement of the elastic deformation of the thrust washers and comparison with their deflection under known loads.

The tester is located in an anechoic chamber (See 1.2.6). The shaft is driven by a quill from a variable speed electric motor outside the anechoic chamber. The rotational speed of the shaft is continuously variable in the range between 1000 and 3600 RPM.

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To measure the vibration of the test bearing, the test bearing outer ring or housing in which the bearing is mounted is contacted by velocity sensitive pickups of **ESF** type MEA-100 (See Progress Report No. 4 of (2)*. The pickups are mounted on a rigid bracket, one for measurement of vibrations in the direction of the applied radial load (vertical) and one for measurement of vibrations perpendicular to the load (horizontal).

1.2.2 Vibration Tester for Measuring Bearing Vibrations under Pure Axial Load

Enclosure 3 shows a photograph of a mechanical test unit **ESF** Model MVB-1 used for this purpose. This tester has been discussed in more detail under U. S. Navy Contract NObs-78593 (2). The tester is equipped with a mechanical loading system for application of the axial load. A pressure element consisting of a rubber cup with a thin steel ring to contact the bearing face is used (See Progress Report No. 4 of (2)). This tester is referred to as the BCV tester in (2). Another version of the tester, operating on exactly the same principle was referred to as the VKL tester in Progress Reports No. 10, 11 and 12 of (2). An **ESF** MEA-100 velocity type pickup is used as vibration sensor. This is normally mounted as shown on Enclosure 2 to measure radial vibrations of the bearing outer ring. The tester is used in conjunction with electronics described in 1.2.7 and 1.2.8.

1.2.3 Race Waviness Tester

Surface waviness is described in the ASA Standard B46.1-1961 as relatively widely spaced irregularities of the nominal surface. To measure the waviness of a ball bearing inner or outer ring, the ring is rotated at a constant speed and the surface being measured is contacted by a velocity sensitive pickup of the type described in 1.2.2. As the ring rotates the waviness irregularities on the surface of the ring displace the pickup stylus radially. The amplitude of the electrical signal produced by the pickup is proportional to the radial velocity of the pickup stylus.

* Literature references are listed at the end of this report.

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This signal is amplified, filtered and metered on a RMS indicating instrument. The waviness is measured in five octave bands corresponding to 3-6, 6-12, 12-24, 24-48 and 48-96 waves per circumference. The readout is expressed in microinches/second at a rotational speed of 1000 RPM. Enclosure 4 shows a photograph of the race waviness tester developed by B&K Industries, Inc. Various sizes of inner and outer rings are accommodated with interchangeable mandrels and pot chucks that are mounted on the spindle nose by a high angle taper held with a draw screw. The spindle used in this tester is of special smooth running quality to minimize the effect of spindle vibrations on waviness readings.

1.2.4 Ball Waviness Tester

The B&K Ball Waviness Tester operates on the same principle as the Race Waviness Tester described in 1.2.3. Since, however, ball waviness, in general, is of a considerably lower amplitude than race waviness additional precautions have been taken to reduce the effect of spindle vibrations on waviness readings. This has been accomplished by mounting the test ball in a seat comprising three stationary balls and rotating the ball with a driver which is flexibly attached to the spindle. The driver is free to accommodate radial motions and therefore will not transmit spindle vibrations to the ball.

Enclosure 5 shows photographs of the mechanical unit of the B&K Ball Waviness Tester. The electronic unit is similar to the one used with the Ball Waviness Tester. It measures ball waviness expressed in microinches/second RMS in five octave bands, corresponding to 4-8, 8-16, 16-32, 32-64 and 64-128 wpc at a spindle speed of 740 RPM.

Additional details on measurement of waviness are given in (4).

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1.2.5 Equipment for Airborne Noise Measurement

Airborne noise measurements were performed using a Bruel and Kjaer Condenser Microphone Type 4111 in conjunction with a Bruel and Kjaer 2602 Microphone Amplifier and a set of Allison filters Model 2ABR. All measurements of airborne bearing noise were performed in an Anechoic Chamber (1.2.6), using the Bearing Vibration Tester described in 1.2.1. The microphone was located in all the tests as shown in Enclosure 6.

1.2.6 Anechoic Chamber

This is a double walled demountable, sound proof room, designed by the Bell telephone Laboratories. It consists of an inner room supported by springs, and an outer room which completely encloses the inner room except that the floor of the outer room consists of the floor of the building. For effective acoustical attenuation the walls of the inner and outer room consist of panels made of two composite sheets of steel cemented to composition board with the interspace filled with rock wool. To reduce sound reverberations in the room, the acoustical treatment of the inner room includes sheets of perforated material on all inner surfaces of the wall and ceiling with an air space separating the perforated sheet from the steel panels. The total wall thickness, including the thickness of the inner and outer room and the air space between them is approximately 14". A 2" hole was provided in the wall for the quill drive of the vibration tester, described in 1.2.1.

The sound levels (in absolute decibels) within the room, measured in octave bands in the 1.25-12800 cps range, with no equipment running inside the room and normal daytime activity outside the room, were as follows:

Frequency Bands (cps)	1.25-2.5	2.5-5	5-10	10-20	20-40	40-80
Decibel Level	42	55	59	70.5	70	59.5
Frequency Bands (cps)	80-160	160-320	320-640	640-1280	1280-2560	2560-5120
Decibel Level	45.5	29.5	59	63	42	30
Frequency Bands (cps)	800-1600	1600-3200	3200-6400	6400-12800		
Decibel Level	<27	<27	<27	<27		

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1.2.7 Equipment for Narrow Band Frequency Analysis

The following equipment, built by Panoramic Radio Products, was used in narrow band bearing vibration analysis:

- a. Sonic Analyzer LP-1A
- b. Subsonic Analyzer LF-1
- c. Power Spectral Density Analyzer PDA-1
- d. Auxiliary Function Unit C-1
- e. Multi Bandwidth Filter CF-1
- f. Recorder RC-2
- g. Sonic Response Indicator G-2

To provide proper gain and for accurate frequency determination the following equipment was used in conjunction with the Panoramic narrow band analyzer:

- a. Ballantine True RMS Electronic Voltmeter Model 320
- b. Hewlett-Packard Amplifier, Model 450-A
- c. Millivac Instruments Hushed Transistor Amplifier, Type VS-64A
- d. Hewlett-Packard Low Frequency Oscillator, Model 202C
- e. Hewlett-Packard Electronic Counter, Model 523-D
- f. Oscilloscope

A block diagram of the arrangement is shown on Enclosure 7. By the use of this instrumentation the frequency axis is accurately scaled by oscilloscopic observation of Lissajous patterns from a precisely known frequency standard and the output of the Sonic Response Indicator, slaved to the Panoramic Analyzer, which gives an exact multiple of the frequency over which the Analyzer sweeps at a given time. Points of frequency coincidence were marked on the graph using a triggered second pen on the recorder.

The Subsonic Analyzer LF-1 was used in the analysis of low frequency spectra in the range up to 200 cps. The Sonic Analyzer LP-1A was used in the higher frequency range.

For details of the procedures used, See Progress Report No. 7, (1) pages 4, 7 and 8.

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1.2.8 Instrumentation for Octave Band and Wide Band Vibration Analysis

For octave band and wide band analysis electronic equipment built by ~~ES~~ **ES** Industries was used to amplify the pickup signal and provide a readout, expressed in microinches/second, in eight octave bands in the 50-12800 cps range and the wider bands 50-300, 300-1800 and 1800-10000 cps (corresponding to the three Anderometer bands). The band pass filters used in these units were built to specification similar to those given for Anderometer filters in (2).

For airborne noise measurements Allison Model 2 AB2 filters were used (See 1.2.5).

1.2.9 Equipment for Testing the Vibration Damping Characteristics of Housing Materials

Enclosure 8 shows a sketch of an arrangement used in testing the vibration damping characteristics of rectangular beams. This equipment is used in conjunction with the vibration tester, 1.2.1. For further details, see Progress Report 9-10, page 8 and Progress Report 11, page 6 and 7, (1).

1.2.10 Tape Recorder

For detailed analysis of vibration signals, especially when narrow band spectra are required, the signal from the pickup was first tape recorded and later analyzed by various methods (See 1.2.7, 1.2.8 and 1.2.11).

The tape recorder used was an Ampex FM Seven Channel Tape Recorder Model F-1107.

1.2.11 Instrumentation for Measuring Axial and Angular Vibrations

The mechanical unit of the vibration tester, 1.2.2 was used for this purpose with two pickups mounted as shown on Enclosure 9. By use of suitable electrical circuitry, as shown on the block diagram of Enclosure 10, the sum and the difference of the outputs from the two pickups were formed, and the spectra of the sum and the difference recorded using a Panoramic narrow band analyzer. If at a given discrete frequency, the vibration is essentially angular, then a peak will appear in the spectrum representing the difference between the two pickup outputs, while the spectrum showing the sum will have near zero amplitude at this frequency. If the vibration is essentially axial translatory motion, the sum of the outputs will be of large amplitude and the difference will be of negligible amplitude. For details see Progress Report No. 11, pages 2-6, (1).

1.2.12 Length Measuring Equipment

Various instruments for contour tracing and dimensional measurements were used such as:

1. Talyrond Roundness Measuring Instrument, Model 50 - Taylor, Taylor and Hobson.
2. Opposed-Head Micro-Ac Comparator, Cleveland Instrument Company.
3. Leitz Perflektometer Length Measuring Instrument.

1.2.13 Digital Computer

The following two computers were used mainly in the numerical computation required for the study of variable elastic compliance motion, reported in the two Special Reports L60L023 and AL61L009, (1):

1. IBM 650
2. RCA 501

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1.3 THE DEEP GROOVE BALL BEARING AS A VIBRATORY SYSTEM

1.3.1 Sources of Bearing Vibration

If a deep groove ball bearing is subjected to an external load, the balls and race grooves deform elastically at the contacts between the races and balls. Due to the non-linearity of the Hertzian contact deformation, the contacts then act as non-linear springs, and the complete bearing behaves as a non-linear vibratory system. In addition to the contact deformations between balls and races, other elastic deformations also take place in the bearing, such as bending of the rings. Due to the complexity of the problem, a complete analytical treatment of the bearing as a vibratory system was not attempted. The bearing vibration was nevertheless studied analytically under various simplifying assumptions. The analytical findings were verified experimentally so that the results presented in this report are based on experimental as well as analytical evidence.

The vibrations generated by a rolling bearing may be divided into the following two main categories:

1. Vibration generated by a geometrically perfect bearing. These vibrations are related to inherent design characteristics of rolling bearings and occur as a consequence of the finite number of balls in these bearings.
2. Vibrations generated by geometrical imperfections of the rolling surfaces.

1.3.2 Vibrations Generated by a Geometrically Perfect Bearing

The vibrations generated by a geometrically perfect bearing may be subdivided into:

1. Vibrations induced by the variable elastic compliance of the bearing.
2. Vibrations caused by bending of the outer ring due to ball loads.

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1.3.3 Vibrations Induced by Variable Elastic Compliance

The vibrations induced by variable elastic compliance occur in a bearing under (constant) radial or combined load but not under pure axial load. The inner and outer ring of the bearing undergo relative motion as a consequence of the fact that the elastic compliance of the bearing varies with the angular position of the ball set relative to the action line of the load (3), (8) - (12).

The vibrations induced by the variable elastic compliance of a radially loaded ball bearing have been discussed in detail in Special Reports L60L023 and AL61L009 (1), under the assumption that:

1. The rings are flexurally rigid and undergo only local deformation due to the contact stresses.
2. The rotational speed is sufficiently low that inertial effects are negligible.
3. Friction forces are negligible.

The motion has the following characteristics:

1. The relative motion of the rings of a radially loaded zero contact angle ball bearing is a periodic two-dimensional motion in a plane perpendicular to the bearing axis.
2. The fundamental frequency of the motion is the frequency of balls passing a given point of the stationary outer ring of the bearing. This frequency is

$$f = \frac{ZN}{120} \left[1 - \frac{D}{d} \right] \quad (1.3.3-1)$$

where

- N = Rotational speed, RPM.
 D = Ball Diameter
 d = Pitch Diameter of ball set.
 Z = Number of balls

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3. Neither of the components of the motion (in the direction of the load and perpendicular to it) is a pure sinusoidal motion. Higher harmonics, which are multiples of the fundamental frequency are therefore present in the vibration spectrum.
4. The amplitudes of the harmonics of the velocity components depend, for a given bearing under pure radial load, on the radial load, the radial looseness, the rotational speed and the order of the harmonic. This relationship is expressed in terms of two parameters:

a. The "load/looseness parameter"

$$G = \frac{F_R}{2 C_d \left(\frac{\epsilon}{2}\right)^{1.5}} \quad (1.3.3-2)$$

where

F_R = Radial load, in lbs

Z = Number of balls

$C_d = \frac{F_R}{\delta^{1.5}}$ = Hertzian coefficient, in lbs/in^{1.5}

δ = Total contact deformation in the bearing under the radial load F_R , in inches.

ϵ = Radial looseness, in inches.

b. Two series of "amplitude/load parameters":

$$C_{ai} = \frac{1}{29.8} \left[\frac{C_d}{F_R} \right]^{\frac{2}{3}} a_i \quad (1.3.3-3)$$

$$C_{bi} = \frac{1}{29.8} \left[\frac{C_d}{F_R} \right]^{\frac{2}{3}} b_i \quad (1.3.3-4)$$

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where

C_{ai} = the "amplitude/load parameter" for the i :th harmonic in the direction of the load.

C_{bi} = the "amplitude/load parameter" for the i :th harmonic in the direction perpendicular to the load.

\bar{a}_i = the RMS value of the velocity component of the i :th harmonic in the direction of the load.

\bar{b}_i = the RMS value of the velocity component of the i :th harmonic in the direction perpendicular to the load.

Knowing the relationship between the "amplitude/load parameter" for any given harmonic and component and the "load/looseness parameter" the vibration amplitudes of the various harmonics can be determined. This requires, however, the solution of a system of transcendental equations. An explicit solution of this system could not be obtained. The analysis showed that the relationship between the parameter G and C_{ai} or C_{bi} depends only on the number of balls, i.e., this relationship is the same for all bearings which have the same number of balls. A numerical procedure, utilizing an IBM 650 computer, was used to determine this relationship. The results for the nine first harmonics of a bearing with seven balls are shown graphically on Enclosures 11-16. For bearings with a number of balls other than seven, similar curves can be obtained using the computational procedure outlined in Report AL61L009(1). The numerical procedure used in obtaining the curves shown on Enclosures 11-16 included, as the last step, a harmonic analysis. At this point an important approximation was introduced. It was assumed that the direction of the relative displacement between the rings coincides with the direction of the applied load. This is not exactly true; for certain angular positions of the ball complement the two directions may deviate by as much as 3°, or approximately 6% of the angular spacing between the balls. The curves of Enclosures 11-16 therefore give only the approximate relationship between the two parameters. The error introduced by this approximation was numerically evaluated for the nine lowest harmonics in the direction of the load for a value of $G = 0.08$. This value of G was selected because, for all nine harmonics, C_{ai} (according to Enclosures 11-16) has a maximum near this point.

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The computations showed that for this value of G the plotted values of C_{ai} are within $\pm 25\%$ of the computed values. This error is not considered excessive and it is therefore believed that the graphs on Enclosures 11-16 can be used to obtain at least order of magnitude estimates of the amplitudes of the various harmonics of the variable compliance vibrations.

5. The amplitude of the i :th ($i=1, 2 \dots 9$) harmonic may be obtained by:

- a. Computing the "load/looseness parameter G " from Equation (1.3.3-2).
- b. Obtaining the value of the "amplitude load parameter" C_{ai} or C_{bi} for the given G value from Enclosures 11-16.
- c. Computing the RMS velocity of the i th harmonic from the equations:

In the direction of the load

$$\bar{a}_i = 29.8 N |C_{ai}| \left[\frac{F_R}{C_g} \right]^{\frac{2}{3}} \quad (1.3.3-5)$$

In the direction of perpendicular to the load

$$\bar{b}_i = 29.8 N |C_{bi}| \left[\frac{F_R}{C_g} \right]^{\frac{2}{3}} \quad (1.3.3-6)$$

6. It is seen from Enclosures 11-16 that the amplitude of any harmonic of either velocity component, parallel or perpendicular to load (but not both components or several harmonics simultaneously), can be made equal to zero, by suitable selection of the radial load and radial looseness. The resultant amplitude of any harmonic can be minimized at a relatively low level.
7. The amplitudes of the harmonics generally decrease with the order of the harmonic.

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8. The results apply strictly only for a bearing under purely radial load. Under purely axial load vibrations due to the variable compliance of the bearing are not generated. If the bearing operates under combined load, the motion depends on the relative magnitudes of the axial and radial loads. If the axial load is small compared to the radial then the contact angle in the load zone approaches zero and the motion can be assumed to be the same as for purely radial load. An increase in axial load tends to equalize the ball loads, which results generally in decreasing amplitudes of the variable compliance vibration. As the ratio between axial and radial load increases these vibrations decrease and become zero for an infinite load ratio.

The amplitudes of the vibration induced by the variable elastic compliance will be compared to the amplitudes generated by other sources in 1.3.23.

1.3.4 Flexural Vibrations of the Outer Ring due to Ball Loads

Flexural vibrations of the outer ring due to ball loads occur in an axially as well as a radially loaded bearing. For details of these vibrations see Special ~~RE~~ ^{USF} Report No. AL61L037.(1).

The flexural vibrations of the outer ring, induced by the ball loads in an axially loaded bearing, have the following characteristics:

1. Any given point of the outer ring undergoes a periodic motion in the radial direction. The fundamental frequency of the motion is the frequency of balls passing a given point of the stationary outer ring of the bearing. This frequency is

$$f = \frac{ZN}{120} \left[1 - \frac{D}{d} \cos \alpha \right] \quad (1.3.4-1)$$

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where

 α = contact angleand Z , N , D and d as defined in 1.3.3.

This frequency is the same as the fundamental frequency of the variable compliance vibrations.

2. Higher harmonics, which are multiples of the fundamental frequency, are also present.
3. In a thin ring (such as the outer ring of most bearings) of mean radius R , and second moment of area of the ring cross-section I , loaded by a axial load F_A the RMS velocity amplitude of the k th harmonic of the flexural vibrations is given by

$$\bar{A}_k = \frac{F_A R^3 k Z N [1 - \frac{D}{d} \cos \alpha]}{60 \sqrt{2} E I [(kZ)^2 - 1]^2 \sin \alpha} \quad (1.3.4-2)$$

where E is the modulus of elasticity of the ring material and Z , N , D , d and α are as defined in 1.3.3 and 1.3.4.

It is seen that the velocity due to this type of vibratory motion is directly proportional to the applied load and speed and decreases inversely with the cube of Z and with the cube of the order k of the harmonic. The fundamental frequency is therefore highly predominant. The velocity amplitude of the second harmonic is approximately 12% and the third approximately 3.5% of that of the fundamental. Higher harmonics have successively lower amplitudes.

In a bearing of a given configuration the vibration level decreases with increasing stiffness I/R^3 of the outer ring.

4. Although the analysis applies strictly only when the ring is loaded by equally spaced, equal radial loads such as occur when the bearing supports purely axial load, the vibration due to radial load can be estimated by using an equivalent value of F_A which produces the same maximum ball load as the radial loading

F_R .

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This value of F_A is given approximately by

$$F_A \approx \frac{5F_R}{\tan \alpha}$$

where F_R is the applied radial load, Equation (1.3.4-2) as applied to radial loading then becomes

$$\bar{A}_k \approx \frac{F_R R^3 k_z N \left[1 - \frac{D}{d} \cos \alpha\right]}{12\sqrt{2} EI [k_z]^2 \tan \alpha} \quad (1.3.4-3)$$

The amplitudes of these vibrations will be compared to vibration amplitudes generated by other sources in 1.3.23

1.3.5 Vibrations Generated by Geometrical Imperfections of the Rolling Surfaces

In every ball bearing small deviations from the perfect geometrical shape of the balls and rolling paths of the races exist. These imperfections include:

- a. Random deviations from the perfect circular shape of the rolling paths of races and balls. These deviations are called waviness.
- b. Eccentricity (waviness of the order: one wave per circumference).
- c. Two point out-of-roundness (waviness of the order: two waves per circumference).
- d. Variations between the average diameters of the different balls in the bearing.

1.3.6 General Description of Vibratory Motion of the Outer Ring

When the surface imperfections are rolled over in the rotating bearing they generate outer ring vibrations by:

- a. Displacing the outer ring as a rigid body (its center moves under the influence of the varying ball loads).

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- b. Deforming the outer ring elastically, mainly by bending it and thus inducing flexural vibrations in addition to those generated by a geometrically perfect bearing.

The dynamic system consisting of the non-rotating rigid outer ring supported by the balls is a system with five degrees of freedom, i.e., the ring is free to move radially and axially as well as angularly. The total displacement of the ring is defined by the following five displacement components, as illustrated by Enclosure 17:

- a. The vertical displacement of the outer ring center, x_v .
- b. The horizontal displacement of the outer ring center, x_h where the selection of "vertical" and "horizontal" directions can be arbitrary.
- c. The axial displacement of the outer ring center, y .
- d. The angular displacement of the outer ring in the horizontal direction, defined by the angle λ_h in Enclosure 17.
- e. The angular displacement of the outer ring in the vertical direction, defined by the angle λ_v in Enclosure 17.

In addition to this rigid body motion, the vibratory motion of the outer ring is also affected by the motion due to elastic deformation of the outer ring. Of the elastic vibrations the flexural vibrations in the plane of the ring have been found to be predominant and will be considered along with the rigid body vibrations.

1.3.7 Contact Deformations in Ball Bearings

Both the rigid body vibration and the flexural vibrations of the outer ring are influenced by the contact deformations between balls and races. These deformations follow the non-linear Hertzian law:

$$P = c_s \delta^{3/2}$$

(1.3.7-1)

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where

P = load at the contact

δ = elastic deformation in the direction of the load P .

C_δ = constant, related to the shape and elastic properties of the contacting bodies*.

In the theoretical treatment of the outer ring vibrations it was found advantageous to use the following linear approximation of Equation (1.3.7-1)

$$P = k_N(\delta - \bar{\delta}) + \bar{P} \quad (1.3.7-2)$$

where

$\bar{\delta}$ = average deformation occurring under a load \bar{P} .

k_N = linear coefficient of contact deformation, related to the Hertzian coefficient by

$$k_N = \left(\frac{\partial P}{\partial \delta} \right)_{\delta = \bar{\delta}} = \frac{3}{2} C_\delta \bar{\delta}^{1/2} \quad (1.3.7-3)$$

Equations (1.3.7-2) and (1.3.7-3) are valid in the neighborhood of the point $\delta = \bar{\delta}$, $P = \bar{P}$, and Equation (1.3.7-2) thus gives a satisfactory approximation of Equation (1.3.7-1) as long as the range $|\delta - \bar{\delta}|$ is small compared to the average deformation $\bar{\delta}$.

For a bearing with Z balls and contact angle α operating under an axial load F_A , the linearized coefficient k_N may be expressed as

$$k_N = \frac{\frac{3}{2} \left[\frac{F_A}{Z \sin \alpha} \right]^{1/3}}{\left(\frac{1}{C_1} \right)^{2/3} + \left(\frac{1}{C_2} \right)^{2/3}} \quad (1.3.7-4)$$

* For the analytical expression relating C_δ to geometry and material parameters of the contact, see e.g., Palmgren (5).

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where C_i and C_o are the Hertzian coefficients at the ball contacts with the inner and outer races, respectively.

Enclosure 18 lists numerical values of the quantity $k_N F_A^{-\frac{1}{3}}$, computed for a few common deep groove ball bearing sizes. From this tabulation the value of k_N can be obtained for any given axial load F_A , and for three selected values of the contact angle α . The table is based on an inner and outer ring groove conformity of 51.75%. For any given groove conformity, contact angle, and number of balls, k_N is mainly influenced by the ball diameter and only to a minor degree by the groove diameters of the inner and outer rings. A good approximation of k_N for any bearing size (accurate within 1%) can therefore be obtained by disregarding the differences in groove diameters between various bearing sizes and considering the effect of ball size only. For a given load, contact angle, groove conformity and number of balls, k_N may then be regarded as a function of ball diameter only. This is illustrated by Enclosure 19 which shows graphically the quantity

$$\beta_N = \left[\frac{Z \sin \alpha}{F_A} \right]^{\frac{1}{3}} k_N \quad (1.3.7-5)$$

computed in this manner as a function of ball diameter, for various groove conformities in the range between 50.5 and 60%. In the computations the groove diameters of an "average size" bearing were used. The 6207 size was selected for this purpose. It is seen that for a constant axial load and number of balls k_N increases with ball size. For constant ball diameter and axial load k_N is inversely proportional to the cube root of the number of balls. It is also seen from Enclosure 19 that tightening the conformity increases k_N . Enclosure 19 is based on equal inner ring and outer ring conformities. If these differ from each other, Enclosure 19 may be applied to the average of the two conformities. The error, introduced by this procedure is less than 4%, for conformities in the range between 51 and 56%, but may amount to as much as 20% for the total range between 50.5 and 60%.

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1.3.8 Frequencies of Rigid Body Vibrations Generated by Geometrical Imperfections

The vibration spectrum of the rigid body motion of the bearing is characterized by a great number of discrete peaks. These peaks may be classified into two categories:

1. Linear peaks, the frequencies of which have been predicted theoretically under the assumption of linearity of the contact deformation according to Equation (1.3.7-2). They have been verified experimentally and found to dominate the rigid body vibration spectrum.
2. Non-linear peaks which occur due to non-linear elastic behaviour in a bearing with large values of ball diameter variation, eccentricity or of low order waviness, if the bearing is not operating under a sufficiently heavy load to render insignificant the "non-linear" effect of these imperfections, i.e., when the condition $|\delta - \delta_0| \ll \delta_0$ no longer holds.

The predominant vibration peaks generated by various orders of waviness under the assumption that the balls act as linear springs are tabulated in Enclosure 20.

It is seen from Enclosure 20 that:

1. Inner ring, outer ring and ball waviness produce radial, angular and axial vibrations of the outer ring of the bearing. Ball diameter variations produce radial and angular vibrations, but not axial. Only certain orders of race and ball waviness generate vibrations according to the linear theory, while other orders have no effect at all.
2. The radial and angular vibration spectra are characterized by vibration peaks at exactly the same frequencies. The following predominant peaks appear in these spectra:
 - a. The ball passage frequency over the outer ring and all multiples of it. These vibrations are induced by outer race waviness of the orders $nZ-1$ and $nZ+1$, where $n = 1, 2, 3 \dots$ and Z is the number of balls.

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- b. The rotational frequency of the cage (no multiples), induced by diameter variations among the balls.
 - c. The rotational frequency of the inner ring (no multiples), induced by inner ring eccentricity.
 - d. Pairs of frequencies centered around all multiples of the ball passage frequency over the inner ring and spaced one inner ring rotation frequency from the center point (no peak at the center point). These are induced by inner ring waviness of the orders $nZ \pm 1$.
 - e. Pairs of frequencies centered around all even multiples of the ball polar rotation frequency (in a system attached to the cage) and spaced one cage rotation frequency from their center point (no peak at the center point). These are induced by all even orders of ball waviness.
3. The axial vibration spectrum is characterized by peaks at the following frequencies:
- a. The ball passage frequency over the outer ring, and all multiples of it. These vibrations are induced by outer race waviness of the orders nZ .
 - b. The ball passage frequency over the inner ring, and all multiples of it. These vibrations are induced by inner race waviness of the orders nZ .
 - c. Even multiples of the ball polar rotation frequency. These vibrations are induced by all even orders of ball waviness.

Vibration peaks in the radial, angular and axial vibration spectrum are introduced by non-linear rigid ring effects, as well as by the linear effects of rigid ring theory. The peaks appearing in the radial spectrum due to non-linear effects are also tabulated in Enclosure 20. (The non-linear effects on axial and angular vibrations have not been studied.) It is seen that non-linear peaks appear at the following frequencies:

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1. Every multiple of the cage rotation frequency, (over the outer ring) not appearing in the linear case, is induced non-linearly by outer ring waviness.
2. Pairs of frequencies centered around all multiples of the cage rotation frequency over the inner ring except around the multiples appearing in the linear case, and spaced one cage-over-outer ring frequency from their center point. (No peak at the center point.) These are non-linearly induced by inner ring waviness of order $K \neq nZ \pm 1$.

Synthetic spectra, computed from linear rigid ring theory for a 6305 bearing rotating at 1800 RPM are shown on Enclosures 21 and 22. These spectra are intended to show frequency, but not amplitude relationships.

Enclosures 23-25 show a few typical experimentally obtained spectra for bearings running at 1800 RPM. The angular and axial spectra were recorded using two pickups and instrumentation as described in 1.2.11.

The peaks explainable by linear rigid ring theory are marked on these spectra. It appears that most of theoretically predicted frequencies according to the linear rigid ring theory can be verified, at least in the 0-600 cps frequency range.

1.3.9 Frequencies of Flexural Vibrations Induced by Low Order Ring Waviness

The frequencies generated by various surface imperfections, as given above, apply to bearings with flexurally rigid rings. If the outer ring is assumed to bend under the influence of the ball loads then not only the rigid body motion, but also the flexural vibrations of the outer ring, induced by the rolling surface imperfections, must be considered.

In addition to the flexural vibrations due to ball loads, discussed above, which are induced even in a bearing with geometrically perfect rolling surfaces, the surface imperfections also produce forced flexural vibrations of the outer ring. These vibrations have the following frequency characteristics:

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1. Inner ring waviness of the order K wpc produces flexural vibrations of the outer ring with a predominant peak of K times the rotational frequency. The amplitudes at other frequencies are small compared to the peak at times the rotational frequency, i.e., inner ring waviness of the order of 2 wpc generates vibrations at twice the rotational frequency, 3 wpc at three times the rotational frequency, etc. The amplitudes of the vibrations are significant only for low orders of waviness.
2. Low order outer ring waviness affects the amplitudes of the vibrations at ball passage frequency over the outer ring, and higher harmonics of this frequency. This effect is due to the finite number of balls and decreases as the number of balls increases.

An experimentally obtained spectrum of a bearing with comparatively high inner ring two and three point out of roundness is shown on Enclosure 26. The spectrum shows predominant peaks at twice and three times the rotational frequency.

1.3.10 Complete Spectrum

Enclosure 27 shows a complete synthetic spectrum of the radial vibrations of an axially loaded 6305 bearing with peaks according to linear and non-linear rigid ring theory as well as the peaks induced by flexural vibrations due to low order inner race waviness. (The flexural peaks up to five times the rotational frequency are shown.) For completeness the peaks induced by ball loads due to the finite number of the balls are also shown (three harmonics have been included). This spectrum is not intended to show amplitudes, but linear peaks are shown at higher amplitude than non-linear peaks to indicate their predominance.

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1.3.11 Relationship Between Waviness and Vibration Frequencies Measured in Finite Bands

Since both vibration and waviness commonly are measured in finite bands, it is often advantageous to compute the approximate waviness bands that correspond to given vibration bands.

Enclosures 28 and 29 are tabulations of the ranges of orders of inner ring, outer ring and ball waviness that generate vibrations in the three Anderometer bands, 50-300 cps, 300-1800 cps and 1800-10000 cps for a bearing rotating at 1800 RPM. A few common bearing sizes in the 6200 and 6300 series are listed. The ranges include vibration generated by both the linear and non-linear rigid ring theory. Flexural vibrations due to low order ring waviness are also listed. The ranges are only approximate since they are not based on the exact discrete frequency peaks generated by each order of waviness, but only on average values. These results are based on a 15° contact angle.

To compute the approximate ranges of orders of waviness in Enclosures 28 and 29, the following equations were used:

$$k_o = \frac{f}{f_c} \quad (1.3.11-1)$$

$$k_i = \frac{f}{f_i} \quad (1.3.11-2)$$

$$k_b = \frac{f}{f_b} \quad (1.3.11-3)$$

where k_i , k_o and k_b are the orders of inner ring, outer ring and ball waviness respectively, generating vibrations of frequency f cps. f_c is the rotational frequency of the cage with respect to outer ring f_i the frequency of the cage with respect to the inner ring and f_b the polar rotation frequency of the ball, in cps.

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I.3.12 Amplitude of Vibrations Generated by Geometrical Imperfections

The amplitude of the vibration generated at any given frequency by geometrical imperfections of the rolling surfaces depends on:

1. The amplitude of the geometrical imperfection generating vibrations at the given frequency.
2. The vibration transmission characteristics of the bearing which relate the "input" amplitude (waviness) to the "output" amplitude (vibration) as follows:

$$x = Tw \quad (1.3.12-1)$$

where

x = displacement amplitude of vibrations at a given frequency.

w = displacement amplitude of geometrical imperfection generating the vibration amplitude x .

T = amplification factor.

In a linear system T is a constant independent of w . For a bearing operating under constant load and speed, T may be assumed to be independent of w for the linearly induced rigid ring vibrations as well as the flexural vibrations induced by low order waviness. For these vibrations the vibration amplitudes at any given frequency are proportional to the amplitudes of the geometrical imperfections generating this frequency.

The proportionality factor T for rigid ring vibration depends on the following principal parameters:

- a. The frequency of the induced vibration compared to the resonant frequency of the bearing.
- b. The origin of the vibration; whether induced by inner or outer ring waviness, by ball waviness or by ball diameter variation.

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- c. If the bearing is mounted in a radially and angularly elastic housing, T depends on the radial and angular spring constants of the housing.

The amplification factor T for flexural vibration induced by low order inner ring waviness depends on:

1. The order of waviness
2. The outer ring rigidity
3. The number of balls
4. The spring constant of the balls

The amplification factors will be discussed in more detail in I.3.16.

I.3.13 Natural Frequencies

Like any other elastic system a ball bearing has a number of natural modes of vibration. If the bearing is excited at a frequency corresponding to any of these modes, comparatively high amplitudes are expected at this frequency.

According to the modes of vibration, the natural frequencies of the outer ring supported by the balls may be divided into two categories:

1. Natural frequencies of the rigid body motion of the outer ring (the outer ring deforms elastically only at its contacts with the balls).
2. Natural frequencies of the elastic vibrations of the outer ring (the outer ring deforms elastically, e.g., by bending).

I.3.14 Natural Frequencies of the Rigid Body Motion

The natural frequencies of the rigid body motion have been studied for the free outer ring supported only by the balls and also for the outer ring supported by the balls and mounted in a housing which is elastic both radially and angularly.

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The natural frequencies of the rigid body motion of the outer ring are influenced by the following parameters:

1. The radial spring constant k_R of the housing. It is assumed that the housing is linearly elastic with equal spring constant in every radial direction, i.e., the radial force required to produce a radial displacement x of the outer ring (not supported by balls) in an arbitrary angular direction Θ , is given by the equation:

$$F_R = k_R x \quad (I.3.14-1)$$

2. The angular spring constant k_λ . It is assumed that the housing acts as a linear spring, producing a restoring moment proportional to the displacement angle λ . For any angular direction Θ the moment M_λ is given by

$$M_\lambda = k_\lambda \lambda \quad (I.3.14-2)$$

3. The linearized coefficient of contact deformation k_W , as given by Equation (I.3.7-3).
4. The outer ring mass M . Ball mass is assumed to be negligible.
5. The moment of inertia I_M of the outer ring with respect to a diameter through the center plane of the bearing.
6. The radius R from bearing center to outer ring groove center.
7. The radius R_M from bearing axis to point of contact between ball and outer ring.
8. The contact angle α .
9. The number of balls, Z .

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There are three natural frequencies of rigid body motions and they are given, in cps by:

$$f_{R1} = \frac{1}{2\pi} K_1 \Omega_0 \quad (I.3.14-3)$$

$$f_{R2} = \frac{1}{2\pi} K_2 \Omega_0 \quad (I.3.14-4)$$

$$f_A = \frac{1}{2\pi} K_A \Omega_0 \quad (I.3.14-5)$$

The frequencies f_{R1} and f_{R2} apply to both the radial and the angular mode of vibration, f_A applies to the axial mode only.

In these equations

$$\Omega_0 = \sqrt{\frac{2k_N}{2M}} \quad (I.3.14-6)$$

$$K_1 = \left[\frac{1}{2} (H_1 + \xi_R + \xi_\lambda) - \sqrt{\frac{1}{4} (H_2 + \xi_R - \xi_\lambda)^2 + H_3} \right]^{\frac{1}{2}} \quad (I.3.14-7)$$

$$K_2 = \left[\frac{1}{2} (H_1 + \xi_R + \xi_\lambda) + \sqrt{\frac{1}{4} (H_2 + \xi_R - \xi_\lambda)^2 + H_3} \right]^{\frac{1}{2}} \quad (I.3.14-8)$$

$$K_A = \sqrt{2} \sin \alpha \quad (I.3.14-9)$$

where

$$H_1 = \cos^2 \alpha + \left(\frac{R}{\rho}\right)^2 \eta \sin^2 \alpha \quad (I.3.14-10)$$

$$H_2 = \cos^2 \alpha - \left(\frac{R}{\rho}\right)^2 \eta \sin^2 \alpha \quad (I.3.14-11)$$

$$H_3 = \left(\frac{R}{\rho}\right)^2 \eta \sin^2 \alpha \cos^2 \alpha \quad (I.3.14-12)$$

$$\xi_R = \frac{2k_R}{2k_M} \quad (I.3.14-13)$$

$$\xi_\lambda = \frac{2k_\lambda}{2k_M \rho^2} \quad (I.3.14-14)$$

$$\eta = \frac{R_M}{R} \quad (I.3.14-15)$$

$$S = \sqrt{\frac{I_n}{A}} = \text{radius of gyration of outer ring about a diameter} \quad (I.3.14-16)$$

The bearing has only one finite resonant frequency in the radial and angular mode when

- a. The outer ring is radially and angularly free (the spring constants k_R and k_λ are zero). In this case

$$f_{R1} = 0 \quad (I.3.14-17)$$

$$f_{R2} = \frac{1}{2\pi} \left\{ 1 + \left[\left(\frac{R}{\rho}\right)^2 \eta - 1 \right] \sin^2 \alpha \right\}^{\frac{1}{2}} \sqrt{\frac{2k_M}{2M}} \quad (I.3.14-18)$$

- b. The housing is radially stiff (infinite spring constant k_R)

For $k_R \rightarrow \infty$ and $k_\lambda = 0$

$$f_{R1} \rightarrow \infty \quad (I.3.14-19)$$

$$f_{R2} = \frac{1}{2\pi} \left(\frac{R}{\rho}\right) \sqrt{\eta} \sin \alpha \sqrt{\frac{2k_M}{2M}} \quad (I.3.14-20)$$

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c. The housing is angularly stiff (infinite spring constant k_λ)

For $k_\lambda \rightarrow \infty$ and $k_R = 0$

$$f_{R1} = \frac{\cos \lambda}{2\pi} \sqrt{\frac{2k_n}{2M}} \quad (I.3.14-21)$$

$$f_{R2} \rightarrow \infty \quad (I.3.14-22)$$

I.3.15 Natural Frequencies of Elastic Vibrations

The natural frequencies of the elastic vibrations of the outer ring have been computed under the assumption that the inner ring is rigid and joined to the outer ring by a set of equally spaced, identical, elastic balls. The outer ring is presumed to be an elastic circular ring which performs only flexural vibrations in its own plane. Other ring motions such as pure radial (expansional) circumferential, and torsional vibrations are known to be of small amplitude and consequently are of little interest in this investigation. Only the case of the free outer ring supported by the balls (not mounted in a housing) has been treated.

The study shows that a ring with multiple elastic supports (such as bearing outer ring supported by the balls) has a sequence of natural frequencies of flexural vibrations which are not integral multiples of a fundamental frequency and are dependent on ring mass and on the elastic properties of ring and supports. These natural frequencies in cps are given by the equation

$$f_{Fn} = \frac{1}{2\pi} \left[\frac{(n^2-1)^2 \frac{\pi E I}{R^3} + \frac{k_n \pi}{2}}{\pi \rho_0 A R (1 + 1/n^2)} \right]^{\frac{1}{2}} \quad (I.3.15-i)$$

$n \geq 2n+1$

where

f_{Fn} = natural frequency in cps

E = Young's modulus of elasticity

I = second moment of area of the ring cross-section

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R = mean radius of ring

k_n = linearized Hertzian coefficient (See Equation 1.3.7-4)

ρ_0 = mass density of ring

A = cross section area of ring

Z = number of balls

n = any positive integer > 0

The restriction on Z in Equation (1.3.15-1) is a restriction on the validity of the equation, but not on the existence of higher natural frequencies.

In Enclosure 30, computed natural frequencies of a few typical bearings are listed. The computations were performed for 15° contact angle and an axial load of 25 lbs.

Since the spring constant k_n depends on the axial load and on the contact angle of the bearing, the natural frequencies are influenced by these parameters. With the exception of very light loads and small contact angles these effects are comparatively small.

1.3.16 Displacement Amplification Factors for Rigid Body Motion

The rigid body displacement amplification factors T introduced in Equation (1.3.12-1) are different for radial, angular and axial vibrations, and are given by the following equations:

For radial vibrations

$$T_R = \left| g \frac{\left(\frac{\Omega_0}{2\pi}\right)^2 \cos \alpha \left[\epsilon_R \left(\frac{\Omega_0}{2\pi}\right)^2 - f^2 \right]}{[f^2 - f_{R1}^2][f^2 - f_{R2}^2]} \right| \quad (1.3.16-1)$$

For angular vibrations

$$T_A = \left| g \left(\frac{\Omega_0}{2\pi}\right)^2 \left(\frac{\eta R}{\rho}\right)^2 \sin \alpha \frac{\left[\epsilon_R \left(\frac{\Omega_0}{2\pi}\right)^2 - f^2 \right]}{[f^2 - f_{R1}^2][f^2 - f_{R2}^2]} \right| \quad (1.3.16-2)$$

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For axial vibrations

$$T_A = 2g \sin \alpha \left| \frac{\left(\frac{D_o}{2\pi}\right)^2}{f^2 - 2\left(\frac{D_o}{2\pi}\right)^2 \sin^2 \alpha} \right| \quad (I.3.16-3)$$

where the parameter g and the frequency f are given by the table below:

	Radial and Angular Vibration T_R and T_A			Axial Vibration T_A		
	g	f	k	g	f	k
Inner Ring Waviness	1	$k f_L \pm f_c$	$n \pm 1$	1	$k f_i$	$n \pm 1$
Outer Ring Waviness	1	$(k \pm 1) f_c$	$n \pm 1$	1	$k f_c$	$n \pm 1$
Ball Waviness	$\sqrt{\frac{n}{2}}$	$k f_b \pm f_c$	$2n$	$\sqrt{\frac{n}{2}}$	$k f_b$	$2n$
Ball Diameter Variation	$\frac{2}{2}$	f_c				

The coefficients g , for ball waviness and ball diameter variation are based on the following simplifying assumptions:

1. The variation of waviness from ball to ball between various diameter on the same ball has been neglected, by using average values of the amplitudes of the various ball waviness harmonics (See Report AL61L032 page 23 (1)).
2. The factor g for ball diameter variation was computed under the assumption that all the balls in the bearing, except for one, have the same diameter. (See Report AL61L032 page 32 (1)).

Typical curves showing the amplification factor as a function of frequency are given in Enclosures 31-38 for different values of the elastic constants ξ_R and ξ_A . It is seen that the resonant frequencies of the system can be varied within a wide range by changing the elastic constants of the housing. It is also possible to select the values of ξ_R and ξ_A so that, for any given frequency, either the radial or angular vibration amplitudes are zero.

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I.3.17 Velocity Band Width Amplification Factors for Rigid Body Motion

In the practice of bearing vibration and waviness testing these quantities are measured as total velocity levels within frequency bands of finite width. It is, therefore, important to determine the relationship between waviness measured in a finite band and vibration measured in a "corresponding" band. This can be done by using "velocity band width amplification factors", defined by

$$U = \frac{V}{W} \quad (I.3.17-1)$$

where V is the RMS value of the vibration velocity measured in a given finite band, and W is the RMS value of the waviness also expressed in units of velocity, and measured in a band corresponding to the vibration band, i.e., the waviness band is selected so that the waviness measured will generate vibration frequencies within the band used in the vibration measurements.

The following approximate relationship holds for the "band level amplification factors" U_o , U_i and U_b for outer ring, inner ring and ball waviness, respectively:

For radial vibration

$$U_o = \sqrt{\frac{2}{Z}} \frac{f_o}{f_{wo}} \cos \alpha \quad (I.3.17-2)$$

$$U_i = \sqrt{\frac{2}{Z}} \frac{f_i}{f_{wi}} \cos \alpha \quad (I.3.17-3)$$

$$U_b = \sqrt{\frac{2}{Z}} \frac{f_b}{f_{wb}} \cos \alpha \quad (I.3.17-4)$$

For axial vibration

$$U_o = \sqrt{\frac{1}{Z}} \frac{f_o}{f_{wo}} \frac{1}{\sin \alpha} \quad (I.3.17-5)$$

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$$U_i = \sqrt{\frac{1}{2}} \frac{f_i}{f_{wi}} \frac{1}{\sin \alpha}$$

(1.3.17-6)

$$U_b = \sqrt{\frac{1}{2}} \frac{f_b}{f_{wb}} \frac{1}{\sin \alpha}$$

(1.3.17-7)

Equations 1.3.17-2 to 1.3.17-7 take into account the effect of the linearly induced peaks only. The frequencies f_{wo} , f_{wi} and f_{wb} are the rotational frequencies used in the measurements of outer ring, inner ring and ball waviness, respectively.

The Equations 1.3.17-2 to 1.3.17-7 are strictly valid only under the following conditions:

1. The waviness harmonics of a given bearing element are all of the same velocity amplitude within bands of the width considered.
2. The bands are narrow enough that an amplification factor at any frequency within the band can be approximated by the same amplification factor taken at the (arithmetic) center frequency of the band. (This presupposes that the band does not contain any of the resonant frequencies.)
3. The bands are wide enough to cover a large number of "lines" of the waviness or vibration spectrum.

It is seen from Equations 1.3.17-2 to 1.3.17-7 that the ratios $U_o : U_i : U_b$ are the same for radial and axial vibrations. The same relationship also holds for angular vibrations.

It also appears that the velocity band width amplification factors for outer and inner ring and ball waviness are all proportional to $\sqrt{1/2}$, where Z is the number of balls.

An amplification factor for ball diameter variation can also be computed. Since its formula is based on somewhat restrictive assumptions, it is not cited here, but can be found in Special Report AL62L005 (1). This amplification factor is proportional to $1/Z$.

Enclosure 39 lists the amplification factors U_o , U_i and U_b for a few bearing sizes. The tabulation is based on a rotational speed of 1000 RPM for waviness testing inner and outer rings, and 740 RPM for waviness testing balls. The rotational speed of the bearing at vibration testing is 1800 RPM.

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1.3.18 Amplification Factor for Flexural Vibrations Induced by Inner Ring Waviness of the Order Two Waves Per Circumference

The displacement amplification factor T_Y for flexural outer ring vibrations due to 2 wpc inner ring waviness is given by

$$T_Y = \frac{Y}{1+Y} \quad (1.3.18-1)$$

where

$$Y = \frac{2k_N R^3}{18TEI} \quad (1.3.18-2)$$

k_N = linearized Hertzian coefficient as given by Equation (1.3.7-4)

R = mean radius of outer ring.

I = second moment of area of outer ring cross-section.

It was shown in Equation (1.3.7-4) that for a given axial load k_N is proportional to $Z^{-1/2}$. Y is therefore proportional to $Z^{1/2}$ and T_Y increases with increasing Z , but not according to a simple power law.

It was also shown in Equation (1.3.7-5) and Enclosure 19 that for a given number of balls and a given axial load k_N increases with ball diameter. Y and T_Y therefore also increase with ball diameter.

Since Y is inversely proportional to the rigidity I/R^3 of the outer ring, T_Y decreases with increasing rigidity I/R^3 .

1.3.19 Effect of Load on Bearing Vibration

The main effects of the applied load on bearing vibration are summarized below. These findings are based on experimental and theoretical analysis of 6305 and 6310 bearings, and apply to an axial load range of 20 to 200 lbs. The radial load ranges are 25 to 250 lbs for the 6305 bearing and 50 to 500 lbs for the 6310 bearing. The results have been verified for test speeds in the range between 1000 and 3600 RPM.

Axial Load

1. The linearized Hertzian coefficient k_N is proportional to the cube root of the axial load, F_A . The rigid body mode resonant frequency is proportional to $\sqrt{k_N}$, and hence, also proportional to $\sqrt[3]{F_A}$. This resonant frequency therefore increases with axial load, and a change in the axial load may therefore change the appearance of the bearing vibration spectrum due to the shift in the resonant frequency. This effect is, however, very small for small changes in the load and even an increase in load by a factor of 10 produces an increase in the resonant frequencies of only 45%. The resonant frequencies of the flexural modes of vibration are even less affected by changes of k_N , as seen from Equation (1.3.15-1).
2. Another effect of the change in k_N due to changes in the axial load is to influence the amplitude of the flexural vibration due to inner ring waviness of two wpc. An increase in the axial load, according to Equations (1.3.18-1) and (1.3.18-2), increases the amplification factor T_Y . This effect is small for most bearings within the load ranges normally used in vibration-critical applications. An increase in the axial load on a 6305 bearing from 20 to 200 lbs, results in an increase of approximately 24% in T_Y .
3. The experimentally found effect of axial load on the bearing vibration, measured in octave bands in a radial or axial direction, is in general small and erratic. For details, see Progress Report No. 8 (1). The effect of axial load on the vibration level measured in the three Anderometer bands is also briefly discussed in Progress Reports No. 6, 7 and 8 (2), where the effect of axial load applied to 6204, 6305 and 6207 bearings in the 6 to 40 lbs range was found to be small.

Radial Load

1. The basic findings in the vibration theory for a bearing with rigid rings operating under axial load also apply to a bearing under radial or combined load. The vibration frequencies induced by various surface imperfections are the same for both loading conditions. The effect of radial load on resonant frequencies and flexural outer ring vibrations is comparable to that of the axial load.
2. The effect of radial load, measured in octave bands in a radial or axial direction, is, in general, small and erratic. For details see Progress Reports No. 4, 6 and 8.

I.3.20 The Effect of Rotational Speed on Vibration Level

The frequencies of the vibrations induced by various sources (such as geometrical imperfections of the rolling surfaces, flexural vibrations due to the ball loads and variable compliance vibrations) are all proportional to the rotational speed of the bearing.

Let $V_1(f)$ represent the RMS vibrational velocity measured in an octave band with midband frequency f cps, at a rotational speed of N_1 RPM. $V_2(f)$ represents the reading in the same octave band at a rotational speed of N_2 RPM.

In a non-resonant region, assuming the vibration transmission characteristics of the bearing to be the same at the two rotational speeds

$$V_2(f) = \frac{N_2}{N_1} V_1\left(\frac{N_1}{N_2} f\right) \quad (I.3.20-1)$$

This equation follows directly from the proportionality between vibration frequencies and rotational speed, i.e., vibration components occurring at a frequency f at the rotational speed N_2 will occur at frequency $\frac{N_2}{N_1} f$ at a rotational speed N_1 . The factor $\frac{N_2}{N_1}$ is due to the fact that, for given displacements, the velocity amplitudes are proportional to the frequency, and thus also to the rotational speed.

Equation (1.3.20-1) shows that:

1. The shape of the spectrum is speed independent in non-resonant regions. The octave band spectrum at a speed N_2 is obtained from the spectrum at the speed N_1 by multiplying both the ordinate and abscissa scales by the speed ratio N_2/N_1 .
2. The relationship between vibration level and speed can be determined at any given frequency from Equation (1.3.20-1) provided that the spectrum at one speed is known.

Enclosures 40-43 show experimentally obtained octave band vibration spectra of 6305 and 6310 bearings. These spectra are typical only of the bearings used in the tests reported in Progress Reports No. 4 and 5 (1), and it should be realized that if other manufacturing methods are used the spectra could be different from these. The spectra shown on Enclosures 40-43 represent average values, measured under six different radial loads. The vibration was measured in the direction of the applied radial load and perpendicular to it, at rotational speeds of 1800 and 3600 RPM.

The spectra of Enclosures 40-43 may be used to study the validity of Equation (1.3.20-1) for the bearings used in the tests. Since the two rotational speeds used differ by a factor of 2, the vibration amplitude in any given octave band at 3600 RPM is, according to Equation (1.3.20-1), twice the amplitude of the adjacent lower octave band at 1800 RPM. The vibration levels in the various octaves at 3600 RPM were computed in this manner from the 1800 RPM spectrum, and the computed values are also shown on Enclosures 40-43. A comparison of the computed and experimental spectrum at 3600 RPM shows good agreement between the two spectra of the 6305 bearing in the direction normal to the load, for octaves in the 100-1600 cps range. The agreement in the direction of the load is somewhat poorer. A similar tendency is observed for the 6310 bearing, although the agreement is somewhat poorer than for the 6305 bearing. It appears that Equation (1.3.20-1) can be used to obtain order of magnitude estimates of the octave band vibration levels at various rotational speeds provided that the spectrum at one speed is known. It should, of course, be realized that for a more accurate evaluation, resonances in the system must be taken into account. For frequencies above 1600 cps, the estimates obtained by using Equation (1.3.20-1) give values which are too large for the higher rotational speeds, which could indicate that the bearing is more highly damped in this frequency range at the higher rotational speeds than at the lower speed and that, therefore, the amplitudes in these (resonant) bands do not increase proportionately with speed. Another possible explanation is that the damping is non-linear increased with amplitude.

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If an analytical expression for the relationship between vibration level and frequency is available, the speed dependence of the vibration level in any given octave band can be computed from Equation (I.3.20-1) within the range of validity of this Equation.

For instance, assume that the relationship between vibration level and frequency is linear at a speed N_1 , i.e., that

$$V_1 = V_0 + Kf \quad ; \quad f_1 < f < f_2 \quad (I.3.20-2)$$

where V_1 is the vibration level measured in an octave band with midband frequency f , V_0 is a positive constant and K a positive or negative constant and f_1 and f_2 define the frequency limits of the validity of Equation (I.3.20-1). Then

$$V_2(f) = \frac{N_2}{N_1} [V_0 + K \frac{N_1}{N_2} f] = \frac{N_2}{N_1} V_0 + Kf \quad ; \quad \frac{N_2}{N_1} f_1 < f < \frac{N_2}{N_1} f_2 \quad (I.3.20-3)$$

i.e., the relationship between vibration level, in a given octave, and speed is also linear. If V_1 is constant over the band then V_2 is also constant and the vibration level increases proportionately with the rotational speed.

If the spectrum at speed N_1 follows a power law

$$V_1 = Kf^\beta \quad ; \quad f_1 < f < f_2 \quad (I.3.20-4)$$

where K and β are constants,

then

$$V_2(f) = \frac{N_2}{N_1} K \left(\frac{N_1}{N_2} f \right)^\beta = K f^\beta \left(\frac{N_2}{N_1} \right)^{1-\beta} \quad ; \quad \frac{N_2}{N_1} f_1 < f < \frac{N_2}{N_1} f_2 \quad (I.3.20-5)$$

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It should be noted, that the resonant frequencies of a bearing are speed independent and the analysis therefore is not valid in regions where the spectrum contains significant resonances.

It is seen from Enclosures 40-43 that the octave band vibration levels at a given speed generally decrease with frequency. Since the vibration spectrum varies considerably from bearing to bearing, no good general analytical expression can be written for this relationship. Using average values for the 6305 and 6310 bearings reported in Progress Reports No. 4 and 5 (1), the following equation has been found to represent a reasonably good approximation for these particular bearings:

$$V = \frac{K}{\sqrt{f}} \quad (1.3.20-6)$$

This is illustrated by the curve, representing Equation (1.3.20-6) for $K = 570000$, plotted on Enclosure 40. It is seen that this curve is in good agreement with the experimental points of the spectrum at 3600 RPM, in the frequency range up to 1600 cps.

Equation (1.3.20-6) represents the special case of Equation (1.3.20-4) for which $\beta = -0.5$. For this value of β , Equation (1.3.20-5) which expresses octave band level in any given band as a function of speed, becomes

$$V_2(f) = K f^{-0.5} \left(\frac{N_2}{N_1} \right)^{1.5} \quad (1.3.20-7)$$

which more generally may be expressed as

$$V = k N^{1.5} \quad (1.3.20-8)$$

where k is a constant and N the rotational speed.

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Experimental results with 6305 and 6310 bearings under light axial and radial loads have shown that the vibration level as measured in octave bands approximately follows the power law

$$V = kN^{\alpha}$$

(I.3.20-9)

where k and α are positive constants and N the rotational speed. The exponent α depends to some extent on the frequency band examined and also on the direction of the vibration measurement. The average value of α was found to be approximately 1.3.

For this average value of α Equation (I.3.20-9) is in agreement with Equation (I.3.20-8) which shows that at least a rough estimate of the amplitude-speed relationship in any given octave band can be obtained if the amplitude-frequency relationship at one speed is known.

Enclosure 44 shows experimental values of the exponent α as a function of frequency. The shape of the curve and the average value of α is approximately the same for vibrations in the direction of the radial load and normal to it. This also applies reasonably well to vibrations measured in the axial direction.

I.3.21 Comparison of Vibration Levels in Different Measuring Directions

The following general trends have been observed in comparing octave band vibration levels in different measuring directions:

1. The octave band vibration level in the axial direction is, in general, higher than in the radial direction for low frequencies (below 800 cps), and approximately the same in the higher frequency range (above 800 cps). This applies to radial vibrations measured both in the direction of the radial load and perpendicular to it. It applies to axial vibrations measured at one point of the outer race, thus including both parallel translatory motion and angular vibrations.

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2. The vibration level measured in the direction of the radial load is, in general, of a somewhat smaller (15%) magnitude than in a direction perpendicular to the load.

The fact that the vibrations, measured in the axial direction in the non-resonant bands below 800 cps, generally are of a higher amplitude than vibrations in the radial direction is explainable by rigid ring theory; as a consequence of the fact that the ratio between the velocity bandwidth amplification factors in the axial and radial direction is $\frac{1}{\sqrt{2} \sin \alpha}$ which is always much larger than 1.

A more detailed analysis shows that for a bearing with free outer ring ($\bar{e}_R = \bar{e}_\lambda = 0$), the ratios between the vibratory energy of axial, radial and angular vibrations in a finite non-resonant band are given approximately by

$$E_A : E_R : E_\lambda = \frac{[1 + \frac{1}{2} \sin^2 \alpha]^2}{2 \sin^2 \alpha} : \cos^2 \alpha : 4 \sin^2 \alpha \quad (I.3.21-1)$$

This relationship holds exactly for a bearing with

$$\left(\frac{R}{\xi}\right)^2 \eta = 2 \quad (I.3.21-2)$$

$$\left(\frac{R}{\xi}\right)^2 \eta = 1.5 \quad (I.3.21-3)$$

where R , ξ and η are dimensional parameters defined in I.3.14.

The conditions given by Equations (I.3.21-2) and (I.3.21-3) represent average values for deep groove ball bearings of conventional design.

For $\alpha = 15^\circ$

$$E_A : E_R : E_\lambda = 0.87 : 0.10 : 0.03 \quad (I.3.21-4)$$

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which shows that the amplitudes of the axial vibrations are considerably higher than those of the radial and angular vibrations. It is also seen that the measured vibrations in the axial direction are mainly influenced by axial translatory motion and to a minor degree by angular motion.

For a bearing mounted in an angularly and radially elastic housing the distribution of vibratory energy among the three modes of vibration can be computed from Equations given in Report AL62L005 (1).

For the special case of a bearing mounted in a housing which is angularly rigid ($\xi \rightarrow \infty$), but with no elastic radial constraint ($\xi_R = 0$), the following applies under the conditions of Equations (I.3.21-2) and (I.3.21-3).

$$E_A : E_R : E_\lambda = (1 + \frac{1}{2} \cot^2 \alpha) : (1 + 2 \tan^2 \alpha) : 0 \quad (I.3.21-5)$$

which for $\alpha = 15^\circ$ becomes

$$E_A : E_R : E_\lambda = 0.87 : 0.13 : 0 \quad (I.3.21-6)$$

Again the vibrations in the axial direction are predominant although the angular component in this case is zero.

For a bearing mounted in a housing which is radially rigid ($\xi_R \rightarrow \infty$), but with no elastic angular constraint ($\xi_\lambda = 0$), the following applies under the condition of Equations (I.3.21-2 and (I.3.21-3) and for $\eta = 1.2$.

$$E_A : E_R : E_\lambda = 1 : 0 : 3 \quad (I.3.21-7)$$

In this case the energy distribution is independent of contact angle.

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The radial vibrations are now of zero amplitude, and only vibrations in the axial direction are measured.

The foregoing applies only to vibrations in a non-resonant band of finite width. For measurements in frequency ranges which are influenced by resonances the following applies:

At any fixed frequency the distribution of the vibratory energy among the axial, radial and angular modes of vibration depend for a given bearing on the spring constants of the housing. The distribution is highly frequency dependent. By proper selection of the spring constants of the housing, the amplitudes in either the radial or the angular mode of vibration can be minimized at any given frequency. A housing with zero angular spring constant (the outer ring angularly free) gives low amplitudes of the radial vibration at low frequencies, while a housing with zero radial spring constant (the outer ring radially free) gives low amplitudes of the angular vibration at low frequencies. This is illustrated by Enclosures 45-48 which shows the distribution of radial and angular vibratory energy for a 6305 bearing as a function of frequency, for a few values of $\frac{F_A}{F_R}$ and $\frac{F_A}{F_A}$. It is seen that if $\frac{F_A}{F_R} = \frac{F_A}{F_A}$ the distribution of vibratory energy is independent of frequency. For further details see Special Report AL62L005 (1).

I.3.22 Comparison of the Effects of Outer Ring, Inner Ring and Rolling Element Waviness on the Bearing Vibration Level

This comparison applies to vibration and waviness measurement made in finite bands, sufficiently wide to contain several linear and non-linear vibration and waviness peaks.

If it is assumed that rolling surface waviness is the only cause of bearing vibration then the vibration in any given frequency band, may be expressed by the equation

$$V^2 = U_o^2 W_o^2 + U_i^2 W_i^2 + U_b^2 W_b^2 \quad (I.3.22-1)$$

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where

V = the RMS value of the bearing vibration measured in any given frequency band.

W_o, W_i, W_b = outer ring, inner ring and rolling element waviness, measured in the frequency bands corresponding to the vibration frequency band.

U_o, U_i, U_b = velocity band width amplification factors of outer ring, inner ring and rolling element waviness, respectively. These are, in a non-resonant band, given by Equations (I.3.17-2) through (I.3.17-7).

The relative contribution of outer ring, inner ring and rolling element waviness respectively to the bearing vibration level may be expressed by the use of the following "contribution factors".

$$Z_o = \frac{U_o W_o}{V} \quad (I.3.22-2)$$

$$Z_i = \frac{U_i W_i}{V} \quad (I.3.22-3)$$

$$Z_b = \frac{U_b W_b}{V} \quad (I.3.22-4)$$

which may be written in terms of quantities that can directly be derived from the bearing design, test speed and the measured waviness, as follows:

$$Z_o = \left[1 + \left(\frac{W_i}{W_o} \right)^2 \left(\frac{f_i}{f_c} \right)^2 \left(\frac{f_{wo}}{f_{wi}} \right)^2 + \frac{\pi}{2} \left(\frac{W_b}{W_o} \right)^2 \left(\frac{f_b}{f_c} \right)^2 \left(\frac{f_{wo}}{f_{wb}} \right)^2 \right]^{-\frac{1}{2}} \quad (I.3.22-5)$$

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$$Z_i = \left[1 + \left(\frac{w_o}{w_i} \right)^2 \left(\frac{f_c}{f_i} \right)^2 \left(\frac{f_{wi}}{f_{wo}} \right)^2 + \frac{\pi}{2} \left(\frac{w_b}{w_i} \right)^2 \left(\frac{f_b}{f_i} \right)^2 \left(\frac{f_{wi}}{f_{wb}} \right)^2 \right]^{-\frac{1}{2}} \quad (I.3.22-6)$$

$$Z_b = \left[1 + \left(\frac{w_o}{w_b} \right)^2 \left(\frac{f_c}{f_b} \right)^2 \left(\frac{f_{wb}}{f_{wo}} \right)^2 + \frac{2}{\pi} \left(\frac{w_i}{w_b} \right)^2 \left(\frac{f_i}{f_b} \right)^2 \left(\frac{f_{wb}}{f_{wi}} \right)^2 \right]^{-\frac{1}{2}} \quad (I.3.22-7)$$

It should be noted that

$$Z_o^2 + Z_i^2 + Z_b^2 = 1 \quad (I.3.22-8)$$

and that the squared factors Z_o^2 , Z_i^2 and Z_b^2 directly indicate the additive contribution from outer ring, inner ring and rolling element waviness, as a fraction of the total (squared) vibration level.

Equations (I.3.22-5), (I.3.22-6) and (I.3.22-7) are useful in developing bearings with improved vibration characteristics. Improvements in the vibration level can be accomplished most efficiently by reducing the waviness of the element having the highest value of Z^2 , while reduction of the waviness of parts with low Z^2 value is of little effect.

The waviness ranges corresponding to a given vibration frequency band may be computed from Equations (I.3.11-1), (I.3.11-2) and (I.3.11-3) or the tables of Enclosures 28 and 29 may be used.

I.3.23 Comparison of Amplitudes of Vibration Generated by Various Sources

To compare the amplitudes of the vibrations generated by the various sources that have been identified within the bearing, numerical computations were performed for a 6305 bearing. Vibrations generated by the following sources were considered:

1. Variable compliance
2. Bending of the outer ring due to the finite number of balls
3. Flexural outer ring vibrations due to inner ring waviness of two wpc.

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4. Rigid body vibrations due to waviness of the rolling surfaces.

1. Variable Compliance:

Enclosure 49 shows a graph of the computed maximum amplitudes of the nine first harmonics of the variable compliance vibrations for a 6305 bearing rotating at 1800 RPM. The values apply to a bearing with 0.0006 inch radial looseness, and to radial loads in the range from 20 to 2000 lbs. The values shown are only approximate; the higher harmonics especially should be used only for comparison of orders of magnitude. The radial load corresponding to the maximum amplitudes both in the direction of the load and perpendicular to it are shown on the enclosure. For comparison, the vibration level corresponding to MIL-B-17391B (Ships) vibration limits in the low (50-300 cps) and medium (300-1800 cps) Anderometer bands is also shown on the graph.

Since the amplitudes of the variable compliance vibrations represent the maximum amplitudes of each harmonic which occur under different radial loads, the total contribution, is a wide band, of the variable compliance vibrations to the bearing vibration level cannot be obtained directly from Enclosure 49. This contribution is, however, judged with certainty to be significant whenever at least one of the harmonics has an amplitude of the order of half of the vibration level generated by other sources, or higher.

It should be realized that the variable compliance vibration do not enter vibration measurements (according to MIL-B-17391B) as these are performed under axial load, but they may affect the vibration level of very quiet running bearings unless there is predominant axial load.

It is seen that the maximum vibration amplitudes of the first and second harmonics of the variable compliance vibrations in the direction of the radial load are in excess of the MIL-B-17391B vibration limit in the low band. The third and fourth harmonic exceed 50% of the limit. The four lower harmonics may therefore significantly affect the vibration level of a quiet running bearing in the low Anderometer band. It is also seen that the fifth to ninth harmonics in the direction of the load are of significant magnitude compared to the MIL-B-17391B limit, in the medium band.

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Since the amplitude of the harmonics generally decrease with the order of the harmonic, orders higher than the ninth are not believed to be of significant amplitude. High Anderometer band vibrations (1800-10000 cps) should therefore not be affected by the variable compliance. The variable compliance vibrations in the direction of the radial load are of lower amplitude. Only the first two harmonics of these vibrations may conceivably have a significant effect on a bearing with a vibration level satisfying MIL-B-17391B limits. The fact that the amplitudes of the variable compliance vibrations in the direction perpendicular to the radial load are higher than in the direction of the load may explain the higher vibration levels, measured in octave bands, in the direction perpendicular to the load (See 1.3.19).

2. Bending Vibrations of Outer Ring due to Ball Loads:

The amplitude of the flexural vibrations induced by bending due to ball loads are also influenced by the applied load. Enclosure 50 shows graphically for a 6305 bearing with 15° contact angle how the amplitude of the first and second harmonics depend on the applied axial and radial loads. (The radial load values are only rough estimates computed from Equation (1.3.4-3).)

It is seen from Enclosure 50 that the vibrations due to ring bending as a result of ball loads are relatively insignificant compared to the MIL-B-17931B limits for small loads such as normally used in vibration testing. For an axial load of 50 lbs or 140 lbs radial load the amplitude of the first harmonic of these vibrations is approximately 25% of the MIL-B-17391B low band limit. However, for loads exceeding these values the effect of the flexural vibration due to ball loads may become an important contributor to the low band vibration level. The second and higher harmonics are of insignificant amplitude.

3. Flexural Bending due to 2 wpc Inner Ring Waviness:

The amplitudes of the vibrations induced by flexural bending due to 2 wpc waviness of the inner ring are shown graphically on Enclosure 51 as a function of the RMS amplitude of inner ring waviness of the order 2 wpc. The abscissa shows the 2 wpc displacement amplitude as well as the 2 wpc velocity amplitude measured on a waviness tester at 1000 RPM rotational speed. For comparison the computed approximate vibration level in the 50-300 cps band due to rigid body vibrations induced by waviness of the inner and outer race and the balls is also shown as a function of waviness velocity level, measured at a speed of 1000 RPM for race waviness and 740 RPM for ball waviness in the waviness bands corresponding to the indicated vibration band.

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The MIL-B-17931-B limit in the 50-300 cps band is also shown on the graph. It is seen that a 2 wpc RMS displacement amplitude of approximately 15 microinches is required to generate a vibration level equal to the MIL-B-17931B limit. This value of the 2 wpc inner ring waviness is of an order of magnitude commonly found in quiet running deep groove ball bearings of this size, and two point out-of-roundness of the inner ring may therefore highly affect the bearing vibration in the 50-300 cps band.

4. Rigid Body Vibrations:

It is seen from Enclosure 51 that an outer ring waviness level of approximately 10,000 microinches/second, an inner ring waviness level of approximately 6000 microinches/second and a ball waviness level of 1000 microinches/second, respectively, are required to generate rigid ring vibrations of a level equal to the MIL-B-17931B limit in the 50-300 cps band. Assuming equal contribution to the bearing vibration from outer ring, inner ring and ball waviness, this corresponds to an outer ring waviness level of approximately 6000 microinches/second, inner ring waviness of approximately 3500 microinches/second and ball waviness of 600 microinches/second. These figures relate to waviness measurements in a $2\frac{1}{2}$ octave band corresponding to the 50-300 cps band. If the waviness is measured in octave bands with equal influence from each octave, the waviness levels are:

3700 Microinches/second for outer ring
2200 Microinches/second for inner ring
400 Microinches/second for balls

These figures are again of an order of magnitude frequently found in quiet running bearings and waviness of the rolling surfaces is therefore judged to be an important contribution to the low band vibration level. In the higher frequency range, which is less affected by the other sources discussed, waviness of the rolling surfaces is the predominant cause of vibration.

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To summarize, the predominance of any given vibratory source in any given frequency range depends on the relative magnitude of the various surface imperfections of the rolling surfaces, on the load and speed and on bearing design geometry. In general, in the low frequency range (up to approximately 10 times the rotational frequency) of a radially loaded bearing the following sources should be considered:

1. Variable compliance vibrations
2. Vibrations due to ball loads, if the bearing operates under comparatively heavy load
3. Flexural vibrations due to low order race waviness
4. Rigid body vibrations, due to race and ball waviness and inner ring eccentricity.
5. Two point out-of-roundness of balls

In the higher frequency range (above 10 times the rotational frequency) the only vibratory sources with significant effect are race and ball waviness.

This also applies to an axially loaded bearing with the exception that variable compliance vibrations are not induced under axial load.

I.3.24 Internal Damping in Ball Bearings

In the theoretical analysis on which the synthetic spectra shown on Enclosures 21, 22 and 27 are based, the effect of damping on the bearing vibration was neglected. This may be justified at frequencies sufficiently remote from the resonant frequencies of the system, but near the resonant frequencies the vibration is highly influenced by damping. A method of evaluating the internal damping directly from the spectrum was given in Progress Report No. 8.(1). Using this method, the damping time constant and the logarithmic decrement were computed for the vibratory modes corresponding to the natural frequencies found in the spectra of a few bearings in the 6203-6312 size range.

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The damping time constant is the time required to reduce the amplitude of transient free vibrations to $1/e$ (=37%) of its initial value.

The logarithmic decrement is defined as the natural logarithm of the ratio of any two successive amplitudes of the free vibration.

The damping time constant was found to vary comparatively little, the minimum being 0.7 m sec, the maximum 2.0 m sec and the average 1.3 m sec. The logarithmic decrement varies between 0.15 and 0.80, the average being 0.37. The ratio between two consecutive amplitudes of the free vibration is obtained by taking the antilogarithm of the logarithmic decrement. For the lowest resonant frequencies (in the 300-1000 cps range) this ratio was found to be of the order of 2, i.e., after 3-4 cycles the free vibration has decayed to less than 10% of its initial value. This occurs within approximately 4 m sec. For higher resonant frequencies (in the 5000-10000 cps range) the amplitude ratio is of the order of 1.2 which means that approximately 13 cycles are required to reduce the amplitude of the free vibrations to 10% of its initial value. This corresponds to less than 3 m sec. On the basis of these results a small ball bearing can be considered a relatively well, though subcritically damped system.

The effect of damping on the bearing vibration level can be evaluated if the damping time constant or logarithmic decrement are known.

The velocity amplification factor U_c of the damped bearing may be expressed by the equation (See Progress Report No. 9-10, pages 14-16) (1).

$$U_c = \frac{1}{\sqrt{[1 - (\frac{f}{f_n})^2]^2 + (\frac{\delta}{2})^2 (\frac{f}{f_n})^2}} \quad (1.3.24-1)$$

The factor U_c here indicates the amplification of the vibration amplitude at a given frequency f , due to the effect of a resonant frequency f_n of the system. δ is the logarithmic decrement.

In a non-resonant region where $f \ll f_n$, $U_c \rightarrow 1$.

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Enclosure 52 shows \bar{U}_c as a function of f/f_n for three values of the logarithmic decrement δ ; i.e., $\delta = 0.15$, 0.37 and 0.80 which are the minimum, average and maximum experimental values of δ .

At the resonant frequency $f = f_n$

$$\bar{U}_c = \bar{U}_{cmax} = \frac{\pi}{\delta} \quad (1.2.34-2)$$

$$\begin{array}{ll} \text{For } \delta = 0.15, & \bar{U}_{cmax} = 21.0 \\ \delta = 0.37, & \bar{U}_{cmax} = 8.5 \\ \delta = 0.80, & \bar{U}_{cmax} = 3.9 \end{array}$$

To evaluate the effect of the resonance on the vibration level measured in a band of finite width Δf , the bandwidth amplification factor \bar{U}_c may be computed from the equation

$$\bar{U}_c = \sqrt{\frac{1}{\Delta f} \int_{f_1}^{f_2} \frac{df}{[1 - (\frac{f}{f_n})^2]^2 + (\frac{\delta}{\pi})^2 (\frac{f}{f_n})^2}} \quad (1.3.24-3)$$

or using the notations

$$\frac{f}{f_n} = \mu \quad (1.3.24-4)$$

$$\frac{\Delta f}{f_n} = \Delta \mu \quad (1.3.24-5)$$

$$\bar{U}_c = \sqrt{\frac{1}{\Delta \mu} \int_{\mu_1}^{\mu_2} \frac{d\mu}{[1 - \mu^2]^2 + (\frac{\delta}{\pi})^2 \mu^2}} \quad (1.3.24-6)$$

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By evaluating the integral, the value of \bar{U}_c can be computed for any given values of μ_1 , μ_2 and $\Delta\mu$.

The bandwidth is often expressed as the number of octaves ϕ contained within the band. If the band is selected so that the resonant frequency f_n coincides with the midband frequency of the band, then the following relationships hold:

$$\frac{f_2}{f_1} = \frac{\mu_2}{\mu_1} = 2^\phi \quad (\text{I.3.24-7})$$

$$\frac{\mu_1 + \mu_2}{2} = 1 \quad (\text{I.3.24-8})$$

which solved for μ_1 and μ_2 give

$$\mu_1 = \frac{2}{2^\phi + 1} \quad (\text{I.3.24-9})$$

$$\mu_2 = \frac{2^{\phi+1}}{2^\phi + 1} \quad (\text{I.3.24-10})$$

Equation (I.3.24-6) may now be expressed as

$$\bar{U}_c = \sqrt{\frac{2^\phi + 1}{2(2^\phi - 1)} \int_{\frac{2}{2^\phi + 1}}^{\frac{2^{\phi+1}}{2^\phi + 1}} \frac{d\mu}{[1 - \mu^2]^2 - \left(\frac{\sigma}{\pi}\right)^2 \mu^2}} \quad (\text{I.3.24-11})$$

Results of a numerical evaluation of Equation (I.3.24-11) are shown graphically on Enclosure 53 which gives \bar{U}_c as a function of ϕ for $\Delta = 0.15$, 0.37 and 0.80 . It is seen that for bandwidths $\phi = 1/3, 1$ and $2\frac{1}{2}$ octaves (which are commonly used in vibration testing) the following values of \bar{U}_c are obtained:

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Bandwidth Amplification Factor \bar{U}_c

ζ	$\delta = 0.15$	$\delta = 0.37$	$\delta = 0.80$
1/3	11.5	6.2	4.0
1	7.2	3.8	2.4
2 1/4	4.9	2.8	1.8

The amplification factors listed above appear to be high, but it should be noted that they apply only to vibrations in the same mode as the resonant frequency contained within the band, e.g., for a band containing the resonant frequency of the rigid body mode of vibration only these rigid body vibrations are being amplified. Correspondingly for a band containing a flexural resonant frequency only the flexural outer ring vibrations will be amplified. It is known that in a non-resonant region the amplitudes of the flexural vibrations are very small compared to those of the rigid body motion and the increase in overall vibration level in a band containing a flexural resonance is therefore not nearly as great as indicated by the amplification factors \bar{U}_c in Enclosure 53.

If the amplitude V_f of the flexural vibrations in any given non-resonant band are expressed by the equation

$$V_f = k_f V \quad (1.3.24-12)$$

where V is the total vibration level and k_f is a positive constant, $k_f < 1$, then the bandwidth amplification factor given by Equation (1.3.24-11) may be modified as follows for bands including flexural resonances.

$$\bar{U}_{cf} \approx \sqrt{1 + k_f^2 \bar{U}_c^2} \quad (1.3.24-13)$$

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An accurate evaluation of k_f requires experiments not performed in this investigation. On the basis of available experimental results it would, however, appear that k_f , in general, does not exceed 1/4. Using this value of k_f Equation (1.3.24-14) yields the order of magnitude estimate:

$$\bar{U}_{cf} = \sqrt{1 + \left(\frac{\delta}{4}\right)^2} \quad (1.3.24-14)$$

For the average value of $\delta = 0.37$ the following orders of magnitude of \bar{U}_{cf} are obtained from Equation (1.3.24-14)

δ	\bar{U}_{cf}
1/3	1.85
1	1.38
2 1/4	1.22

For rigid body vibrations \bar{U}_c as obtained from Enclosure 53 or Equation (1.3.24-11) can be used directly, without modification. It appears, on the basis of experimental results, that the logarithmic decrement δ is larger for the rigid body mode than for the flexural modes and the best estimate of \bar{U}_c in a band containing the resonant frequency of the rigid body mode may therefore be obtained by using the maximum value of $\delta = 0.80$.

These results indicate that the rigid body resonant frequency, in general, has a greater effect on the vibration level than the flexural resonances. For instance, in a 2 1/2 octave band, the rigid body resonance increases the level by a factor of 1.8 while the flexural resonance increases it by a factor of 1.22. These factors appear to be in agreement with experimental values.

Enclosure 53 was derived for the case that the resonant frequency f_n coincides with the midband frequency of the band examined. Sufficiently accurate order of magnitude estimates may, however, be obtained from this enclosure even if this is not the case, as long as the resonant frequency falls well inside the band examined. In case of more than one resonant frequency within the band, an estimate of the total amplification factor may be obtained as the product of the individual amplification factors computed for each resonant frequency.

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I.3.25 Vibration Attenuating Bearing Housing

Bearing vibration may be attenuated at least in certain frequency ranges by selection of proper elastic and damping media and increase in housing mass. This can be accomplished, for instance, by using arrangements similar to those shown on Enclosure 8. Test results and analytical evaluation of these mounting arrangements show that:

1. Vibrations near zero frequency are difficult to attenuate, since this would require nearly infinitely soft springs.
2. In the 50-100 cps band attenuation can be accomplished simply through added mass of the housing. Laminates or other vibration damping structures are no more effective in this band than solid steel elements.
3. In the frequency range above 100 cps comparatively soft laminated elements (with low natural frequency) give best overall results. Laminates of thin steel strips and layers of soft damping material (rubber, cork and glue) have been found effective in this frequency range. The following average values were obtained for this type of laminated beam installed in the housing.

Frequency Band (cps)	Vibration level in Percent of the Level Without Damped Housing
50- 200	22
200-1600	54
1600-12800	11
Overall	32

It is noted that these attenuation values are rather substantial.

4. Further possibilities for the reduction of amplitudes of very low frequencies are suggested by the findings mentioned in I.3.21 that in a bearing, mounted so that it can vibrate both radially and angularly, the amplitudes of the radial vibrations may be theoretically reduced to near zero at low frequencies, by using a mounting arrangement which is highly flexible in the angular direction.

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A housing built on this principle could be useful if only radial vibrations must be attenuated while angular vibrations can be tolerated. (If it is desirable to attenuate the angular vibrations, the housing should be radially flexible.)

1.3.26 Effect of Fit Between Bearing O.D. and Housing

Test results with housing to outer ring fits in the tolerance range given by housings with bore diameter between ISO F6 and ISO M6 and using a bearing with O.D. within ISO h5 have shown that the effect of fit between housing and bearing O.D. is rather small and erratic. The effect is the same whether a lubricant between housing and O.D. was used or not.

1.3.27 Airborne Noise Characteristics of Ball Bearings

Airborne noise measurements were performed using equipment described in 1.2.5. The test rig accommodates two test and two support bearings. If it is assumed that the noise from the four bearings is additive by squares and that each individual bearing generates the same noise (using bearings from the same lot in all positions), the noise level of one bearing should be 6 db lower than the level reported.

If there is ambient noise the noise level D generated by the four bearings alone (in db) may be computed from the equation:

$$D = D_2 + 10 \log_{10} \left[10^{\frac{D_1 - D_2}{10}} - 1 \right] \quad (1.3.27-1)$$

where

D_1 = ambient noise and bearing noise together (in db)

D_2 = ambient noise (in db)

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All noise levels reported here refer to the noise level D of the bearings alone as determined by Equation (1.3.27-1).

For frequencies below 200 cps the background noise of the anechoic chamber, without the tester running, was, at the time of these tests, the same order of magnitude as the noise level with the tester running. Meaningful readings could therefore not be obtained in this range.

The following conclusions regarding the airborne noise emitted by the bearings can be drawn in the range above 200 cps:

1. The spectral distribution of the airborne noise, measured in octaves in the 200-12800 cps range is shown on Enclosures 54 and 55 for a sample of 6305 and 6310 bearings. A comparison between the structureborne vibration and the airborne noise spectra shows that at the same rotational speed the vibration and airborne noise spectra are similar.
2. The airborne noise level increases with speed with a trend similar to that of structure borne vibration. A somewhat smaller value of the speed-amplitude coefficient α is applicable for the airborne noise than for the structure borne vibration due to the slight difference in their spectral distributions.
3. The effect of radial and axial loads on the airborne noise is as small as the effect on the structure borne vibrations.
4. The correlation between airborne noise and structure borne vibration levels is sufficiently good to justify for most purposes the use of the much simpler vibration measurements alone in evaluating all quiet running characteristics of ball bearings. Of course, for most purposes, structure borne vibration of the bearing is the primary variable to be controlled.

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I.3.28 Additional Parameters Affecting Bearing Vibration and Airborne Noise

The effect of various parameters such as bearing dimensions, number of balls, surface micro-geometry, contact angle, radial looseness, load, speed and mounting parameters was previously discussed. Other parameters which may influence the bearing vibration but which will not be discussed in detail are listed here for completeness:

- a. Lubricant: The damping characteristics of the bearing are influenced by the lubricant used and it is known that the bearing vibration level to some extent depends on the type lubricant used.
- b. Land Height: If the lands adjacent to the grooves in the inner and outer rings are not sufficiently high, the balls will run near the edge of the groove. Under sufficient axial load the contact ellipse may extend to the edge of the groove which is likely to cause rough running.
- c. Groove Profiles: Excessive variation in the groove profiles from a circular contour, may result in increased vibration amplitudes.
- d. Misalignment: If a bearing operates under misalignment the contact angle varies along the circumference of the bearings. If the misalignment is sufficient the balls may run from near one edge of the groove to the other passing an unloaded zone between these two extreme positions. Since the maximum contact angle in a misaligned bearing is larger than in a perfectly aligned bearing, the misalignment has the same effect as insufficient land height. Another cause of rough running in a misaligned bearing is the fact that the "loose" balls passing the unloaded zone may generate vibrations which do not occur in an aligned bearing operating under centered thrust load, where all the balls are under load.

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- e. Localized Damage on the Rolling Surfaces: This generates vibration peaks which may be objectionable in a smooth running bearing. SKF Industries, Inc. has analyzed these peaks and developed instrumentation for their detection (6). Damage may, among other means, be induced by plastic deformation of the races on balls in case of improper assembly of the bearing into machinery. A method for evaluating this type of damage has been presented (7).
- f. Dirt on Rust: Dirt causes random vibration peaks which add to the vibration level of the bearing. Another effect of dirt is that it may cause permanent damage to the bearing in the form of small dents on the rolling surfaces of the races and balls. These dents increase the vibration level of the bearing as they are rolled over by the balls. Rust has the same effect as dirt; it may also form pits which produce vibration peaks as they are rolled over by the balls.
- g. Cage: The cage, if operating properly, is believed to have only a minor effect on the structure borne vibrations of small ball bearings. Its effect on the airborne noise level may be more noticeable and can be controlled by design.
- h. Shields and Seals: These, in general, influence the airborne noise level, but hardly the structure borne vibration. A secondary effect of some shields is to deform the outer ring, thus generating additional vibrations due to ring out-of-roundness.

PART II

STUDY OF VIBRATION AND NOISE CHARACTERISTICS OF
ROLLER BEARINGS WITH BORE SIZES IN THE RANGE OF 200-300 MM

2.1 INTRODUCTION

This part of the study covers roller bearings with bore sizes in the range between 200 and 300 mm, typifying bearings, suitable for use in naval propeller shaft and other main drive applications subject to moderate and high stationary radial loads. The speed range of interest is wide, from very slow rotation to speeds of the order of 1000 RPM. The following three basic types of roller bearings were included in the study:

1. Spherical roller bearings (double row)
2. Cylindrical roller bearings (single row)
3. Tapered roller bearings (single row)

Test loads ranged up to 100,000 lbs radial and axial, speeds between 100 and 1000 RPM.

Sketches of the various bearing sizes used, with main dimensions and other data are shown on Enclosures 56-61.

As for the smaller bearings, the tests and analysis were performed for a stationary outer ring and inner ring rotating at a constant speed.

Test equipment used in the experimental study of large roller bearings is described in Section 2.2.

Various sources of bearing vibrations are examined in Sections 2.2.1 - 2.3.4.

A brief discussion of contact deformations in roller bearings is given in Section 2.3.5.

Amplitude and frequency characteristics of various vibratory sources are discussed in Sections 2.3.6 - 2.3.10.

The effect of load and speed is covered in Section 2.3.11.

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A comparison of the vibration levels in different measuring directions, generated by various sources is given in Sections 2.3.12 - 2.3.14.

The effect of radial looseness is briefly discussed in Section 2.3.15.

Airborne noise measurements are described in Section 2.3.16.

Additional parameters affecting the vibrations and airborne noise of large roller bearings are listed in Section 2.3.17.

A comparison of the vibratory characteristics of different types of large roller bearings is given in Section 2.3.18.

2.2 TEST EQUIPMENT

Equipment used in the experimental study and analysis of the vibration characteristics of large roller bearings with 200-300 mm bore, is listed below:

2.2.1 Large Bearing Vibration Tester

The test machine used for measuring the vibration of large roller bearings with 200-300 mm bore is shown on Enclosure 62. The tester consists of a large shaft supported radially on two Kingsbury journal bearings. Axial loads which are applied to the shaft via the test bearing are carried by a Kingsbury tilting shoe thrust bearing located in the rear journal bearing housing. The test bearing is mounted on a tapered seat on the opposite end of the shaft from the Kingsbury thrust bearing.

The large bearing vibration test machine is presently equipped with two shafts and bearing housing assemblies. One shaft and housing assembly is designed to test bearings with bore sizes in the neighborhood of 200 mm, while the other permits testing of bearings with bore sizes of 280 mm and slightly larger. The test machine can be made to accommodate bearings with bore sizes up to 500 mm by re-designing the test shaft and housing. The test shaft is belt driven from a variable speed drive. The drive unit consists of a 200 HP induction motor coupled to an eddy current clutch. The output speed of the clutch is controlled by a speed setting potentiometer and maintained at the desired speed within $\pm 2\%$ by a transistorized controller. The test machine can be operated at speeds ranging from 0 to 1000 RPM.

Test bearing loading is achieved by means of two cylindrical hydraulic rams which are loaded by an air driven oil pump. Each ram is coupled to a loading rod which transmits the load to the test bearing housing. The radial load rod is coupled to the radial portion of the housing. The load rod for axial loading goes through the center of the hollow test shaft and imposes its load on the axial load cap which transmits the load directly to the outer ring of the test bearing. Loads ranging from 0 to 100,000 lbs in both the radial and axial direction can be applied to the test bearing.

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The Kingsbury radial and thrust bearings are capable of carrying full loads at speeds as low as 100 RPM with hydrostatic assistance. The hydrostatic assist can also be used to permit starting and stopping under full load, variation of oil film thickness, and variation of the spring constant and damping of the support bearings. To operate the hydrostatic assist of the support bearings, a large volume of high pressure oil (approximately 28 gpm) at a maximum pressure of 3500 psi is supplied by a Denison pump unit.

All speed, load and hydraulic controls are housed in the control console. This console also contains a strain indicator calibrated to read the radial and axial loads applied to the test bearing and temperature indicators which monitor the operating temperatures of the test bearing and the support bearings.

Enclosure 63 is a picture showing the layout of the large bearing vibration tester and associated equipment.

Ambient vibrations generated by the machine accessories have a significant effect on octave band readings in the low frequency range, especially at low rotational speeds. At speeds below 100 RPM meaningful octave band readings can be obtained only in the bands above 200 cps. At higher rotational speeds (800 RPM) meaningful octave band readings may be obtained in the frequency range above 25 cps.

The ambient spectrum generated by the machine accessories and surrounding machinery is characterized by a few peaks at discrete frequencies. Test bearing vibrations in the frequency range below 200 cps may be studied without interference, using narrow band analysis, at all frequencies other than the discrete frequencies of the ambient spectrum.

The effect of the ambient sources is illustrated by Enclosure 64 which shows narrow band vibration spectra measured at the front journal bearing under various conditions, and also by the octave band readings measured at various test bearing speeds (including zero), at the test bearing housing. From Enclosure 65 it is seen that the vibrations measured with the test bearing stationary (ambient vibrations) are of comparable order to the bearing generated vibrations in the octaves below 200 cps*.

* These ambient data were recorded at the time of measurements made for this contract. Since then, the test facility was relocated to the SKF Industries Engineering and Research Center and placed on a heavy (200,000 lbs) inertial block. Ambient vibration levels under the new conditions are generally lower.

The tester is used in conjunction with instrumentation described in 2.2.2 - 2.2.7.

2.2.2 Vibration Transducers

Piezo-electric accelerometers are utilized as the vibration sensing device in the vibration measurement of large roller bearings. The accelerometers are attached to the test bearing housing and measure the vibration of the bearing in three mutually perpendicular planes as shown in Enclosure 66.

Two types of accelerometers were used for the test measurements:

1. High sensitivity accelerometers: Endevco Model 2219 and Columbia Research Laboratories Model 410 with sensitivities greater than 300 MV/g were used for precise measurement of vibration amplitudes in the frequency range below 2500 cps.
2. Low sensitivity accelerometers: Columbia Research Laboratories Model 504 accelerometers were used for vibration measurements at frequencies up to 12000 cps.

The accelerometers were always used in conjunction with a Columbia Model 6006 Six channel cathode follower and amplifier.

2.2.3 Equipment for Narrow Band Frequency Analysis

The Panoramic narrow band spectrum analysis system, miscellaneous signal conditioning and frequency calibration equipment listed in 1.2.7 were used in the analysis of large bearing vibration data. An instrumentation layout showing the vibration analyzing equipment as utilized in the large bearing vibration studies is given on Enclosure 67.

2.2.4 Equipment for Octave Band and Wide Band Analysis

The electronic equipment (in addition to instruments listed in 2.2.2) used for octave and wide band analysis consisted of the following:

1. Krohn-Hite variable band pass filters Model 330M (frequency range 0.2 - 20,000 cps)
2. Ballantine True RMS Voltmeter Model 320.

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The bearing vibration was measured in twelve octave bands in the 3.125-12,000 cps frequency range. Wider frequency bands corresponding to the three Anderometer bands were also used. The vibration readout of this system is in RMS millivolts and is converted to acceleration by using known values of the pickup sensitivity and amplifier gain. The acceleration measurements were expressed in inches/second² RMS.

This equipment is shown in the instrumentation layout of Enclosure 67.

2.2.5 Instrumentation for Measuring Shaft Vibrations

The shaft vibrations of the large bearing vibration tester, were determined by measuring the electrical capacitance of a narrow air gap formed between the rotating shaft and a capacitance probe. The following equipment was used to measure the capacitance variation produced as the shaft vibrates:

1. Robersshaw-Fulton Proximity Meter No. 9501.
2. Krohn-Hite Ultra-Low Band Pass Filter Model 330-A.
3. Ballantine True RMS Voltmeter Model 320.

The capacitance measuring system and test configuration is shown in Enclosure 68. For details on the tests see Progress Report No. 13 pages 2-11 (1).

2.2.6 Tape Recorder

For detailed analysis of vibration data, vibration signals were recorded on magnetic tape for subsequent playback into the Panoramic spectrum analyzing equipment. The tape recorder used is an Ampex FM Recorder FR-1107 having seven input channels.

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2.2.7 Equipment for Airborne Noise Measurement

The airborne noise measurements of large roller bearings were made with a Bruel and Kjaer Condenser Microphone Type 4111. The microphone output was fed into a Bruel and Kjaer 2602 Microphone Amplifier with an external set of Allison Band Pass Filters, Model 2ABR. Since the airborne noise measurements were conducted in an area having a high ambient background, a test chamber was constructed to reduce the background noise. The noise test chamber and electronic equipment used in these tests are shown in Enclosure 69.

The ambient noise levels, expressed in absolute decibels, in the octave bands between 25 and 12800 cps, measured at the microphone location in the noise test chamber are tabulated below:

Frequency Band, cps	25-50	50-100	100-200	200-400	
Noise Level, db	82	77	78	70	
Frequency Band, cps	400-800	800-1600	1600-3200	3200-6400	6400-12800
Noise Level, db	69	62	61	50	42

2.2.8 Large Race Waviness Test Equipment

Waviness measurements of large bearing rings (200-300 mm bore) were obtained with a large vertical wavometer which uses a rotating pickup and a stationary specimen. The mechanical unit is a Micrometrical Manufacturing Co. Wavometer Type PE-65. It is equipped with an **ESF** MEA-100 velocity pickup and the associated electronic equipment was built by **ESF**.

Since the bearing rings tested on this instrument are much larger than the small ball bearing rings tested on the equipment described in 1.2.3 and since the rotational speed at which the large roller bearings are tested is lower than that used for small bearings, higher orders of race waviness than measured with the equipment described in 1.2.3, becomes important for the large roller bearings. Therefore the frequency range of the instrument was extended by adding eight octaves, covering the range from 15 to 3840 wpc, to the original five octaves between 3 and 96 wpc.

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Further details of this test equipment are given in Progress Report No. 8 pages 7-8.(1).

2.2.9 Roller Waviness Tester

The Race Waviness Tester described in 1.2.4 was used with special tooling to hold the rollers at a rotational speed of 740 RPM to measure roller waviness in the octave bands corresponding to 4-8, 8-16, 16-32, 32-64 and 64-128 wpc.

2.2.10 Length Measuring Equipment

The same equipment described in 1.2.12 was used for length and geometry measurements.

2.3 THE LARGE ROLLER BEARING AS A VIBRATORY SYSTEM

2.3.1 Sources of Bearing Vibration

The sources within the bearing itself, generating vibrations in large roller bearings, are essentially the same as those for deep groove ball bearings, discussed in 1.3. As for ball bearings, roller bearings vibrations are generated in a geometrically perfect bearing as well as by geometrical imperfections of the rolling surfaces. The majority of the analytical findings developed for ball bearings apply to roller bearings also.

2.3.2 Vibrations Generated by a Geometrically Perfect Roller Bearing

These include vibrations induced by variable compliance and by flexural ring bending due to roller loads.

2.3.3 Vibrations Induced by Variable Elastic Compliance

Due to the design differences between ball bearings and the roller bearings here considered, the variable compliance vibrations for roller bearings differ in the following respects from those of the ball bearings discussed in 1.3.3:

- a. In a cylindrical or tapered roller bearing the races and rolling elements are approximately in line contact, while they are in point contact in a ball bearing. The contact deformation in a cylindrical or tapered roller bearing therefore follows the nearly linear law (Palmgren (5)).

$$P = c_d \delta^{1.1} \quad (2.3.3-1)$$

instead of the $3/2$ power law of Equation (1.3.7-1) valid for ball bearings.

In a perfectly linear system where the displacements at the contacts between rolling elements and races are proportional to the loads at the contacts, the relative displacement between the rings is always in the direction of the applied radial load and proportional to the load (this is not the case in a non-linear system).

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The total displacement in a linear system is therefore constant, which means that variable compliance vibrations are not induced. Therefore, the amplitudes of the variable elastic compliance vibration in a cylindrical or tapered roller bearing, which is nearly linear, are expected to be small compared to those in a ball bearing.

- b. In a spherical roller bearing the races and rolling elements are in point contact, as in a ball bearing, and the variable compliance vibrations therefore should follow the same law for these two types of bearings. The coefficient C_J is, however, considerably higher for the roller bearing than for a ball bearing with the same rolling element diameter. The exact effect of this difference in Hertzian coefficients depends on the load applied and cannot be determined without extensive analysis.
- c. The number of rolling elements in the roller bearings here considered is much larger than in deep groove ball bearings of conventional design. This is expected to reduce the amplitudes of the variable compliance vibrations in roller bearings, since the amplitudes of these vibrations decrease with the number of rolling elements.
- d. The overall effect of the design difference between roller and ball bearings on the variable compliance vibrations is a reduction of the amplitudes of these vibrations in the roller bearings. This is in agreement with experimental results which for roller bearings, show no significant difference between the amplitudes measured in the direction of the radial load and perpendicular to it (See 2.3.12). For ball bearings the vibration amplitudes perpendicular to the radial load are generally of higher amplitude than those in the direction of the load which is attributed to the effect of the variable compliance vibrations.

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2.3.4 Flexural Vibrations of the Outer Ring Due to Roller Loads

Since the amplitudes of these vibrations are inversely proportional to the cube of the number of rolling elements, the large number of rollers in the roller bearings have a pronounced reducing effect on these amplitudes. Enclosure 70 show the computed amplitudes of these vibrations at 800 RPM rotational speed as a function of axial as well as radial load, computed from Equations (1.3.4-2) and (1.3.4 3). The amplitudes are expressed in terms of acceleration since acceleration measurements were used in all large roller bearing tests. Typical amplitudes of bearing vibration, measured in an octave band containing the frequency at which the flexural vibration occurs (the roller passage frequency), are shown for comparison. It is seen that, even under heavy loads the vibrations due to roller loads are of low amplitude compared to the overall vibration level. This effect is even smaller if the bearing is mounted in a housing, as was done in the measurements of the overall vibration level. These vibrations can therefore be neglected in most cases.

2.3 .5 Contact Deformation in Roller Bearings

As mentioned in 2.3.3 the deformation at the contact between the rollers and races in spherical roller bearings may be assumed to follow the non-linear Hertzian law, for point contacts given by Equation (1.3.7-1).

For cylindrical and tapered roller bearings the deformation is nearly linear, as given by Equation (2.3.3-1).

As for ball bearings, it is advantageous to use a linear approximation of the Hertzian law as given by Equation (1.3.7-2). The linear coefficient of contact deformation k_N for spherical roller bearings can then be computed from Equation (1.3.7-4) for a bearing under axial load or from the following more general equation in which the coefficient k_N is expressed in terms of rolling element load

$$k_N = \frac{\frac{3}{2} P^{1/3}}{\left(\frac{1}{C_i}\right)^{2/3} + \left(\frac{1}{C_o}\right)^{2/3}} \quad (2.3.5-1)$$

Formulae for numerical computation of C_i and C_o can be found in (5).

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For cylindrical and tapered roller bearings the linear coefficient of deformation k_N may be expressed in terms of the rolling element load P as

$$k_N = \frac{1.1 P^{0.09}}{\left(\frac{1}{C_1}\right)^{0.9} + \left(\frac{1}{C_2}\right)^{0.9}} \quad (2.3.5-2)$$

The values of k_N for the various roller bearings used in the tests reported here are plotted as a function of roller load in Enclosures 71 and 72. It is seen that for the cylindrical and tapered roller bearings k_N is relatively load independent, while k_N for the spherical roller bearing varies considerably with the load.

2.3.6 Vibrations Generated by Geometrical Imperfections of the Rolling Surfaces

As in a ball bearing, imperfections of the rolling surfaces of roller bearings induce vibrations of the outer race. These vibrations include rigid body motion and vibrations caused by elastic deformation (bending) of the outer ring.

The rigid body vibrations comprise radial, axial and angular vibrations. Due to the differences in design of the various types of bearings one mode may be more predominant in one bearing type than in another. These differences will be pointed out later.

2.3.7 Rigid Body Vibrations Generated by Geometrical Imperfections

The rigid ring vibration theory originally developed for deep groove ball bearings has been experimentally verified for large roller bearings.

Frequencies predicted by the theory have been found in experimental spectra of radially as well as axially loaded large roller bearings. Enclosure 73 shows a narrow band spectrum of the radial vibrations of a 23256 spherical roller bearing operating under 20,000 lbs radial load. The spectrum contains peaks at frequencies, coinciding with those that are, according to linear theory, induced by inner ring, outer ring and roller waviness. The spectrum includes frequencies only up to 170 cps, since the most significant effects of race waviness have been observed in this range.

Enclosure 74 shows a radial vibration spectrum of an NJ256 cylindrical roller bearing operating under 20,000 lbs radial load. In this bearing roller waviness was high compared to inner and outer ring waviness as evidenced by peaks induced by roller waviness at frequencies predicted from linear rigid ring theory.

Enclosures 75-77 are tabulations of orders of inner ring, outer ring and roller waviness that generate vibrations in given octave bands at selected rotational speeds. The ranges include vibrations generated by both the linear and non-linear theory of rigid ring vibrations, and apply to radial, axial and angular vibrations.

In a bearing with two rows of rolling elements (such as the spherical roller bearings used in the tests) both rows contribute to the bearing vibration. This case was not covered by the mathematical analysis in (1), and a detailed analysis will therefore be given here.

To illustrate the effect of two rows of rolling elements only the linear radial vibrations will be analyzed under the assumptions made in Special Report AL61L032 (1), i.e., the bearing is constrained in such a manner that the outer ring vibrates only radially, but not angularly or axially. Angular and axial vibrations of double-row bearings will not be analyzed here.

We first consider the effect of inner ring waviness on the radial vibrations of a double-row bearing. The subscript I will be used to refer to quantities related to the first row of rolling bodies, and the subscript II for the second row, on the assumption that both rows of rolling bodies carry the same load. Equation (30) of Report AL61L032 (1) may then be expressed as

$$\sum_{j=1}^{z_f} \left\{ k_{wI} \{ W_{jI} - x_H \cos[\omega_c t + \frac{2\pi(j-1)}{z_f}] \} - x_V \sin[\omega_c t + \frac{2\pi(j-1)}{z_f}] \right\}^2 + P_f \cos[\omega_c t + \frac{2\pi(j-1)}{z_f}] \quad (2.3.7-1)$$

$$+ \sum_{j=1}^{z_g} \left\{ k_{wII} \{ W_{jII} - x_H \cos[\omega_c t + \frac{2\pi(j-1)}{z_g} + \theta] \} - x_V \sin[\omega_c t + \frac{2\pi(j-1)}{z_g} + \theta] \right\}^2 + P_g \cos[\omega_c t + \frac{2\pi(j-1)}{z_g} + \theta] = M \ddot{x}_H$$

This, using Equation (41) - (48) of Report AL61L032 can be simplified as

$$M \ddot{x}_H + \frac{z_f}{2} k_{wI} x_H + \frac{z_g}{2} k_{wII} x_H = k_N \sum_{j=1}^{z_f} W_{jI} \cos[\omega_c t + \frac{2\pi(j-1)}{z_f}] \quad (2.3.7-2)$$

$$+ k_N \sum_{j=1}^{z_g} W_{jII} \cos[\omega_c t + \frac{2\pi(j-1)}{z_g} + \theta]$$

where z_I and z_{II} are the numbers of rolling elements in row I and row II, respectively (only the case $z_I = z_{II}$ will be considered),

x_H is the radial displacement in the horizontal direction; θ is the phase angle between the rolling elements of the two rows (the polar angle between the arbitrarily numbered "rolling element No. 1" of row I and "rolling element No. 1" of row II). w_{jI} and w_{jII} are according to Equation (61) of Report AL61L032 (1) defined for inner ring waviness by

$$w_{jI} - \bar{w}_{jI} = \sum_{k=1}^{\infty} A_{kI} \sin k \left[\omega_i t + \alpha_{kI} - \frac{2\pi(j-1)}{z_I} \right] \cos \left[\omega_c t + \frac{2\pi(j-1)}{z_I} \right] \quad (2.3.7-3)$$

$$w_{jII} - \bar{w}_{jII} = \sum_{k=1}^{\infty} A_{kII} \sin k \left[\omega_i t + \alpha_{kII} - \frac{2\pi(j-1)}{z_{II}} + \theta \right] \cos \left[\omega_c t + \frac{2\pi(j-1)}{z_{II}} + \theta \right] \quad (2.3.7-4)$$

where α_{kI} and α_{kII} are the phase angles of the waviness function for the two rows, measured from a common angular position for both rows.

A_{kI} and A_{kII} are the amplitudes of the waviness harmonics.

Using trigonometric identities and Equations (32)-(35) of Report AL61L032 (1) Equation (2.3.7-4) can be expressed as

$$w_{jII} = \sum_{k=1}^{\infty} A_{kII} \left\{ S_1(k, z) \sin k(\omega_i t + \alpha_{kII} + \theta) \cos(\omega_c t + \theta) + S_2(k, z) \cos k(\omega_i t + \alpha_{kII} + \theta) \sin(\omega_c t + \theta) \right\} \quad (2.3.7-5)$$

where the fact that terms containing $S_3(k, z)$ and $S_4(k, z)$ vanish, has been considered. The functions $S_1(k, z)$, $S_2(k, z)$, $S_3(k, z)$ and $S_4(k, z)$ are defined by Equation (32) - (35) in Report AL61L032 (1).

Equation (2.3.7-4) can further be written as

$$W_{jII} = \frac{1}{2} \sum_{k=1}^{\infty} A_{kII} \left\{ [S_1(k, z) - S_2(k, z)] \sin[(k\omega_i - \omega_c)t + k\alpha_{kII} + (k+1)\theta] \right. \\ \left. + [S_1(k, z) - S_2(k, z)] \sin[(k\omega_i + \omega_c)t + k\alpha_{kII} + (k-1)\theta] \right\} \quad (2.3.7-6)$$

This equation is the equivalent for double row bearings of Equation (63) of (1).

The solution of Equation (2.3.7-2) may now be expressed as follows, using Equation (2.3.7-6), the identities (43) and (44) of Report AL61L032 (1), and the method outlined on pages 16-21 of the same report:

$$x_{HI} = \frac{1}{2} \sum_{\substack{n=1 \\ k=nZ_1}}^{\infty} \frac{A_{kI}}{1 - \left(\frac{k\omega_i - \omega_c}{\omega_f}\right)^2} \sin[(k\omega_i - \omega_c)t + k\alpha_{kI}] \\ + \frac{1}{2} \sum_{\substack{n=0 \\ k=nZ_1-1}}^{\infty} \frac{A_{kI}}{1 - \left(\frac{k\omega_i + \omega_c}{\omega_f}\right)^2} \sin[(k\omega_i + \omega_c)t + k\alpha_{kI}] \quad (2.3.7-7) \\ + \frac{1}{2} \sum_{\substack{n=1 \\ k=nZ_1-1}}^{\infty} \frac{A_{kII}}{1 - \left(\frac{k\omega_i - \omega_c}{\omega_f}\right)^2} \sin[(k\omega_i - \omega_c)t + k\alpha_{kII} + (k+1)\theta] \\ + \frac{1}{2} \sum_{\substack{n=0 \\ k=nZ_1}}^{\infty} \frac{A_{kII}}{1 - \left(\frac{k\omega_i + \omega_c}{\omega_f}\right)^2} \sin[(k\omega_i + \omega_c)t + k\alpha_{kII} + (k-1)\theta]$$

This solution is the double-row bearing equivalent of Equations (74, 78-80) of (1). Note that in the present equation, a factor 1/2 appears ahead of the summations which is not present in the single row solution. In a non-resonant range

where

$$\left(\frac{k\omega_i \pm \omega_c}{\omega_f} \right)^2 \ll 1 \quad (2.3.7-8)$$

Equation (2.3.7-7) may be simplified as

$$x = \frac{1}{2}(x_I + x_{II}) =$$

$$\frac{1}{2} \sum_{\substack{n=1 \\ k=n-1}}^{\infty} x_{kI} \sin[(k\omega_i - \omega_c)t + k\alpha_{kI}] + \frac{1}{2} \sum_{\substack{n=0 \\ k=n+1}}^{\infty} x_{kI} \sin[(k\omega_i + \omega_c)t + k\alpha_{kI}] \quad (2.3.7-9) \\ + \frac{1}{2} \sum_{\substack{n=1 \\ k=n-1}}^{\infty} x_{kII} \sin[(k\omega_i - \omega_c)t + k\alpha_{kII} + (k-1)\theta] + \frac{1}{2} \sum_{\substack{n=0 \\ k=n+1}}^{\infty} x_{kII} \sin[(k\omega_i + \omega_c)t + k\alpha_{kII} + (k-1)\theta]$$

Here x_x represents the instantaneous displacement of a bearing consisting of row I only (the first two summations in Equation (2.3.7-9)), and x_{II} the displacement of a bearing consisting of row II only (the last two summations in Equation (2.3.7-9)). x is the instantaneous displacement of the double-row bearing.

x_{kI} is the amplitude of the k :th harmonic of the vibrations generated by a bearing, having only the first row of rolling elements, and x_{kII} the corresponding amplitude generated by the second row.

The equations for outer ring and rolling element waviness can be obtained from Equation (2.3.7-9) by substituting for the frequencies $k\omega_i \pm \omega_c$, the characteristic frequencies for vibration induced by outer ring waviness: $(k \pm 1)\omega_c$ and ball waviness: $k\omega_b \pm \omega_c$, and by using the values of k applicable to outer ring and ball waviness, respectively (See Enclosure 20).

Equation (2.3.7-9) is based on the assumption that both rows of rolling elements carry equal loads. Since this is not necessarily the case, Equation (2.3.7-9) gives generally only an approximation of actual conditions. If, for instance, the bearing operates under heavy thrust load and no radial load, the effect of the loaded row is predominant while the other row contributes comparatively little to the bearing vibration. For most operating conditions used in the tests on large spherical roller bearings Equation (2.3.7-9) is, however, believed to give a good approximation of the bearing vibration.

The interpretation of Equation (2.3.7-9) depends on the relative magnitudes of the parameters x_{kI} , x_{kII} , α_{kI} , α_{kII} and θ . The following cases may be considered:

1. $\alpha_{kI} \neq \alpha_{kII}$. In this case the waviness patterns of the two rows are unrelated, with a random phase shift between the same harmonics of the two rows. The frequencies in the vibration spectrum are, in this case, exactly the same as for a single row bearing. The RMS vibration level in a wide band is

$$\bar{x} = \frac{1}{2} \sqrt{\bar{x}_I^2 + \bar{x}_{II}^2} \quad (2.3.7-10)$$

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where the bars indicate RMS values.

If $\bar{x}_I = \bar{x}_G$ (which may be considered to be the case if both rows are finished by the same process), then

$$\bar{x} = \frac{1}{\sqrt{2}} \bar{x}_I \quad (2.3.7-11)$$

i.e., the vibration level of the double row bearing is $1/\sqrt{2}$ times that of the same bearing equipped with only one row of rolling elements. The case $\alpha_{kI} = \alpha_{kG}$ always applies to vibrations generated by rolling elements, and may also be used in most cases where the two rows of rolling elements run in separate grooves such as in a spherical roller bearing inner ring. This result is independent of phase angle θ .

2. $\alpha_{kI} = \alpha_{kG}$ and $x_{kI} = x_{kG}$. In this case, the waviness patterns of the two rows are the same. This case may be approximated for instance, by the outer ring of a spherical roller bearing where both rows of roller run on the same spherical surface. The resultant vibration now depends on the phase angle θ . Using trigonometric identities, Equation (2.3.7-9) may be expressed as

$$x = \sum_{\substack{n=1 \\ k=nZ-1}}^{\infty} x_{kI} \sin[(k\omega_i - \omega_c)t + k\alpha_k + \frac{k+1}{2}\theta] \cos \frac{k+1}{2}\theta \\ + \sum_{\substack{n=0 \\ k=nZ+1}}^{\infty} x_{kI} \sin[(k\omega_i + \omega_c)t + k\alpha_k + \frac{k-1}{2}\theta] \cos \frac{k-1}{2}\theta \quad (2.3.7-12)$$

It is seen that x_k , the vibration amplitude generated by k th waviness harmonic of the double row bearing may be expressed in terms of the corresponding amplitude of the single row bearing x_{kI} by the equations

$$x_k = x_{kI} \cos \frac{1}{2}(k+1)\theta \quad ; k = nZ-1 \quad (2.3.7-13)$$

$$x_k = x_{k\Gamma} \cos \frac{1}{2}(k-1)\theta \quad ; \quad k = n\pm 1 \quad (2.3.7-14)$$

or

$$x_n = x_{n\Gamma} \cos \frac{1}{2}n\theta \quad ; \quad n = 0, 1, 2, 3 \dots \quad (2.3.7-15)$$

Equation (2.3.7-15) shows the dependence of the vibration amplitudes on the phase angle θ . Two cases will be considered:

- a. Bearings with both rows of rolling elements held in one common cage. In this case the phase angle θ is constant. If $\theta = 0$, the angular positions of the rolling elements of the two rows coincide. In this case $x_n = x_{n\Gamma}$ and the vibration spectrum of the double row bearing is identical with that of the single-row bearing. If $\theta = \pi/2$, the rolling elements are staggered so that the rolling elements of one row fall exactly at the midpoints between the rolling elements of the other row. In this case, according to Equation (2.3.7-6), $x_n = 0$, for $n = 1, 3, 5 \dots$ and $x_n = x_{n\Gamma}$ for $n = 2, 4, 6 \dots$, i.e., the number of peaks of the double row bearing spectrum is only 1/2 of that of the single-row bearing. The frequencies of these peaks correspond to race waviness harmonics of the orders $n/2 \pm 1$ where 2 is the total number of rolling elements in the bearing (both rows). The vibration spectrum of the double-row bearing is identical with the spectrum of a single-row bearing having the same number of rolling elements as the total number of rolling elements of the double-row bearing.

For other values of the phase angle θ the amplitudes of the various vibration harmonics may be computed from Equation (2.3.7-15). Enclosure 78 shows how the amplitudes of the harmonics corresponding to $n = 1, 2, 3$ and 4 depends on the phase angle θ .

It follows from Equation (2.3.7-6) that the RMS value of the vibration amplitudes of the double-row bearing, in a wide band, may be expressed as

$$\bar{x} = \frac{\bar{x}_{\Gamma}}{\sqrt{2}} \quad \text{for} \quad \theta \neq 0$$

$$\bar{x} = \bar{x}_{\Gamma} \quad \text{for} \quad \theta = 0 \quad (2.3.7-16)$$

where the bars indicate RMS values.

It should be noted that these equations are strictly valid only for an infinitely wide band ($\gamma \rightarrow \infty$). For sufficient accuracy the band should contain at least $\pi/2\phi$ harmonics, which means that for small values of ϕ a very wide band is required.

- b. Bearings with separate half-cages for the two rows of rolling elements. In this case relative slip always occurs between the half-cages, i.e., the phase angle varies, and amplitudes x_j are therefore also subject to time variations. It follows from Equation (2.3.7-15) that the long term average RMS values of the vibration amplitudes \bar{x} of the double-row bearing, in a wide band, is, in this case:

$$\bar{x} = \frac{\bar{x}_1}{\sqrt{2}} \quad (2.3.7-17)$$

A comparison of Equation (2.3.7-10), (2.3.7-15) and (2.3.7-16) shows that, with one exception, the amplification factor for the radial vibrations of double-row bearings with equal load sharing between the two rows is $1/\sqrt{2}$ times that of the corresponding single-row bearing. Equation (1.3.17-2), (1.3.17-3) and (1.3.17-4) may therefore be used both for bearings with one and two rows of rolling elements, if Z is taken as the total number of rolling elements in the bearing. The one exception to this rule is the case $\phi = 0$, given by Equation (2.3.7-15), when the angular position of the rolling elements of the two rows coincide, and the rolling element in both rows run on a common ring surface. In this case the double-row and single-row bearing have the same amplification factor, i.e., the number of rolling bodies per row is to be used in the formulas.

If the bearing operates under heavy axial load so that only one row is loaded, the amplification factors are the same as for the corresponding single-row bearing, i.e., $\sqrt{2}$ times the values for the double row bearing.

In Enclosure 79 the velocity bandwidth amplification factors U_o , U_i and U_b for outer ring, inner ring and roller waviness for the roller bearings under study are tabulated. The factors refer to a rotation speed of 100 RPM of the bearing in vibration testing, 200 RPM in waviness testing inner and outer rings and 740 RPM in roller waviness testing.

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The amplification factors at speeds other than 100 RPM may be obtained by multiplying the values in Enclosure 79 by $N/100$ where N is the rotational speed in RPM. Acceleration amplification factors relating waviness measured in terms of velocity to vibration measured in terms of acceleration may be obtained by multiplying the values in Enclosure 79 by $2\pi f$, where f is the midband frequency of the band in which the vibration is measured. Therefore the ratio between U_a , U_v and U_b is the same for acceleration as for velocity measurements. All the factors given in Enclosure 79 apply to measurements in comparatively wide non-resonant band. They are based on the assumption of equal load distribution between rows.

It can be shown similarly to the reasoning given above that Equations (1.3.17-5), (1.3.17-6) and (1.3.17-7) which give the amplification factors for the axial vibrations of single-row bearings are also valid for double-row bearings operating with separate half-cages (variable ϕ), and equal load sharing between the rows. In these equations Z again indicates the total number of rolling elements in the bearing.

The angular vibrations of double-row bearings are believed to be highly influenced by bearing design, e.g., whether the bearing is a ball bearing or spherical roller bearing. These vibrations for double-row bearings have not been included in the present study.

2.3.8 Flexural Vibrations Induced by Low Order Race Waviness

The narrow band spectrum of the roller bearing in Enclosure 73 shows peaks at twice and three times the rotational frequency. As for ball bearings, vibrations at these frequencies can be explained as flexural vibrations induced by 2 wpc and 3 wpc inner ring waviness. Since the amplitude of these peaks, according to Equation (1.3.18-2) increases with the number of the rolling elements and with the elastic spring constant of the rollers, this effect is expected to be of importance in large roller bearings.

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If, in a double-row bearing, the waviness patterns of the two inner ring tracks are identical then the flexural vibrations induced are similar to those in a single row bearing. If the waviness patterns are not identical then the resultant elastic vibrations depend not only on the flexural rigidity but also on the torsional rigidity of the outer ring. These vibrations have not been studied.

2.3.9 Natural Frequencies of the Outer Ring

The natural frequencies of the vibrations of the outer ring may be computed from Equation (1.3.15-1) where the lowest mode gives the natural frequency of the radial rigid body mode of the free outer ring supported by the rolling elements and the higher modes represent flexural vibrations. The natural frequencies of axial rigid body motions may be computed from Equations (1.3.14-3), (1.3.14-4) and (1.3.14-5).

The natural frequencies in the range between 0 and 10,000 cps of the roller bearings used in the study are tabulated in Enclosure 80. The rigid body resonant frequency of a system, consisting of the bearing mounted in its housing on the vibration tester (described in 2.2.1) with radial loading bar and piston attached to the housing, is also listed in the enclosure. (For details see Progress Report No. 17 (1)). Enclosures 81 and 82 show typical narrow band spectra of spherical roller bearings, recorded at several rotational speeds. In Enclosure 83 the spectra obtained at the various speeds have been superimposed on the same graph. It is seen that the spectra contain peak regions which are reasonably speed independent indicating the probability of resonances in these regions. These regions are seen to coincide well with the computed natural frequencies.

2.3.10 The Effect of Load on the Vibration of Large Roller Bearings Spherical Roller Bearings

Radial load, in the 20,000 to 80,000 lbs range has a small and erratic effect on the vibrations measured in octave bands between 50 and 1600 cps in radial and axial directions. In the octave bands above 1600 cps the vibration level increases with load.

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This effect is most pronounced in the highest band studied (6400-12800 cps). An increase in axial load from 5000 to 25000 lbs was found to significantly increase the vibration level only in the two octave bands between 3200 and 12800 cps. Below 3200 cps the effect of axial load is insignificant.

Cylindrical Roller Bearings

The effect of radial load on the vibration level of cylindrical roller bearings is so small and erratic that significant trends are difficult to detect.

Tapered Roller Bearings

The effect of load on the vibrations of tapered roller bearings was not studied experimentally.

2.3.11 The Effect of Speed on the Vibration of Large Roller Bearings

It was shown in Section 1.3.20 that the relationship between vibration level measured in octave bands, and rotational speed, for small ball bearings may be expressed by Equation (1.3.20-9). The vibration level of these bearings was measured in terms of velocity. Experimental results have shown that the same equation, with different values of the exponent α , also applies to large roller bearings for which the vibration level is expressed in terms of acceleration.

Enclosure 84 shows typical experimentally obtained values of α for roller bearings as a function of frequency. The curve shown on Enclosure 84 gives the approximate average values of α for a spherical roller bearing under radial load. The curves for cylindrical and tapered roller bearings show the same tendency. For all three types of bearings α is approximately 0.7 in the 50-400 cps range and approximately 1.5-2.0 in the higher octave bands.

As for small ball bearings, the relationship between vibration level and speed for large roller bearings depends on the shape of the spectrum. As in the case of ball bearings the vibration level in any given octave band at any given rotational speed may be computed provided that the octave band spectrum at one speed is known.

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In a non-resonant region, assuming the vibration transmission characteristics of the bearing to be the same at the two speeds, the following applies for measurements expressed in terms of acceleration:

$$V_{2A}\left(\frac{N_2}{N_1}f\right) = \left(\frac{N_2}{N_1}\right)^2 V_{1A}(f) \quad (2.3.11-1)$$

where $V_{1A}(f)$ is the RMS vibrational acceleration measured in an octave band with midband frequency f cps, at a rotational speed of N_1 RPM.

$V_{2A}(f)$ represents the reading in the same band at a rotational speed of N_2 RPM. The squared factor $(N_2/N_1)^2$ in Equation (2.3.11-1) is due to the fact that, for given displacements, the acceleration amplitudes are proportional to the square of the frequency and thus also to the square of the rotational speed. It follows from Equation (2.3.11-1) that the octave band acceleration spectrum at a speed N_2 is obtained from the spectrum at the speed N_1 by multiplying the ordinate scale by $(N_2/N_1)^2$ and the abscissa by N_2/N_1 .

The value of the acceleration-speed exponent α_A equals that of the velocity-speed exponent α to the degree of approximation to which, for the octave bands under consideration.

$$V_A = 2\pi f V \quad (2.3.11-2)$$

where f is the midband frequency of the octave band.

While Equation (2.3.11-2) is not generally exact, it may be used for rough estimates of the vibration levels, and therefore, the values of α and α_A are approximately the same for the same bearing.

A comparison of Enclosures 44 and 84 shows that the exponent α for small ball bearings is approximately the same as the exponent α_A for large roller bearings in the frequency range above 400 cps. (In this range the average α for the ball bearings is approximately 1.4 and the average α_A for the roller bearings is approximately 1.7.

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In the 50-400 cps the average α_A is only 0.7 while α for the ball bearings is 1.7.) It should, however, be realized that in the low frequency range the results on the large roller bearings are influenced by ambient vibrations (Section 2.2.1) which tends to increase the vibration readings, especially at low speeds. This has a reducing effect on the values of α_A in the 50-400 cps bands, which may at least partly, explain the low values of α_A in this range.

Considering the differences in size, design and rotational speed it is, of course, not surprising to find some differences between the values of α for ball bearings and large roller bearings. A comparison of Enclosures 28, 29 and 79 shows that the waviness orders generating vibrations of the roller bearings under the speeds here considered are much higher than those in the ball bearings. This, no doubt, accounts for some of the differences between the vibration spectra of the bearings.

While the vibration-speed exponents for acceleration and velocity can be considered to be approximately the same for the same bearing, the shape of the acceleration spectrum is widely different from that of the velocity spectrum. The velocity octave band spectrum (in a non-resonant region) corresponding to $\alpha = 1.5$ was found to follow Equation (1.3.20-6), i.e., the octave band vibration readings decrease with frequency. The corresponding octave band acceleration spectrum follows the equation

$$V_A = K\sqrt{f}$$

(2.3.11-3)

which gives vibration levels increasing with frequency.

Enclosures 85 and 86 show typical octave band acceleration spectra of a 23256 spherical roller bearing rotating at 300 and 500 RPM. It is seen that, in all three measuring directions there is a tendency for the vibration levels to increase with frequency as suggested by Equation (2.3.11-3).

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It should be realized that Equation (2.3.11-3) describes a spectrum corresponding to a value of $\alpha_A = 1.5$, in non-resonant regions only. Since, according to 2.3.9 the spectra of large roller bearings are characterized by several resonant peaks, considerable deviations from the approximate spectral shape (2.3.11-3) are expected, even for bearings for which $\alpha \approx 1.5$. This is clearly illustrated by a comparison of the octave band spectra of several spherical, cylindrical and tapered roller bearings which will be discussed in more detail in 2.3.14. It will be shown that for the octaves in the 50-1600 cps range the vibration level of all bearing types tested increases with frequency. In the frequency range above 1600 cps a decreasing tendency is observed in a few cases.

2.3.12 Comparison of Vibration Levels in Different Measuring Directions

The following general trends have been observed in comparing octave band vibration readings in different measuring directions:

Spherical Roller Bearings

1. The vibration levels measured in octave bands in the range from 3 to 6400 cps in the direction of the applied load and perpendicular to it are approximately the same, for bearing rotational speeds in the range between 100 and 800 RPM. Since the frequency range studied includes the frequencies of all the important harmonics of the variable compliance vibration, the results may be interpreted to mean that, the effect of the variable compliance vibrations on the total vibration level is small, as expected on the basis of the discussion in 2.3.3.
2. The vibration level in the axial direction is, generally, approximately twice as high as in the radial direction. This is in agreement with the values of the amplification factors ψ_r , ψ_a and ψ_b in the radial and axial direction as shown on Enclosure 79.

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Cylindrical Roller Bearings

1. Significant trends are somewhat more difficult to detect than for the spherical roller bearings, due to more erratic variations in the vibration levels. Since the cylindrical roller bearing is more sensitive to misalignment than the spherical roller bearing, these variations may be due to unintentionally induced misalignment in applying the load.
2. Generally, the octave band vibration levels are approximately the same in the direction of the radial load, perpendicular to the load and in the axial direction.

Tapered Roller Bearings

1. The vibration levels in the axial direction are, in general, higher than in the radial directions in octave bands up to 800 cps, as expected from the value of v_a , v_r and v_b listed in Enclosure 79. In the bands above 800 cps the axial vibrations are of lower amplitude than the vibrations measured in the radial directions.
2. The vibration amplitudes in the direction of the radial load and perpendicular to it are approximately the same in octave bands up to 800 cps. In the bands above 800 cps the amplitudes of vibrations in the direction of the load are approximately twice as high as in the direction perpendicular to it.

2.3.13 Comparison of the Effects of Inner Ring, Outer Ring and Rolling Element Waviness on the Bearing Vibration Level

The method given in 1.3.22 may advantageously be applied to large roller bearings, particularly in studying the effects of improvements of the rolling surface waviness.

The theoretically expected reduction in vibration level, due to improvements in waviness, may be computed from the equation

$$\left(\frac{v_B}{v_A}\right)^2 = Z_o^2 \left(\frac{w_{oB}}{w_{oA}}\right)^2 + Z_i^2 \left(\frac{w_{iB}}{w_{iA}}\right)^2 + Z_b^2 \left(\frac{w_{bB}}{w_{bA}}\right)^2 \quad (2.3.13-1)$$

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where the subscript A refers to the unimproved bearing and the subscript B to the improved bearing. V_A and V_B are the theoretically expected vibration levels in any given band and the quantities W_o ,

W_i and W_b are waviness readings in bands corresponding to the frequency band of vibration. Z_o , Z_i and Z_b are given by Equations (1.3.22-5), (1.3.22-6) and (1.3.22-7).

Enclosures 87-92 give experimental vibration and waviness data for unimproved and improved spherical, cylindrical and tapered roller bearings. The subscript A always refers to the unimproved bearing and B and C, respectively to improved bearings of two degrees of refinement. (The manufacturing improvements will be further discussed in 2.3.18.) Enclosures 93-95 show a graphical comparison of the computed and measured V_B/V_A ratios. It is seen that for the spherical bearings the experimental ratio, in general, is somewhat higher than that computed, while for the cylindrical and tapered roller bearings the opposite is true. The relative influence of improvements in outer ring, inner ring and roller waviness for the various bearings used may be obtained from a comparison of the Z_o^2 , Z_i^2 and Z_b^2 factors for the different bearings. This is illustrated by Enclosure 96. It is recalled that the value of the contribution factors Z depends not only on design, but also on the relative magnitude of waviness of the different parts found in the reference (A) bearing.

The fact that the reduction in vibration level due to improvements in waviness is smaller for the spherical bearing than for the cylindrical and tapered bearings could be explained by assuming that the vibration level of the bearing is also influenced by other causes than waviness (including the sources discussed in 2.3.3 and 2.3.4). Since the vibration level of the unimproved spherical roller bearing tested was considerably lower than that of the cylindrical and tapered bearings, the contribution to the total vibration level from sources other than waviness is likely to be greater for the spherical bearing than for the two other types. The vibration level of the spherical bearing was therefore less influenced by reductions in the waviness level of the parts than the two other types of bearings.

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2.3.14 Comparison of the Amplitudes of Vibration Generated by Various Sources

Of the four vibratory sources mentioned in 2.3.1 through 2.3.4 the variable compliance vibrations and bending of the outer ring due to the finite number of rolling elements are judged to be less important in the large roller bearings than in ball bearings, according to the discussions in 2.3.3. The two other sources, i.e., flexural bending due to low order inner ring waviness and rigid body vibrations induced by waviness of the rolling surfaces are considered the main contributors to the bearing vibration.

The effects of 2 wpc inner ring waviness on the flexural vibration of the outer ring is illustrated by Enclosure 97, for a 23240 spherical roller bearing.

The results apply to a rotational vibration test speed of 800 RPM. The vibration level is given in terms of acceleration. For comparison the average vibration level measured in the 25-50 cps band, (which contains the frequency generated by this source) is also shown. It is seen that 2 wpc with an RMS amplitude of 175 micro-inches (corresponding to 450 microinches maximum diameter difference) is required to produce a vibration amplitude equal to the measured level. Since this amount of inner ring two point out-of-roundness is of an order of magnitude commonly found in bearings of this size the two point out of roundness is expected to be a significant contributor to the low frequency vibration level of the bearing. This is also verified by the experimental spectrum of a 23256 bearing shown on Enclosure 98, which shows a peak at twice the rotational frequency. Inner ring waviness at 3 wpc also has a similar effect. A peak at three times the rotational frequency corresponding to this order of inner ring waviness is also seen in the spectrum shown on Enclosure 98.

The theoretically expected effect of outer ring, inner ring and roller waviness of a 23240 spherical roller bearing is illustrated by Enclosure 99. This Enclosure shows the computed vibration levels in terms of acceleration (according to linear rigid ring theory) induced by the three types of waviness in the 400-800 cps band in a bearing rotating at 800 RPM. The graph shows the vibration levels generated by outer ring, inner ring and roller waviness separately, as a function of the waviness level in a corresponding waviness band. The amplification factors computed from Equation (1.3.17-2), (1.3.17-3) and (1.3.17-4) were used in this analysis and the conversion to acceleration was performed by multiplying by the midband frequency of the 400-800 cps band. For comparison the measured average vibration level in the 400-800 cps band is also shown. The waviness levels of the rollers, the inner ring and the outer ring measured in bands approximately corresponding to the 400-800 cps vibration band are marked on the graph. It is seen that the vibration levels computed from the measured values of the outer ring, inner ring and roller waviness, in each case, are a significant portion of the measured vibration level. -94-

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2.3.15 The Effect of Radial Looseness on the Vibration of Roller Bearings

Tests performed on spherical roller bearings have shown that there is a tendency for the vibration level to increase with decreasing radial looseness. The effect of radial looseness in the positive looseness range is, however, very small. A preloaded bearing (no radial looseness) has, in general, a higher vibration level than the same bearing mounted with loose fit. For further details, See Progress Report No. 14 (1).

2.3.16 Airborne Noise Measurements of Large Roller Bearings

The following tendencies concerning the airborne noise of large roller bearings were observed:

1. The airborne noise level, measured in octave bands generally decreases slowly with increasing frequency.
2. The airborne noise and structure borne vibration velocity spectra of each bearing are similar in shape.
3. The airborne noise readings correlate reasonably well with the structure borne vibration readings. This correlation holds both for comparisons of the same bearing at different speeds and in different octave bands and for comparison of different bearings tested with the same setup.
4. The airborne noise generally increases with speed. The exact relationship between airborne noise level and speed cannot be determined on the basis of available test results. There is, however, no indication that this relationship would greatly differ from the corresponding relationship between structure borne vibration level and speed.

Detailed results are given in Progress Report No. 17 (1).

2.3.17 Additional Parameters Affecting the Vibration and Airborne Noise of Large Roller Bearings

In addition to the various sources discussed in Sections 2.3.1 through 2.3.16, the vibration and airborne noise emitted by large roller bearings are influenced by a number of other parameters. Some of these parameters, such as lubrication, damage on the rolling surfaces, dirt and rust were discussed in 1.3.28 for ball bearings. The effect of these parameters for large roller bearings is judged to be similar to that for ball bearings. Other effects occur in large roller bearings, due to design and size differences between them and small ball bearings.

Some of these effects, typical of large roller bearings are:

a. The Contact Between Roller End and Race Flange:

The geometrical shape and micro-geometry of both the roller end and the flange may influence both the axial and radial vibrations of the bearings. These effects have not been studied.

b. Roller Skewing:

In applying the analytical results in Section 1.3 to large roller bearings, it was assumed that the axes of the rollers of the rotating bearing remain parallel to the axis of rotation of the bearing. This is not necessarily the case. The rollers may, at least under certain conditions, operate with their axes misaligned with respect to the bearing axis. The skewing of the rollers is likely to affect the vibration characteristics of the bearings. The amount of roller skewing is influenced by factors such as flange and roller geometry and surface micro-geometry, roller crowning and conformity, cage geometry, lubrication, load and speed.

c. Effects Peculiar to Double Row Bearings:

Some of these effects were discussed in 2.3.7. Additional effects may be induced by inaccuracies in the relative location of the two race ways (the grooves may be displaced radially or angularly with respect to each other) or by dimensional differences between the two grooves. The fact that under many loading conditions the two rows are not equally loaded also influences the vibration of the bearing.

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d. Cage Effects:

The heavy cages generally used in large roller bearings may generate forces of high enough magnitude to significantly affect the bearing vibration. The cage also influences roller skewing. It may directly emit air-borne noise which is not reflected on vibrations of the outer ring.

2.3.18 Comparison of Different Types of Large Roller Bearings

Enclosures 100 and 101 give a comparison of observed vibration levels of spherical, cylindrical and tapered roller bearings. Enclosure 100 shows the vibration levels of standard production quality bearings manufactured with polished or ground race grooves and rollers. The bearings shown on Enclosure 101 are of improved quality. The improved spherical 23240 roller bearing is manufactured with honed inner race groove, polished outer race groove and honed rollers. The improved NJ240 cylindrical roller bearing is manufactured with honed inner and outer race grooves and lapped rollers. The improved 96900/96140 tapered roller bearing was loaned to the SKF Industries, Inc. Research Laboratory by the U. S. Naval Engineering Experiment Station, Annapolis, Maryland with the indication that it represents the best quiet running quality tapered roller bearings. The octave band readings shown graphically in Enclosures 100 and 101 are tabulated in Enclosures 90-92.

From Enclosures 100 and 101, it is obvious that the tapered roller bearing tested has the highest vibration levels of the standard quality bearings. This bearing reads more than five times higher than the two other bearing types in the 400-6400 cps range. The spherical roller bearing has the lowest vibration levels in most bands and the cylindrical bearing reads up to four times higher than the spherical in the frequency range below 1600 cps.

A comparison of the vibration levels of the improved bearings presented in Enclosure 101 shows that the improved tapered roller bearing tested reads as much as four times higher than the other two bearings of improved quality. The spherical roller bearing again has the lowest vibration levels of the three bearing types. Although the improved cylindrical bearing reads higher than the spherical bearing, the difference in vibration level between the two is less than in the case of the corresponding standard bearings.

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From the waviness test results of the standard and improved bearings presented in Enclosures 87-89, it is seen that the spherical bearing parts in general have lower waviness than the others while the tapered roller bearing parts read the highest.

To determine whether the differences between the vibration levels of the three bearing types are explainable by the differences in their waviness levels only, the following equation may be used:

$$\frac{V_x}{V_y} = \frac{(f_c)_x \sqrt{Z_y} [W_o^2 + (\frac{U_i}{U_o})^2 W_i^2 + (\frac{U_b}{U_o})^2 W_b^2]_x^{\frac{1}{2}}}{(f_c)_y \sqrt{Z_x} [W_o^2 + (\frac{U_i}{U_o})^2 W_i^2 + (\frac{U_b}{U_o})^2 W_b^2]_y^{\frac{1}{2}}} \quad (2.3.18-1)$$

where the subscripts x and y refer to the two bearing types being compared, e.g., spherical, cylindrical or tapered roller bearings.

f_c is the rotational frequency of the cage, Z is the total number of rolling elements in the bearing.

V = the theoretical RMS value of the bearing vibration in any given frequency band.

W_o, W_i, W_b = outer ring, inner ring and rolling element waviness, measured in the frequency bands corresponding to the vibration frequency band.

U_o, U_i, U_b = velocity bandwidth amplification factors, relating waviness velocity amplitudes to vibration amplitudes.

If the theoretical ratio V_x/V_y is not significantly different from the corresponding experimental ratio, it may be concluded that the differences in the vibration level of the various types of bearings are due to differences in waviness levels only. If, however, the theoretical and experimental ratios are significantly different, effects other than those predictable from rigid ring theory enter. In this case differences in waviness are not sufficient to explain the differences in vibration level; or there are waviness effects not covered by the rigid ring theory (such as flexural outer ring vibrations, roller end and flange effects, unaccounted-for double-row effects, etc).

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The theoretical and experimental V_x/V_y ratios are given in Enclosure 102, for the standard production and improved bearings. The ratios between the theoretical and experimental vibration ratios are also given in this enclosure.

Examination of Enclosure 102 shows that for the bearings of improved quality, the experimental and theoretical vibration ratios generally are in good agreement except for the 50-100 cps band where the spherical bearing reads higher than expected. For the standard quality bearings the experimental ratio between cylindrical and spherical roller bearings is two to five times as high as the theoretical and the experimental ratio between tapered and spherical roller bearings 1.3 to thirteen times as high as the theoretical in the 100-1600 cps range. These results seem to indicate that, for bearings with comparatively high waviness values (standard production), effects other than those predictable from rigid ring theory occur. For the lower waviness levels of the improved bearings, rigid ring theory applies reasonably well. This may be explainable by large imperfections, other than rolling path waviness, in the standard production bearings.

It can be concluded that a valid, reasonably accurate comparison of the vibration levels of bearings of the same geometry can be made on the basis of their waviness readings alone, by applying the rigid ring theory. A comparison of bearings of different geometry becomes more complex. Due to the design differences, the vibration transmission characteristics of the bearings are different which must be taken into account along with the waviness effects.

In the present comparisons of vibration levels the effect of flange waviness of the cylindrical and tapered roller bearings was not considered. Effects due to such causes as roller skewing, the exact influence of the double row of rollers instead of a single row and flexural outer ring vibrations have not been taken into account. These effects may depend not only on the design and operating conditions but also on the waviness levels.

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Even with these qualifications, the comparison between the three types of large roller bearings tested seems to indicate that the differences between the vibration levels of the three types of bearings cannot be fully explained by differences in the measured waviness levels alone. It would appear that the spherical roller bearing of standard production quality offers some advantages over the two other types of bearings of corresponding quality. in addition to the advantage due to the double row effect discussed in 2.3.7. For the bearings of improved quality this additional advantage of the spherical roller bearing is not apparent.

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PART III

DESIGN GUIDELINES

3.1 INTRODUCTION

The parameters influencing the structure borne vibrations and airborne noise, generated by a rolling bearing mounted in a piece of machinery, may be divided into the following three main categories:

1. Bearing parameters, such as dimensions, number of balls, macro and micro-geometry.
2. Operating parameters, such as load and speed.
3. Mounting parameters, such as masses attached to the bearing, the elastic properties of these masses, and housing and bore fit and geometry.

In developing a bearing with quiet running characteristics in a given application, these parameters must all be considered. By proper combination of the parameters it is usually possible to reduce the vibrations at least in selected critical frequency ranges below the levels obtainable without parameter adjustment.

The operating parameters (load and speed) in any given application, are, of course, selected mostly on the basis of considerations other than vibration. The sole operating parameter that can often be selected on the basis of bearing vibration requirements is axial preload. Still, preload adjustments can yield substantial advantages in vibration level. Aside from preload, the parameters available for adjustment to obtain quiet running, are the bearing parameters and mounting parameters.

In Part III of this report the effects of the various parameters on bearing vibration are reviewed and guide-lines given for selection of these parameters. The guide lines apply, in general, to large roller bearings as well as to the smaller ball bearings. Whenever this is not the case, it has been mentioned.

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In Section 3.2 the bearing parameters are discussed. The effects of the various bearing parameters on bearing vibration is summarized in 3.2.1. The parameters investigated include bearing design parameters such as the number and size of balls, ring dimensions, groove conformity and contact angle, as well as manufacturing parameters (micro-geometry of the rolling surfaces). Guide lines for the selection of bearing parameters are given in 3.2.2.

The operating parameters are discussed in Section 3.3. The effects of load are reviewed in 3.3.1 and a brief discussion on the effect of speed is given in 3.3.2.

In Section 3.5, a numerical example is given of designing a bearing to meet given vibration limits.

3.2 BEARING PARAMETERS

3.2.1 Summary of the Effects of Bearing Parameters on Vibration

1. Boundary Dimensions:

Boundary dimensions are usually determined by considerations other than the quiet running characteristics of the bearing, such as bearing load capacity and available space.

Since, however, the absolute magnitudes of the micro-geometrical imperfections generally increase with bearing size and since the total vibratory energy of a bearing also increases with size, it is usually advantageous for vibration control to select a bearing of as small a size as permitted by other considerations, even though specific bearing load will be higher.

2. Bearing Geometry:

a. Number of Rolling Elements Z :

The number of rolling elements has the following effects:

1. The bearing vibration level due to rigid ring motion generated by ring and rolling element waviness is approximately proportional to $\sqrt{1/Z}$, where Z is the total number of rolling elements (including both rows of a double-row bearing). An increase in the number of rolling elements therefore results in a reduced vibration level. This affects the vibration level in all frequency bands.
2. The amplitudes of the flexural vibrations due to rolling body loads are approximately proportional to $1/Z^3$. The amplitudes of these vibrations are therefore highly reduced by increasing the number of rolling bodies. This is a low frequency effect; (for a bearing rotating at 1800 RPM a significant effect is expected only in the low Anderometer band, 50-300 cps).
3. The amplitudes of the flexural vibrations induced by low order inner ring waviness increase with the number of rolling elements. For inner ring two point out-of-roundness these amplitudes are proportional to the quantity $T_Y = \frac{1}{1+\gamma}$ where γ is given in Equation (1.3.18-2). Since γ is proportional to the quantity $k_N Z$ and since (for ball bearings), for a given axial load F_A , is proportional to $Z^{-2/3}$, then γ for any given load F_A is proportional to $Z^{1/3}$ and T_Y proportional to the quantity $\frac{C Z^{1/3}}{1 + C Z^{1/3}}$ (where C is a constant) from which follows

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that the amplitudes of the flexural vibrations induced by 2 wpc inner ring waviness increase with number of balls.

This effect of 2 wpc inner ring waviness occurs at a vibration frequency equal to twice the rotational frequency of the bearing.

The effect of inner ring 3 wpc on the flexural outer ring vibrations is similar, although the value of T_y is different. Assuming the axial load constant, the maximum radial deflection of a ring under three equally spaced loads is approximately 1/6 of that under two equally spaced loads. An order of magnitude estimate of the 3 wpc inner ring waviness effect may be then obtained by using a value of γ in Equation (1.3.18-1) which is 1/6 of the value applicable to 2 wpc. This approximate value of γ for 3 wpc inner ring waviness is

$$\gamma_3 \approx \frac{2k_n R^3}{108 \pi E I} \quad (3.2.1-1)$$

The flexural vibrations induced by 3 wpc inner ring waviness occur at three times the rotational frequency. Since T_y for 3 wpc inner ring waviness is smaller than that of 2 wpc the effect of 3 wpc is generally smaller than that of 2 wpc inner ring waviness. The effect of the number of balls Z in the 3 wpc case is similar to that of the 2 wpc case, but the differences in the value of T_y must be taken into account in estimating the relative effects in both cases. The effects of higher order of inner ring waviness may be estimated using an approach similar to the one used for 3 wpc. The influence of inner ring waviness of orders higher than 3 wpc is, however, in most cases small, and can be neglected unless the waviness of the orders 4, 5, 6 ... is very predominant.

4. The natural frequencies of the rigid body vibrations are proportional to \sqrt{Z} . The natural frequencies of the flexural vibrations of the outer ring are approximately proportional to \sqrt{Z} . An increase in the number of rolling bodies therefore increases the natural frequencies of the bearing.

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5. The number of rolling elements affects the variable elastic compliance of the bearing. The expected overall effect of an increase in the number of rolling elements is a reduction of the amplitude of the variable compliance vibrations. The frequency of each harmonic increases proportionately with the number of rolling elements.

To evaluate the overall effect of Z , due to the various sources listed above, it is necessary to consider not only the various amplification factors but also the relative magnitudes of the vibrations generated by the different sources. For this purpose, consider a bearing with Z rolling elements. The total vibration level in a comparatively wide band, in a frequency range influenced both by rigid ring and flexural vibrations, is \bar{V} . The vibration level of the rigid ring vibration in this band is \bar{V}_R , the vibration level of flexural vibrations, due to two and three wpc inner ring waviness \bar{V}_{Y2} , and \bar{V}_{Y3} , respectively and the vibration level of flexural outer ring vibrations due to ball loads is \bar{V}_F . Variable compliance vibrations are not being considered since the bearing is assumed to operate under axial load. If the number of rolling elements is changed from Z to Z then the vibration levels listed above change to V , V_R , V_{Y2} , V_{Y3} and V_F , respectively.

The following equation relates the vibration level V_{Y2} to the level \bar{V}_{Y2} .

$$\frac{V_{Y2}}{\bar{V}_{Y2}} = \frac{T_{Y2}}{\bar{T}_{Y2}} \quad (3.2.1-2)$$

which using (1.3.18-2) may be expressed as

$$\frac{V_{Y2}}{\bar{V}_{Y2}} = \frac{\gamma_2(1+\bar{\gamma}_2)}{\bar{\gamma}_2(1+\gamma_2)} \quad (3.2.1-3)$$

or since γ is proportional to $Z^{2/3}$

$$\frac{V_{Y2}}{\bar{V}_{Y2}} = \left(\frac{Z}{\bar{Z}}\right)^{\frac{2}{3}} \frac{(1+\bar{\gamma}_2)}{[1+\bar{\gamma}_2(\frac{Z}{\bar{Z}})^{\frac{2}{3}}]} \quad (3.2.1-4)$$

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Expressing \bar{V}_2 in terms of \bar{T}_{Y2} Equation (3.2.1-4) becomes

$$\frac{V_{Y2}}{\bar{V}_{Y2}} = \frac{\left(\frac{2}{Z}\right)^{3/2}}{1 - \bar{T}_{Y2} + \bar{T}_{Y2}\left(\frac{2}{Z}\right)^{3/2}} \quad (3.2.1-5)$$

which gives the relative change in the vibration level of the flexural vibrations induced by two wpc inner ring waviness, due to a change in the number of rolling elements from Z to Z .

The corresponding equation for three wpc inner ring waviness is

$$\frac{V_{Y3}}{\bar{V}_{Y3}} = \frac{\left(\frac{2}{Z}\right)^{3/2}}{1 - \bar{T}_{Y3} + \bar{T}_{Y3}\left(\frac{2}{Z}\right)^{3/2}} \quad (3.2.1-6)$$

The ratios $\frac{V_{Y2}}{\bar{V}_{Y2}}$ and $\frac{V_{Y3}}{\bar{V}_{Y3}}$ depend on the amplification factors \bar{T}_{Y2} and \bar{T}_{Y3} . These ratios can be computed provided that \bar{T}_{Y2} and \bar{T}_{Y3} are known. Enclosure 103 shows V_Y/\bar{V}_Y as a function of Z/Z for various values of \bar{T}_Y (V_Y can here be used to represent V_{Y2} or V_{Y3} and \bar{T}_Y to represent \bar{T}_{Y2} or \bar{T}_{Y3}).

The following equation gives the approximate relationship between V_R and \bar{V}_R .

$$\frac{V_R}{\bar{V}_R} = \left(\frac{2}{Z}\right)^{1/2} \quad (3.2.1-7)$$

and the relationship between V_F and \bar{V}_F is given by

$$\frac{V_F}{\bar{V}_F} = \left(\frac{2}{Z}\right)^3 \quad (3.2.1-8)$$

The total vibration level V may be obtained as the square root of the squared levels V_{Y2} , V_{Y3} , V_R and V_F . This may be expressed as

$$\left(\frac{V}{\bar{V}}\right)^2 = \left[\frac{\left(\frac{Z}{2}\right)^{2/3}}{1 - \bar{T}_{r2} + \bar{T}_{r2}\left(\frac{Z}{2}\right)^{2/3}} \right]^2 \left(\frac{\bar{V}_{r2}}{\bar{V}}\right)^2 + \left[\frac{\left(\frac{Z}{2}\right)^{2/3}}{1 - \bar{T}_{r3} + \bar{T}_{r3}\left(\frac{Z}{2}\right)^{2/3}} \right]^2 \left(\frac{\bar{V}_{r3}}{\bar{V}}\right)^2 + \frac{\bar{Z}}{2} \left(\frac{\bar{V}_R}{\bar{V}}\right)^2 + \left(\frac{\bar{Z}}{2}\right)^6 \left(\frac{\bar{V}_F}{\bar{V}}\right)^2 \quad \text{AL63L023} \quad (3.2.1-9)$$

From Equation (3.2.1-9) the overall effect of changing the number of rolling elements from \bar{Z} to \underline{Z} can be obtained, if \bar{T}_{r2} , \bar{T}_{r3} , \bar{V}_{r2}/\bar{V} , \bar{V}_{r3}/\bar{V} , \bar{V}_R/\bar{V} and \bar{V}_F/\bar{V} are known.

To find the optimum value of Z , i.e., the value of Z that gives a minimum value of V/\bar{V} a graphical method may conveniently be used. This is illustrated by Enclosure 104 which shows $(\bar{V}_{r2}/\bar{V})^2$, $(\bar{V}_{r3}/\bar{V})^2$, $(\bar{V}_R/\bar{V})^2$ and $(\bar{V}_F/\bar{V})^2$ plotted as functions of Z/\bar{Z} . The total squared vibration level represented by $(V/\bar{V})^2$ is also shown. The following values were selected for this example:

$$\begin{aligned} \left(\frac{\bar{V}_{r2}}{\bar{V}}\right)^2 &= \frac{1}{3} & \left(\frac{\bar{V}_R}{\bar{V}}\right)^2 &= \frac{1}{3} & \bar{T}_{r2} &= \frac{4}{5} \\ \left(\frac{\bar{V}_{r3}}{\bar{V}}\right)^2 &= \frac{1}{6} & \left(\frac{\bar{V}_F}{\bar{V}}\right)^2 &= \frac{1}{6} & \bar{T}_{r3} &= \frac{2}{5} \end{aligned} \quad (3.2.1-10)$$

It is seen that for the specific example used in Enclosure 104 $(V/\bar{V})^2$ has a minimum at $Z/\bar{Z} = 1.7$.

Equation (3.2.1-9) and Enclosure 104 apply only if Z is the only bearing parameter which is changed. If rolling element size or other bearing dimensions are also changed this must be taken into account as will be shown later. It should, of course, be realized that, due to design difficulties or other reasons, it may not always be possible to use the optimum value of Z .

b. Rolling Body (Ball or Roller) Diameter D and Diameter of Rolling Body Pitch Circle, d

The rolling body size and diameter of the rolling body pitch circle have the following effects:

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1. A change in rolling body size and pitch circle diameter affects the rigid ring vibration frequencies generated by given orders of waviness. The most predominant effect of this is on the vibration frequencies generated by rolling body waviness. These are approximately inversely proportional to the rolling body diameter. A reduction in rolling body diameter therefore increases the vibration frequencies generated by given orders of rolling body waviness; it also increases the vibration frequencies generated by outer ring waviness and reduces those generated by inner race waviness, but these changes are usually much smaller than those induced by rolling body waviness.

The changes in the rigid body vibration frequencies also result in changes in the amplification factors U_o, U_i and \bar{U} . Consider a bearing with rolling body diameter \bar{D} and rolling body pitch circle diameter d . If the rolling body diameter is changed to D , with other dimensions (including d), remaining the same, the following ratios between the amplification factors for the bearing with rolling body diameter \bar{D} and for the bearing with rolling body diameter D are obtained:

$$\frac{U_o}{\bar{U}_o} = \frac{f_c}{\bar{f}_c} = \frac{1 - \frac{D}{d} \cos \alpha}{1 - \frac{\bar{D}}{d} \cos \alpha} \quad (3.2.1-11)$$

or

$$\frac{U_o}{\bar{U}_o} = \frac{\frac{d}{D \cos \alpha} - \frac{D}{\bar{D}}}{\frac{d}{\bar{D} \cos \alpha} - 1} \quad (3.2.1-12)$$

$$\frac{U_i}{\bar{U}_i} = \frac{f_i}{\bar{f}_i} = \frac{\frac{d}{D \cos \alpha} + \frac{D}{\bar{D}}}{\frac{d}{\bar{D} \cos \alpha} + 1} \quad (3.2.1-13)$$

$$\frac{U_b}{U_b} = \frac{f_b}{f_b} = \frac{\bar{D} \left(1 - \frac{D^2}{d^2} \cos^2 \alpha\right)}{D \left(1 - \frac{D^2}{d^2} \cos^2 \alpha\right)} \quad (3.2.1-14)$$

or, since usually $\left(\frac{D}{d} \cos \alpha\right)^2 \ll 1$,

$$\frac{U_b}{U_b} \approx \frac{\bar{D}}{D} \quad (3.2.1-15)$$

From Equation (3.2.1-11) through (3.2.1-15) the effect of variation in rolling body diameter on the rigid ring vibration level can be computed for any given vibration frequency band, if the waviness level in the corresponding waviness band is known. Since the change in rolling body diameter affects the vibration frequencies generated by given orders of waviness, the waviness bands for the two bearings (with rolling body diameters \bar{D} and D) are not the same, and the two waviness levels to be compared are therefore not necessarily the same.

Denoting the two waviness levels by \bar{w}_0 and \bar{w}_0 , \bar{w}_i and \bar{w}_i , \bar{w}_b and \bar{w}_b for outer ring, inner ring and rolling body waviness, respectively, the following equations hold for the rigid ring vibration levels

$$\frac{V}{\bar{V}_0} = \frac{U_0 \bar{w}_0}{\bar{U}_0 \bar{w}_0} \quad (3.2.1-16)$$

$$\frac{V_i}{\bar{V}_i} = \frac{U_i \bar{w}_i}{\bar{U}_i \bar{w}_i} \quad (3.2.1-17)$$

$$\frac{V_b}{\bar{V}_b} = \frac{U_b \bar{w}_b}{\bar{U}_b \bar{w}_b} \quad (3.2.1-18)$$

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These equations do not take into account the effect of changes in resonant frequencies due to the change in rolling element diameter.

If

$$\frac{W_o}{W_o} = \frac{W_i}{W_i} = \frac{W_b}{W_b} = 1 \quad (3.2.1-19)$$

it follows from Equation (3.2.1-11) through (3.2.1-18) that an increase in rolling body diameter

- a. reduces the rigid ring vibration level induced by outer ring waviness.
- b. increases the rigid ring vibration level induced by inner ring waviness.
- c. reduces the rigid ring vibration level induced by rolling body waviness.

Since, generally $\frac{D}{d} \cos \alpha$ is small compared to 1, the effect (c) is much stronger than (a) and (b). The statements (a), (b) and (c) are valid under the assumptions of Equation (3.2.1-19). These assumptions are, of course, never quite exact. In actual bearings there generally appears to be a tendency for the outer and inner ring waviness level to decrease with increasing frequency and for ball waviness to increase with frequency, i.e.,

$$\frac{W_o}{W_o} < 1 ; \frac{W_i}{W_i} > 1 ; \frac{W_b}{W_b} > 1 \quad (3.2.1-20)$$

for $\frac{D}{d} < 1$

and

$$\frac{W_o}{W_o} > 1 ; \frac{W_i}{W_i} < 1 ; \frac{W_b}{W_b} < 1 \quad (3.2.1-21)$$

for $\frac{D}{d} > 1$

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Under the conditions of Equation (3.2.1-20) the effect of D on V_o and V_i is not quite as strong as suggested by Equations (3.2.1-12) and (3.2.1-13). Statements (a) and (b) still hold if

$$\frac{\frac{d}{B \cos \alpha} - 1}{\frac{d}{B \cos \alpha} + \frac{D}{B}} > \frac{V_o}{W_o}$$

$$\frac{\frac{d}{B \cos \alpha} + 1}{\frac{d}{B \cos \alpha} + \frac{D}{B}} < \frac{W_i}{V_i} \quad (3.2.1-22)$$

If Equation (3.2.1-20) is valid, the statement (c) is even stronger than suggested by Equation (3.2.1-13).

The effect of rolling body pitch circle diameter d , is small, in general, since for given boundary dimensions possible variations in d are usually too small to have a significant effect on V_o , V_i or V_b .

2. The linearized Hertzian coefficient k_N of ball bearings increases with ball size (See Enclosure 19) for constant thrust load. Since γ is according to Equation (1.3.10-2) proportional to k_N , it follows that a reduction in ball size results in a reduction of the amplitude of the flexural vibrations induced by low order (2 wpc and 3 wpc) inner ring waviness in ball bearings. The same tendency applies to roller bearings, although it is less pronounced because the relationship between roller diameter and k_N is different from that for ball bearings.

The flexural vibration levels V_{r2} , V_{r3} , \bar{V}_{r2} and \bar{V}_{r3} are related to amplification factors T_{r2} , T_{r3} , \bar{T}_{r2} and \bar{T}_{r3} as follows:

$$\frac{V_{r2}}{\bar{V}_{r2}} = \frac{T_{r2}}{\bar{T}_{r2}} \quad (3.2.1-23)$$

and

$$\frac{V_{r3}}{\bar{V}_{r3}} = \frac{T_{r3}}{\bar{T}_{r3}} \quad (3.2.1-24)$$

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Here the quantities \bar{V}_{Y2} , \bar{V}_{Y3} , \bar{T}_{Y2} and \bar{T}_{Y3} refer to a bearing with rolling body diameter \bar{D} while the quantities V_{Y2} , V_{Y3} , T_{Y2} and T_{Y3} refer to a bearing with the same dimension d , but with rolling body diameter D .

The ratios T_{Y2}/\bar{T}_{Y2} and T_{Y3}/\bar{T}_{Y3} may, in analogy with Equations (3.2.1-4) and (3.2.1-5), be expressed in terms of the ratio k_N/\bar{k}_N as follows:

$$\frac{T_Y}{\bar{T}_Y} = \frac{k_N/\bar{k}_N}{1 - \bar{T}_Y + \bar{T}_Y \frac{k_N}{\bar{k}_N}} \quad (3.2.1-25)$$

where T_Y and \bar{T}_Y apply to Y_2 as well to Y_3 ; and k_N and \bar{k}_N are the corresponding linearized Hertzian coefficients. The ratio T_Y/\bar{T}_Y can be expressed as a function of D/\bar{D} . To do that, it is first necessary to express k_N/\bar{k}_N as a function of D/\bar{D} which can be done by the use of Enclosure 19, where the quantity is plotted as a function of D for various conformities. For a given load, number of rolling elements, conformity and contact angle, β_N is proportional to k_N ; the ratio $\beta_N/\bar{\beta}_N$ equals the ratio k_N/\bar{k}_N , and T_{Y2}/\bar{T}_{Y2} and T_{Y3}/\bar{T}_{Y3} can then according to Equation (3.2.1-25) be plotted as a function of D/\bar{D} . Such plots are shown on Enclosure 105 for $\bar{D} = 1/2"$, $\bar{T}_{Y2} = 4/5$ and $\bar{T}_{Y3} = 4/10$.

3. Due to the effect of rolling element size on k_N the natural frequencies of the bearing decrease with rolling body size. This applies both to flexural and rigid body vibrations.
4. The variable compliance vibrations depend on the Hertzian coefficient k_N , which again is a function of rolling body size. The exact effect depends, however, on the load, radial looseness and bearing dimensions, and cannot be determined without detailed analysis.

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To compute the overall effect of changes in the rolling body diameter D on the vibration level in a given frequency band, it is again necessary to know the relative magnitudes of the vibration level of the flexural vibrations, and of the rigid ring vibrations. It is also necessary to know the relative contribution from outer ring, inner ring and ball waviness, respectively, to the total rigid ring vibration level. Denoting the total vibration level of a bearing with rolling body diameter D , by \bar{V} , the vibration level of flexural vibrations due to 2 wpc inner ring waviness by \bar{V}_{r2} , that due to 3 wpc inner ring waviness by \bar{V}_{r3} , the rigid body vibrations generated by outer ring, inner ring and ball waviness by \bar{V}_o , \bar{V}_i , \bar{V}_b , respectively, and the level of flexural vibrations due to ball loads by \bar{V}_F , then the following equation holds for a bearing with rolling body diameter D .

$$\left(\frac{\bar{V}}{\bar{V}_D}\right)^2 = \left(\frac{\bar{V}_{r2}}{\bar{V}_D}\right)^2 + \left(\frac{\bar{V}_{r3}}{\bar{V}_D}\right)^2 + \left(\frac{\bar{V}_o}{\bar{V}_D}\right)^2 + \left(\frac{\bar{V}_i}{\bar{V}_D}\right)^2 + \left(\frac{\bar{V}_b}{\bar{V}_D}\right)^2 + \left(\frac{\bar{V}_F}{\bar{V}_D}\right)^2 \quad (3.2.1-26)$$

where all quantities without bar refer to the bearing with rolling body diameter D .

The terms indicating the squared rigid body vibration levels may be computed from Equation (3.2.1-11) through (3.2.1-18), and the squared flexural vibration levels from Equations (3.2.1-23) through (3.2.1-25).

The vibration level \bar{V}_F is proportional to $1 - \frac{D}{d}$ and therefore follows the same trend as \bar{V}_o .

The terms in Equation (3.2.1-26) can now all be expressed as a function of D/B , if the relative magnitudes of \bar{V}_{r2} , \bar{V}_{r3} , \bar{V}_o , \bar{V}_i , \bar{V}_b and \bar{V}_F are known. To illustrate the use of Equation (3.2.1-26), it is assumed that the levels of flexural and rigid ring vibration are according to Equation (3.2.1-10) and also that the contributions from outer ring, inner ring and ball waviness to the total rigid ring vibration are equal. It is further assumed that Equation (3.2.1-19) is valid. Then the following relations hold.

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$$\left(\frac{\bar{V}_1}{\bar{V}}\right)^2 = \frac{1}{9}$$

$$\left(\frac{\bar{V}_2}{\bar{V}}\right)^2 = \frac{1}{3}$$

$$\left(\frac{\bar{V}_3}{\bar{V}}\right)^2 = \frac{1}{9}$$

$$\left(\frac{\bar{V}_{r2}}{\bar{V}}\right)^2 = \frac{1}{6}$$

$$\left(\frac{\bar{V}_p}{\bar{V}}\right)^2 = \frac{1}{9}$$

$$\bar{T}_{r2} = \frac{4}{5}$$

(3.2.1-27)

$$\left(\frac{\bar{V}_r}{\bar{V}}\right)^2 = \frac{1}{6}$$

$$\bar{T}_{r3} = \frac{2}{5}$$

Enclosure 106 is a graphical representation of Equation (3.2.1-26) showing the squared vibration levels due to the various sources, as a function of D/S . The curves shown are based on the curves of $\bar{T}_{r2}/\bar{T}_{r1}$ and $\bar{T}_{r3}/\bar{T}_{r1}$ as a function of D/S , shown on Enclosure 105, and on Equations (3.2.1-11) through (3.2.1-19) for rigid ring vibrations with a value of

$$\frac{d}{S \cos \alpha} = 4$$

(3.2.1-28)

It is seen from Enclosure 106, that for the example used, the function $(V/\bar{V})^2$ varies comparatively little for values of D/S between 1 and 3.5 but increases rapidly with decreasing D/S in the range of $D/S < 1$. $(V/\bar{V})^2$ has a minimum at $D/S \approx 2.3$, which in this case is likely to correspond to a diameter value outside the practical range.

c. Groove Conformity

Groove conformity affects the bearing vibration as follows:

1. The linearized Hertzian coefficient k_H is influenced by the groove conformity. As shown on Enclosure 19, k_H increases as the conformity is tightened. Since γ according to Equation (1.3.18-2) is proportional to k_H , a tight conformity results in increased flexural vibration levels induced by low order inner ring waviness (2 wpc and 3 wpc). The same tendency holds for roller bearings with point contact, although the relationship between k_H and conformity is different from that shown on Enclosure 19.

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The effect of k_n on the amplification factor T_y can be determined from Equation (3.2.1-25) and since the conformity is a function of k_n , T_y/\bar{T}_y may be plotted as a function of conformity. This has been done on Enclosure 107 for a bearing with 1/2" balls.

\bar{T}_y here refers to a bearing with 51.8% conformity for both inner and outer race, T_y to bearings with inner and outer ring conformities as indicated by the abscissa. Enclosure 107 has been plotted for several values of \bar{T}_y . It is seen that a loose conformity is always advantageous in reducing flexural vibrations.

2. Due to the effect of groove conformity on k_n , the natural frequencies of both rigid body and flexural vibrations increase as the conformity is tightened. This effect is small for the range of conformities conventionally used.
3. The variable compliance vibrations depend on the Hertzian coefficient c_d which is a function of conformity. The exact effect depends on the load, radial looseness and bearing dimensions, and cannot be determined without detailed analysis.

Since the band width amplification factors of the rigid ring vibration are not influenced by conformity, the only significant, predictable, effect of conformity is on the flexible vibrations due to low order waviness, which, as shown above, can be reduced by loosening the groove conformity.

d. Thickness of the Outer Ring

The outer ring dimensions influence the bearing vibration as follows:

1. The vibration induced by ball loads are inversely proportional to the flexural rigidity of the outer ring (I/R^3), where R is the mean radius of the ring and I is the second moment of area of the ring cross section. Increasing the rigidity reduces the flexural vibrations.

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2. The flexural vibrations induced by low order inner ring waviness are also inversely proportional to the rigidity I/R^3 , and can be reduced by increasing the rigidity.
3. The rigid body mode natural frequencies are inversely proportional to the square root of the mass of the outer ring M . These natural frequencies are reduced by increasing the mass of the outer ring.
4. The natural frequencies of the flexural vibrations of the outer ring are approximately proportional to $\frac{1}{R} \sqrt{\frac{E}{A}}$, where A is the area of the ring cross-section. It follows that these natural frequencies increase with the thickness of the ring.

The overall effect of the increased thickness of the outer ring is a reduction of the low frequency vibrations due to the increased flexural rigidity. The effect in the higher frequency range cannot be predicted without detailed analysis of the vibration sources and the natural frequencies of the bearings.

e. Radial Looseness:

The radial looseness affects the variable compliance vibrations, and it is possible to minimize these vibrations by proper selection of the radial looseness.

Since the radial looseness affects the contact angle it also has indirect effects on the bearing vibration, as will be seen below.

f. Contact Angle, α :

The contact angle, α has the following effects:

1. Since the axial vibration amplitudes are inversely proportional to $\sin \alpha$, the axial vibrations are highly influenced by changes in contact angle. Radial vibrations, which are proportional to $\cos \alpha$, are affected only to a minor degree, for contact angles generally used in radial bearings.

Angular vibrations are proportional to $\sin \alpha$, and are therefore influenced more than the radial vibrations, but less than the axial vibrations, by changes in α .

2. The resonant frequency of rigid body mode axial vibrations is proportional to $\sin \alpha$ and therefore increases with α . The resonant frequencies of the radial and angular vibrations are generally only slightly influenced by α .
3. For a given axial load F_A the ball loads are inversely proportional to $\sin \alpha$. Vibrations induced by ball loads therefore increase with decreasing α .
4. The flexural vibrations induced by 2 and 3 wpc waviness of the inner ring increase in amplitude with k_N , which again decreases with increasing contact angle. Therefore the flexural vibrations decrease with increasing contact angle.

g. Combined Effect of Bearing Design Parameters

The various bearing design parameters discussed above cannot generally be selected independently. For instance, the three important parameters: rolling body diameter D , number of rolling bodies Z and thickness of outer ring H are, for a bearing with given boundary dimensions and pitch circle diameter d , interrelated. To increase the number of rolling bodies Z , it is usually necessary to reduce the rolling body diameter D , which results in an increased outer ring thickness H .

It is advantageous for load carrying capacity to use the maximum number of rolling bodies of any given size that can be assembled into the bearing. Under these circumstances the product ZD is approximately constant, as long as d is kept constant.

The effect of the outer ring thickness on bearing vibration is due to its influence on the second area moment of the outer ring cross-section I . It is, of course, possible to compute the exact relationship between I and D . To illustrate the tendency a much simpler, approximate relationship will be derived as follows:

The outer ring thickness at the bottom of the groove is

$$H_G = G - \frac{1}{2}D \quad (3.2.1-29)$$

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where

$$G = \frac{1}{2}(d_o - d) \quad (3.2.1-30)$$

d_o = outer diameter of bearing

The outer ring thickness at the outer ring land is

$$H_L = G - (\frac{1}{2} - c_L)D \quad (3.2.1-31)$$

where c_L is the ratio between the land height and rolling body diameter. It is assumed that c_L is independent of D .

For ball bearings of good quiet running quality c_L is usually between 0.15 and 0.20.

I may be expressed as

$$I = \frac{BH^3}{12} \quad (3.2.1-32)$$

where B is the width of the bearing and H defined by

$$H = G - c_H D \quad (3.2.1-33)$$

where $\frac{1}{2} - c_L < c_H < \frac{1}{2}$

c_H is here assumed to be a constant, independent of D . This is usually not exactly true, but a good estimate of I as a function of D may be obtained in this manner if c_H is properly selected.

Of the three parameters D , z and I , only D and z affect the rigid ring vibrations (except for shifts in resonant frequencies). Due to the combined effects of z and D , Equations (3.2.1-12), (3.2.1-13) and (3.2.1-15) are modified as follows, taking into account the effect of z as expressed by Equation (3.2.1-7) and the fact that zD is constant:

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$$\frac{\bar{U}_o}{\bar{U}_b} = \frac{\frac{d}{\bar{D} \cos \alpha} - \frac{D}{\bar{D}}}{\frac{d}{\bar{D} \cos \alpha} - 1} \sqrt{\frac{\bar{D}}{D}} \quad (3.2.1-32)$$

$$\frac{\bar{U}_i}{\bar{U}_b} = \frac{\frac{d}{\bar{D} \cos \alpha} + \frac{D}{\bar{D}}}{\frac{d}{\bar{D} \cos \alpha} + 1} \sqrt{\frac{\bar{D}}{D}} \quad (3.2.1-33)$$

$$\frac{\bar{U}_b}{\bar{U}_b} = \sqrt{\frac{\bar{D}}{D}} \quad (3.2.1-34)$$

The quantities with bars refer to a bearing with \bar{Z} rolling bodies, rolling body diameter \bar{D} , and outer ring thickness \bar{H} (as defined by Equation (3.2.1-32)). This bearing is used as reference, and its vibration level is being compared to that of a bearing with rolling element diameter D , Z rolling bodies and outer ring thickness H .

The flexural vibrations induced by 2 and 3 wpc inner ring waviness are influenced by both Z , D and H . The following equation holds for the parameter γ , defined for 2 wpc by Equation (1.3.18-2) and for 3 wpc by Equation (3.2.1-1).

$$\frac{\gamma}{\bar{\gamma}} = \frac{k_v Z \bar{I}}{k_v \bar{Z} I} = \frac{k_v \bar{D} \bar{I}}{k_v D I} \quad (3.2.1-35)$$

assuming the change in R to be negligible.

The ratio I/\bar{I} may according to Equation (3.2.1-33) be written as

$$\frac{I}{\bar{I}} = \left(\frac{G - c_H D}{G - c_H \bar{D}} \right)^3 \quad (3.2.1-36)$$

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or expressing G in terms of \bar{D} as

$$G = c_a \bar{D} \quad (3.2.1-37)$$

$$\frac{\bar{I}}{\bar{I}} = \left[\frac{c_a \bar{D} - c_h \bar{D}}{(c_a - c_h) \bar{D}} \right]^3 = \left[\frac{c_a - c_h (\frac{D}{\bar{D}})}{c_a - c_h} \right]^3 \quad (3.2.1-38)$$

The ratio k_N/k_N may according to Equation (1.3.7-5) be expressed as

$$\frac{k_N}{k_N} = \frac{\beta_N}{\beta_N} \left(\frac{\bar{Z}}{\bar{Z}} \right)^{\frac{1}{2}} = \frac{\beta_N}{\beta_N} \left(\frac{D}{\bar{D}} \right)^{\frac{1}{2}} \quad (3.2.1-39)$$

β_N as a function of ball diameter is shown graphically on Enclosure 19.

The ratio $\gamma/\bar{\gamma}$ may now, using Equations (3.2.1-38) and (3.2.1-39), be written as

$$\frac{\gamma}{\bar{\gamma}} = \frac{\beta_N}{\beta_N} \left(\frac{D}{\bar{D}} \right)^{\frac{1}{2}} \left[\frac{c_a - c_h}{c_a - c_h (\frac{D}{\bar{D}})} \right]^3 \quad (3.2.1-40)$$

Since β_N , for a given conformity, axial load, contact angle and number of balls, is a function of ball diameter D only, Equation (3.2.1-40) gives $\gamma/\bar{\gamma}$ as a function of D/\bar{D} only. The corresponding ratio T_Y/\bar{T}_Y may be computed from

$$\frac{T_Y}{\bar{T}_Y} = \frac{\frac{\gamma}{\bar{\gamma}}}{1 - \bar{T}_Y + \bar{T}_Y \frac{\gamma}{\bar{\gamma}}} = \frac{V_Y}{\bar{V}_Y} \quad (3.2.1-41)$$

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The flexural vibrations due to 2 wpc and 3 wpc inner ring waviness can be computed from Equations (3.2.1-40) and (3.2.1-41) as a function of ball diameter.

The flexural vibrations due to ball loads follows approximately the equation

$$\frac{V_F}{\bar{V}_F} = \frac{\bar{I}}{I} \left(\frac{\bar{z}}{z} \right)^3 \frac{1 - \frac{D}{d} \cos \alpha}{1 - \frac{D}{d} \cos \alpha} \quad (3.2.1-42)$$

which, using Equations (3.2.1-30) and (3.2.1-12) may be expressed as

$$\frac{V_F}{\bar{V}_F} = \left[\frac{(C_0 - C_H) \left(\frac{D}{B} \right)^3}{C_0 - C_H \left(\frac{D}{B} \right)} \right] \left[\frac{\frac{d}{D} \cos \alpha - \frac{D}{B}}{\frac{d}{B} \cos \alpha - 1} \right] \quad (3.2.1-43)$$

Enclosure 10B shows the squared vibration level ratios $(V_0/\bar{V})^2, (V_1/\bar{V})^2, (V_2/\bar{V})^2, (V_{r2}/\bar{V})^2, (V_{r3}/\bar{V})^2, (V_F/\bar{V})^2$ and $(V/\bar{V})^2$ plotted as a function of D/\bar{D} . The corresponding z/\bar{z} and H/\bar{H} ratios are also marked on the abscissa.

It is seen that the $(V/\bar{V})^2$ function has a sharp minimum at $D/\bar{D} \approx 0.5, z/\bar{z} \approx 2, H/\bar{H} \approx 1.5$

3. Micro-geometry:

Waviness generates both rigid ring and flexural vibrations. The vibration frequencies generated by various orders of waviness, according to rigid ring theory, are listed in Enclosure 20. The ranges of waviness orders that generate vibrations in wide frequency bands may be computed from Equations (1.3.11-1) through (1.3.11-3) and are tabulated in Enclosures 28 and 29 for ball bearings in the 6200 and 6300 series and in Enclosures 75, 76 and 77 for the large roller bearings used in the tests reported.

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The vibration amplitudes of each harmonic generated by waviness is proportional to the corresponding amplitude of waviness. This proportionality also holds approximately for vibrations measured in wide frequency bands (except for vibrations induced by "the geometrically perfect bearing"). The relationship between the vibration level, measured in a finite band, and the corresponding waviness level may be expressed by means of the band width amplification factors. Amplification factors for rigid ring vibrations are given by Equations (1.3.17-2) through (1.3.17-7). Numerical values of band width amplification factors of rigid ring vibrations are tabulated in Enclosure 39 for a few ball bearing sizes and in Enclosure 79 for a few large roller bearings.

The vibrations generated by outer ring, inner ring and ball waviness are additive by their squares, and the relative contribution from these three vibratory sources may be evaluated by means of the contribution factors given by Equations (1.3.22-5) through (1.3.22-7). In improving the quiet running quality of a bearing by reducing waviness, the most effective approach is usually to reduce the waviness of the part with the highest contribution factor.

Flexural vibrations induced by low order inner ring waviness occur at a frequency k times the rotational frequency where k is the order of waviness. Amplification factors for these vibrations are given for 2 wpc inner ring waviness by Equations (1.3.18-1) and (1.3.18-2), and for 3 wpc inner ring waviness by Equation (1.3.18-1) and (3.2.1-1). Higher orders of inner ring waviness generally have negligible effect on flexural vibrations. The main effect of outer ring waviness on the flexural vibrations, is to induce vibrations at the ball passage frequency over the outer ring. This effect, on the basis of experimental evidence, appears to be much smaller than that of inner ring waviness; amplification factors for these vibrations have not been computed.

3.2.2 Selection of Bearing Parameters

On the basis of the foregoing, selection of bearing parameters for low vibration levels may proceed as follows:

1. Select tentatively a basic bearing size on the basis of considerations other than vibration. This bearing will be used as a reference bearing for future comparisons, and the quantities related to this bearing will be denoted by a bar, e.g., the number of rolling bodies by \bar{Z} .
2. Specify the quiet running characteristics of the desired bearing (measured at a specified rotational speed, under a specified load and other specified conditions) such as
 - a. Approximate RMS vibration levels in wide frequency bands, octave bands or at specified discrete frequencies.
 - b. Frequency ranges where vibration peaks, due to resonances or other causes should be avoided.
 - c. Whether the main emphasis should be put on control of radial, axial or angular vibrations.
3. Compute the following bearing constants for the reference bearing:
 - \bar{A} = area of outer ring cross-section
 - \bar{M} = outer ring mass
 - \bar{R} = mean radius of outer ring
 - \bar{I} = second moment of area of outer ring cross-section
 - \bar{k}_N = linearized Hertzian coefficient. For ball bearings, \bar{k}_N may be obtained graphically from Enclosure 19. For roller bearings Equation (2.3.5-1) or (2.3.5-2) may be used. Enclosures 71 and 72 show values of \bar{k}_N for a few roller bearing sizes, plotted as a function of load.
4. Specify on the basis of available results on manufacturing capability of bearings in the size range considered, feasible values of parameters such as:
 - a. Waviness of outer ring \bar{w}_o , inner ring \bar{w}_i , and rolling bodies \bar{w}_b , measured in octave bands (or if feasible in wider bands corresponding to specified vibration bands)

* Feasibility of the wide band approach depends on the absence of resonances in the band and on approximate uniformity of the waviness spectrum.

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- b. Inner ring eccentricity \bar{w}_{i1}
- c. Inner ring 2 wpc waviness \bar{w}_{i2}
- d. Inner ring 3 wpc waviness \bar{w}_{i3}
- e. Rolling body diameter variation \bar{D}_v

5. Compute the approximate waviness bands corresponding to the vibration frequency bands selected in 2a, for the reference bearing selected in 1, from the equations.

$$\bar{k}_o = \frac{f}{f_c} \quad (3.2.2-1)$$

$$\bar{k}_i = \frac{f}{f_i} \quad (3.2.2-2)$$

$$\bar{k}_b = \frac{f}{f_b} \quad (3.2.2-3)$$

where \bar{k}_o , \bar{k}_i and \bar{k}_b are the orders of inner ring, outer ring and rolling body waviness, respectively, generating vibrations of frequency f cps, and

$$\bar{f}_c = \frac{f_r}{2} \left(1 - \frac{\bar{D}}{d} \cos \bar{\alpha} \right) \quad (3.2.2-4)$$

$$\bar{f}_i = \frac{f_r}{2} \left(1 + \frac{\bar{D}}{d} \cos \bar{\alpha} \right) \quad (3.2.2-5)$$

$$\bar{f}_b = \frac{f_r}{2} \frac{d}{\bar{D}} \left(1 - \frac{\bar{D}^2}{d^2} \cos^2 \bar{\alpha} \right) \quad (3.2.2-6)$$

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d = rolling body diameter
 d_r = diameter of rolling body pitch circle
 α = contact angle
 f_r = rotational frequency = $\frac{N}{60}$
 N = specified rotational speed, in vibration testing, RPM

For the bearings listed in Enclosures 28, 29, 75, 76 or 77, the values tabulated in these enclosures may be used instead of Equations (3.2.2-1) through (3.2.2-3).

If the waviness bands selected do not coincide with waviness bands used in actual waviness testing, select actual waviness bands which agree closest with the computed bands. If the computed band is wider than the actual bands, a combination of two or more of the actual bands may be selected as the waviness band corresponding to any given vibration band.

6. Compute the bandwidth amplification factors \bar{U}_0 , \bar{U}_i and \bar{U}_b of radial vibrations for the reference bearing from the equations

$$\bar{U}_0 = \sqrt{\frac{2}{Z}} \frac{\bar{f}_c}{f_{w0}} \cos \bar{\alpha} \quad (3.2.2-7)$$

$$\bar{U}_i = \sqrt{\frac{2}{Z}} \frac{\bar{f}_i}{f_{wi}} \cos \bar{\alpha} \quad (3.2.2-8)$$

$$\bar{U}_b = \sqrt{\frac{2}{Z}} \frac{\bar{f}_b}{f_{wb}} \cos \bar{\alpha} \quad (3.2.2-9)$$

where f_{w0} , f_{wi} and f_{wb} are the rotational frequencies used in the waviness testing of outer ring, inner ring and rolling bodies, respectively, and Z is the total number of rolling bodies in the bearing.

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Similar expressions may be written for axial and angular vibrations. These have not been included here, and will not be treated in the sequence, since they generally are of less interest.

Instead of using Equations (3.2.2-7) through (3.2.2-9) the numerical values of Enclosures 39 and 79 may be used for the bearing sizes tabulated in these enclosures.

7. Compute natural frequencies of the bearing from the equation

$$\bar{f}_n = \frac{1}{\pi} \left[\frac{(\eta^2 - 1)^2 \frac{\pi E \bar{I}}{R^3} + \frac{\bar{k}_n \bar{I}}{2}}{\bar{M} (1 + 1/\eta^2)} \right]^{\frac{1}{2}} \quad (3.2.2-10)$$

where

\bar{f}_n = natural frequency, cps

E = Young's modulus of elasticity

$\bar{R}, \bar{M}, \bar{I}, \bar{k}_n$ = as defined in 3.

$\eta = 1, 2, 3, \dots$ where $\eta = 1$ gives the natural frequency \bar{f}_R of radial and angular rigid body motion of the outer ring, $\eta = 2$ gives the natural frequency \bar{f}_F of the first mode of flexural vibrations, $\eta = 3$ gives the natural frequency \bar{f}_{F2} of the second mode of flexural vibrations, etc.

Equation (3.2.2-10) applies to values of $\eta \leq \frac{Z-1}{2}$.

Equation (3.2.2-10) applies if the outer ring is comparatively free to move radially and angularly, i.e., if the spring constant of the bearing is large compared to that of the mounting. If this is not the case the resonant frequencies of the rigid body motion may be computed according to Section 1.3.14.

8. Compute the "resonance amplification factor" \bar{U}_c for each vibration band from the equation

$$\bar{U}_c = (\bar{U}_{cR})^{\bar{\eta}_R} (\bar{U}_{cF})^{\bar{\eta}_F} \quad (3.2.2-11)$$

where

\bar{n}_R = the number of rigid body resonant frequencies within the vibration band.

\bar{n}_F = the number of flexural resonant frequencies within the vibration band.

U_{CR} and U_{CF} are given by Table 3.2.2-1

Table 3.2.2-1

Vibration Bandwidth in Octaves	U_{CR}	U_{CF}
1/3	4.0	1.8
1	2.4	1.4
2 1/2	1.8	1.2

For bandwidths other than those listed in Table 3.2.2-1, the values of U_{CR} and U_{CF} may be interpolated from Table 3.2.2-1 or the more detailed procedure in Section 1.3.24 may be used.

If any of the resonant frequencies is lower than five times the rotational frequency, the assumption on which Table 3.2.2-1 is based (k_f in Equation (1.3.24-12) $< 1/4$) may not hold. U_{CR} and U_{CF} must then be computed according to Section 1.3.24.

9. Compute the amplification factor \bar{T}_{Y2} for flexural vibration induced by 2 wpc inner ring waviness from the Equation

$$\bar{T}_{Y2} = \frac{\bar{Y}_2}{1 + \bar{Y}_2} \quad (3.2.2-12)$$

where

$$\bar{Y}_2 = \frac{\bar{z} \bar{k}_N \bar{R}^3}{18 \pi E \bar{I}} \quad (3.2.2-13)$$

and \bar{z} , \bar{k}_N , \bar{I} , \bar{R} as defined in Paragraphs 3 and 7.

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10. Compute the amplification factor \bar{T}_{Y3} for flexural vibrations induced by 3 wpc inner ring waviness from the equation

$$\bar{T}_{Y3} = \frac{\bar{Y}_3}{1 + \bar{Y}_3} \quad (3.2.2-14)$$

where

$$\bar{Y}_3 = \frac{\bar{z} \bar{k}_N \bar{R}^3}{108 \pi E I} \quad (3.2.2-15)$$

11. Compute the following theoretical vibration levels of the reference bearing:

a. Rigid ring vibrations, due to outer ring waviness

$$\bar{V}_o = \bar{U}_c \bar{U}_o \bar{W}_o \quad (3.2.2-16)$$

b. Rigid ring vibrations, due to inner ring waviness

$$\bar{V}_i = \bar{U}_c \bar{U}_i \bar{W}_i \quad (3.2.2-17)$$

c. Rigid ring vibrations, due to rolling body waviness

$$\bar{V}_b = \bar{U}_c \bar{U}_b \bar{W}_b \quad (3.2.2-18)$$

The values of \bar{W}_o , \bar{W}_i and \bar{W}_b in Equations (3.2.2-16), (3.2.2-17) and (3.2.2-18) are selected for the waviness bands determined according to 4. \bar{U}_o , \bar{U}_i and \bar{U}_b are obtained from Equations (3.2.2-7), (3.2.2-8) and (3.2.2-9). \bar{U}_c is computed according to paragraph 8.

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- d. Flexural vibrations at twice the rotational frequency induced by 2 wpc inner ring waviness

$$\bar{V}_{r2} = \bar{T}_{r2} \frac{2f_r}{f_{r2}} \bar{W}_{i2} \quad (3.2.2-19)$$

where \bar{W}_{i2} is the RMS amplitude of the 2 wpc inner ring waviness velocity, measured at a rotational frequency f_{r2} . \bar{T}_{r2} is given by Equation (3.2.2-12). f_r is the rotational speed of the bearing in vibration testing.

If the 2 wpc inner ring waviness is expressed as a RMS displacement $(\bar{W}_{i2})_d$

$$\bar{V}_{r2} = 4\pi f_r \bar{T}_{r2} (\bar{W}_{i2})_d \quad (3.2.2-20)$$

- e. Flexural vibrations at three times the rotational frequency induced by 3 wpc inner ring waviness

$$\bar{V}_{r3} = \bar{T}_{r3} \frac{3f_r}{f_{r3}} \bar{W}_{i3} \quad (3.2.2-21)$$

where \bar{W}_{i3} is the RMS amplitude of the 3 wpc inner ring waviness velocity, measured at a rotational frequency f_{r3} . \bar{T}_{r3} is given by Equation (3.2.2-14).

If the 3 wpc inner ring waviness is expressed as a RMS displacement $(\bar{W}_{i3})_d$

$$\bar{V}_{r3} = 6\pi f_r \bar{T}_{r3} (\bar{W}_{i3})_d \quad (3.2.2-22)$$

Equations (3.2.2-19) through (3.2.2-22) are valid if no resonant frequencies, computed according to 7, are lower than $5f_r$. If this is not the case the computed values of \bar{V}_{r2} and \bar{V}_{r3} must be multiplied by the factor \bar{U}_c determined according to 1.3.24.

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- f. Flexural vibrations at the ball passage frequency over the outer ring $\bar{z}\bar{f}_c$ induced by ball loads

$$\bar{V}_F = \frac{F_A \bar{R}^3 \bar{z} N \left[1 - \frac{\bar{D}}{2} \cos \alpha\right]}{60 \sqrt{2} \pi E I [\bar{z}^3 - 1]^2 \sin \alpha} \quad (3.2.2-23)$$

where the notation is as in paragraphs 3, 4, 6 and 7.

- g. Vibrations at the rotational frequency due to inner ring eccentricity

$$\bar{V}_{i1} = 2\pi f_r \bar{W}_{i1}$$

where \bar{W}_{i1} is the inner ring eccentricity (RMS displacement).

- h. Vibrations at the cage frequency \bar{f}_c due to rolling body diameter variation

$$\bar{V}_D = \frac{4\pi \bar{f}_c \bar{D}_v}{\bar{z}}$$

where \bar{D}_v is the maximum value of rolling body diameter variation in the bearing.

- i. The total vibration level

$$\bar{V}_{TH} = \sqrt{\bar{V}_0^2 + \bar{V}_1^2 + \bar{V}_b^2 + \bar{V}_{r2}^2 + \bar{V}_{r3}^2 + \bar{V}_F^2 + \bar{V}_{i1}^2 + \bar{V}_D^2} \quad (3.2.2-24)$$

If vibration test results are available for a bearing of the same size with the same order of magnitude of waviness, compare the computed vibration level \bar{V}_{TH} and the experimentally obtained value \bar{V} of the measured bearing, and modify Equation (3.2.2-24) to read

$$\bar{V} = K_v \sqrt{\bar{V}_0^2 + \bar{V}_1^2 + \bar{V}_b^2 + \bar{V}_{r2}^2 + \bar{V}_{r3}^2 + \bar{V}_F^2 + \bar{V}_{i1}^2 + \bar{V}_D^2} \quad (3.2.2-25)$$

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where

$$K_v = \frac{\bar{V}}{\bar{V}_{TH}} \quad (3.2.2-26)$$

12. Compute the following squared vibration level ratios

$$\bar{R}_0^2 = \left(\frac{K_v \bar{V}_0}{\bar{V}} \right)^2 \quad (3.2.2-27)$$

$$\bar{R}_1^2 = \left(\frac{K_v \bar{V}_1}{\bar{V}} \right)^2 \quad (3.2.2-28)$$

$$\bar{R}_b^2 = \left(\frac{K_v \bar{V}_b}{\bar{V}} \right)^2 \quad (3.2.2-29)$$

$$\bar{R}_{r2}^2 = \left(\frac{K_v \bar{V}_{r2}}{\bar{V}} \right)^2 \quad (3.2.2-30)$$

$$\bar{R}_{r3}^2 = \left(\frac{K_v \bar{V}_{r3}}{\bar{V}} \right)^2 \quad (3.2.2-31)$$

$$\bar{R}_r^2 = \left(\frac{K_v \bar{V}_r}{\bar{V}} \right)^2 \quad (3.2.2-32)$$

$$\bar{R}_{l1}^2 = \left(\frac{K_v \bar{V}_{l1}}{\bar{V}} \right)^2 \quad (3.2.2-33)$$

$$\bar{R}_D^2 = \left(\frac{K_v \bar{V}_D}{\bar{V}} \right)^2 \quad (3.2.2-34)$$

13. Compare the vibration level \bar{V} in the specified vibration frequency bands to the specified limits. If the expected vibration level \bar{V} exceeds the limit in any of the bands, study the effects of changing the number \bar{Z} and diameter \bar{D} of the rolling bodies as follows:

- a. Change the number \bar{Z} and diameter \bar{D} of the rolling bodies simultaneously, so that the product $\bar{Z}\bar{D}$ remains constant, i.e., for any \bar{Z} and \bar{D}

$$\bar{Z}\bar{D} = \bar{Z}\bar{D} \quad (3.2.2-35)$$

while \bar{d} remains constant.

- b. Compute I/R^3 for various values of \bar{D} and plot I/R^3 as a function of \bar{D} , for constant $\bar{d} = \bar{d}$.
- c. Compute the ratios between the amplification factors of a bearing with rolling body diameter \bar{D} and the reference bearing with rolling body diameter \bar{D} , for various values of \bar{D} , under the conditions of Equation (3.2.2-27). Use the following equations:

$$\frac{U_o}{U_o} = \frac{\frac{d}{\bar{D}\cos\alpha} - \frac{\bar{D}}{\bar{D}}}{\frac{d}{\bar{D}\cos\alpha} - 1} \sqrt{\frac{\bar{D}}{\bar{D}}} \quad (3.2.2-36)$$

$$\frac{U_i}{U_i} = \frac{\frac{d}{\bar{D}\cos\alpha} + \frac{\bar{D}}{\bar{D}}}{\frac{d}{\bar{D}\cos\alpha} + 1} \sqrt{\frac{\bar{D}}{\bar{D}}} \quad (3.2.2-37)$$

$$\frac{U_b}{U_b} = \sqrt{\frac{\bar{D}}{\bar{D}}} \quad (3.2.2-38)$$

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- d. Compute the ratios $\gamma_2/\bar{\gamma}_2$ and $\gamma_3/\bar{\gamma}_3$ for various values of D from the equation

$$\frac{\gamma_2}{\bar{\gamma}_2} = \frac{\gamma_3}{\bar{\gamma}_3} = \frac{\beta_N}{\beta_N} \left(\frac{\bar{D}}{D} \right)^{\frac{2}{3}} \frac{\bar{I} \bar{R}^3}{I \bar{R}^3} \quad (3.2.2-39)$$

where β_N is obtained graphically from Enclosure 19 as a function of ball diameter.

For roller bearings use

$$\frac{\gamma_2}{\bar{\gamma}_2} = \frac{\gamma_3}{\bar{\gamma}_3} = \frac{k_N \bar{D} \bar{I} \bar{R}^3}{k_N D I R^3} \quad (3.2.2-40)$$

where k_N is obtained as in paragraph 3.

- e. Compute for various values of D ,

$$\frac{T_{r2}}{\bar{T}_{r2}} = \frac{\frac{\gamma_1}{\bar{\gamma}_2}}{1 - \bar{T}_{r2} + \bar{T}_{r2} \frac{\gamma_1}{\bar{\gamma}_2}} \quad (3.2.2-41)$$

$$\frac{T_{r3}}{\bar{T}_{r3}} = \frac{\frac{\gamma_1}{\bar{\gamma}_3}}{1 - \bar{T}_{r3} + \bar{T}_{r3} \frac{\gamma_1}{\bar{\gamma}_3}} \quad (3.2.2-42)$$

- f. Plot the following squared vibration level ratios as a function of D for each vibration frequency band specified

$$\left(\frac{\bar{V}_0}{\bar{V}} \right)^2 = \bar{R}_0^2 \left(\frac{V_0}{\bar{V}} \right)^2 = \bar{R}_0^2 \left(\frac{U_c}{\bar{U}_c} \right)^2 \left(\frac{U_0}{\bar{U}_0} \right)^2 \left(\frac{W_0}{\bar{W}_0} \right)^2 \quad (3.2.2-43)$$

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$$\left(\frac{V_i}{V}\right)^2 = \bar{R}_i^2 \left(\frac{V_i}{V_i}\right)^2 = \bar{R}_i^2 \left(\frac{U_e}{U_i}\right)^2 \left(\frac{U_i}{U_i}\right)^2 \left(\frac{W_i}{W_i}\right)^2 \quad (3.2.2-44)$$

$$\left(\frac{V_b}{V}\right)^2 = \bar{R}_b^2 \left(\frac{V_b}{V_b}\right)^2 = \bar{R}_b^2 \left(\frac{U_e}{U_b}\right)^2 \left(\frac{U_b}{U_b}\right)^2 \left(\frac{W_b}{W_b}\right)^2 \quad (3.2.2-45)$$

$$\left(\frac{V_{r2}}{V}\right)^2 = \bar{R}_{r2}^2 \left(\frac{V_{r2}}{V_{r2}}\right)^2 = \bar{R}_{r2}^2 \left(\frac{T_{r2}}{T_{r2}}\right)^2 \quad (3.2.2-46)$$

$$\left(\frac{V_{r3}}{V}\right)^2 = \bar{R}_{r3}^2 \left(\frac{V_{r3}}{V_{r3}}\right)^2 = \bar{R}_{r3}^2 \left(\frac{T_{r3}}{T_{r3}}\right)^2 \quad (3.2.2-47)$$

$$\left(\frac{V_F}{V}\right)^2 = \bar{R}_F^2 \left(\frac{V_F}{V_F}\right)^2 = \bar{R}_F^2 \left\{ \frac{\bar{I} R^3}{I R^3} \left(\frac{D}{\bar{D}} \right) \left[\frac{\frac{d}{D \cos \alpha} - \frac{D}{\bar{D}}}{\frac{d}{D \cos \alpha} - 1} \right] \right\}^2 \quad (3.2.2-48)$$

$$\left(\frac{V_{ii}}{V}\right)^2 = \bar{R}_{ii}^2 \quad (3.2.2-49)$$

$$\left(\frac{V_D}{V}\right)^2 = \bar{R}_D^2 \left(\frac{V_D}{V_D}\right)^2 = \bar{R}_D^2 \left(\frac{D}{\bar{D}}\right)^2 \left(\frac{U}{U}\right)^2 \quad (3.2.2-50)$$

where the waviness levels w_0 , w_1 and w_2 relate to the waviness frequency bands corresponding to the specified vibration frequency band, for each value of D . These waviness bands must be determined according to paragraph 4 as a function of diameter D . Since the waviness bands change with D , w_0 , w_1 and w_2 are not necessarily equal to \bar{w}_0 , \bar{w}_1 and \bar{w}_2 . For order of magnitude estimates, it may, however, be assumed that $w = \bar{w}_0$, $w_1 = \bar{w}_1$ and $w_2 = \bar{w}_2$, if the waviness spectra are reasonably flat over the ranges considered. If this is not the case the differences between w_0 and \bar{w}_0 , w_1 and \bar{w}_1 , w_2 and \bar{w}_2 must be taken into account.

The factor $(u_c/\bar{u}_c)^2$ appearing in Equations (3.2.2-43), (3.2.2-44) and (3.2.2-45) is due to the fact that the resonant frequencies of the bearing change as D , z and the thickness of the outer ring vary. To compute the factor $(u_c/\bar{u}_c)^2$ it is simplest to estimate the number of resonances within each of the specified frequency bands, according to paragraph 7, and to compute u_c from Equation (3.2.2-11).

- g. Plot the squared total vibration level ratio $(V/\bar{V})^2$ as a function of D for each vibration frequency band specified.

Use the equation

$$\left(\frac{V}{\bar{V}}\right)^2 = \left(\frac{V_0}{\bar{V}}\right)^2 + \left(\frac{V_1}{\bar{V}}\right)^2 + \left(\frac{V_2}{\bar{V}}\right)^2 + \left(\frac{V_{rs}}{\bar{V}}\right)^2 + \left(\frac{V_{rs}}{\bar{V}}\right)^2 + \left(\frac{V_r}{\bar{V}}\right)^2 + \left(\frac{V_l}{\bar{V}}\right)^2 + \left(\frac{V_p}{\bar{V}}\right)^2 \quad (3.2.2-51)$$

- h. Determine from the plot of Equation (3.2.2-51) the value of D which gives minimum $(V/\bar{V})^2$. This value of D is likely to be different in the various frequency bands specified. Select D to minimize the vibration in the frequency band considered most critical, by comparing with the specified limits. The selected value of D is denoted D_T .

- i. Determine the value z_I of z corresponding to D_I from the equation

$$z_I \approx \frac{\bar{z} \bar{D}}{D_I} \quad (3.2.2-52)$$

14. Study the effect of deviation of z from the value z_I given by Equation (3.2.2-52), by keeping D constant $= D_I$ and varying only z . Since the value z_I approximately represents the maximum value of z for the given D , it is necessary to study only values of z smaller than or approximately equal to z_I .

The subscript I will be used to denote all quantities related to the bearing with $D = D_I$ and $z = z_I$. No subscript will be used for the quantities related to a bearing with $D = D_I$ and an arbitrary number of balls z . Compute the following squared vibration level ratios for various values of z :

$$\left(\frac{V_o}{V_{oI}}\right)^2 = \frac{z_I}{z} \left(\frac{U_c}{U_{cI}}\right)^2 \quad (3.2.2-53)$$

$$\left(\frac{V_i}{V_{iI}}\right)^2 = \frac{z_I}{z} \left(\frac{U_c}{U_{cI}}\right)^2 \quad (3.2.2-54)$$

$$\left(\frac{V_b}{V_{bI}}\right)^2 = \frac{z_I}{z} \left(\frac{U_c}{U_{cI}}\right)^2 \quad (3.2.2-55)$$

$$\left(\frac{V_{r2}}{V_{r2I}}\right)^2 = \left[\frac{\left(\frac{z}{z_I}\right)^{2/3}}{1 - T_{r2I} + T_{r2I} \left(\frac{z}{z_I}\right)^{2/3}} \right]^2 \quad (3.2.2-56)$$

$$\left(\frac{V_{r3}}{V_{r3I}}\right)^2 = \left[\frac{\left(\frac{z}{z_I}\right)^{2/3}}{1 - T_{r3I} + T_{r3I} \left(\frac{z}{z_I}\right)^{2/3}} \right]^2 \quad (3.2.2-57)$$

$$\left(\frac{V_p}{V_{pI}}\right)^2 = \left(\frac{z_I}{2}\right)^6 \quad (3.2.2-58)$$

$$\left(\frac{V_{li}}{V_{liI}}\right)^2 = 1 \quad (3.2.2-59)$$

$$\left(\frac{V_D}{V_{DI}}\right)^2 = \left(\frac{z_I}{2}\right)^2 \quad (3.2.2-60)$$

15. Plot the following squared vibration ratios as a function of z for each specified vibration frequency band

$$\left(\frac{V_o}{V_I}\right)^2 = \left(\frac{V_o}{V_{oI}}\right)^2 \left(\frac{V_{oI}}{V_I}\right)^2 \quad (3.2.2-61)$$

$$\left(\frac{V_i}{V_I}\right)^2 = \left(\frac{V_i}{V_{iI}}\right)^2 \left(\frac{V_{iI}}{V_I}\right)^2 \quad (3.2.2-62)$$

$$\left(\frac{V_b}{V_I}\right)^2 = \left(\frac{V_b}{V_{bI}}\right)^2 \left(\frac{V_{bI}}{V_I}\right)^2 \quad (3.2.2-63)$$

$$\left(\frac{V_{r2}}{V_I}\right)^2 = \left(\frac{V_{r2}}{V_{r2I}}\right)^2 \left(\frac{V_{r2I}}{V_I}\right)^2 \quad (3.2.2-64)$$

$$\left(\frac{V_{r3}}{V_I}\right)^2 = \left(\frac{V_{r3}}{V_{r3I}}\right)^2 \left(\frac{V_{r3I}}{V_I}\right)^2 \quad (3.2.2-65)$$

$$\left(\frac{V_F}{V_I}\right)^2 = \left(\frac{V_F}{V_{FI}}\right)^2 \left(\frac{V_{FI}}{V_I}\right)^2 \quad (3.2.2-66)$$

$$\left(\frac{V_{LI}}{V_I}\right)^2 = \left(\frac{V_{LI}}{V_I}\right)^2 \quad (3.2.2-67)$$

$$\left(\frac{V_D}{V_I}\right)^2 = \left(\frac{V_D}{V_{DI}}\right)^2 \left(\frac{V_{DI}}{V_I}\right)^2 \quad (3.2.2-68)$$

$$\left(\frac{V}{V_I}\right)^2 = \left(\frac{V_o}{V_I}\right)^2 + \left(\frac{V_i}{V_I}\right)^2 + \left(\frac{V_b}{V_I}\right)^2 + \left(\frac{V_{i1}}{V_I}\right)^2 + \left(\frac{V_{i2}}{V_I}\right)^2 + \left(\frac{V_F}{V_I}\right)^2 + \left(\frac{V_{i1}}{V_I}\right)^2 + \left(\frac{V_D}{V_I}\right)^2 \quad (3.2.2-69)$$

where the ratios $(V_o/V_{oI})^2, (V_i/V_{iI})^2, (V_b/V_{bI})^2, (V_{i1}/V_{i1I})^2, (V_{i2}/V_{i2I})^2, (V_F/V_{FI})^2, (V_{i1}/V_{i1I})^2, (V_D/V_{DI})^2$ are obtained from Equation (3.2.2-53) through (3.2.2-60) and the ratios $(V_{oI}/V_I)^2, (V_{iI}/V_I)^2, (V_{bI}/V_I)^2, (V_{i1I}/V_I)^2, (V_{i2I}/V_I)^2, (V_{FI}/V_I)^2, (V_{i1I}/V_I)^2, (V_{DI}/V_I)^2$ from the graph of 14 at the point $D = D_i$.

16. Determine from the plot of Equation (3.2.2-69) the value of Z which gives minimum $(V/V_I)^2$. Select the optimum value of Z , considering all the specified frequency bands.
17. Check the selected values of D , d and Z for design feasibility and make modifications if necessary.
18. Compare the estimated vibration level of the bearing to the specified vibration limits.
19. If the estimated vibration levels still exceed the limits in any of the specified vibration frequency bands, make recommendations for reduction of the limits for micro-geometry. Special emphasis should be put on the reduction of the input from vibratory sources with the highest contribution to the total vibration level, as indicated by the squared ratios given by Equations (3.2.2-61) through (3.2.2-68).

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In particular, to evaluate the relative contribution from outer ring, inner ring and rolling body waviness, compute the following contribution factors:

For outer ring waviness

$$Z_o = \left[1 + \left(\frac{W_i}{W_o} \right)^2 \left(\frac{f_i}{f_c} \right)^2 \left(\frac{f_{wo}}{f_{wi}} \right)^2 + \frac{\pi}{2} \left(\frac{W_b}{W_o} \right)^2 \left(\frac{f_b}{f_c} \right)^2 \left(\frac{f_{wo}}{f_{wb}} \right)^2 \right]^{-\frac{1}{2}} \quad (3.2.2-70)$$

For inner ring waviness

$$Z_i = \left[1 + \left(\frac{W_o}{W_i} \right)^2 \left(\frac{f_c}{f_i} \right)^2 \left(\frac{f_{wi}}{f_{wo}} \right)^2 + \frac{\pi}{2} \left(\frac{W_b}{W_i} \right)^2 \left(\frac{f_b}{f_i} \right)^2 \left(\frac{f_{wi}}{f_{wb}} \right)^2 \right]^{-\frac{1}{2}} \quad (3.2.2-71)$$

For rolling body waviness

$$Z_b = \left[1 + \left(\frac{W_o}{W_b} \right)^2 \left(\frac{f_c}{f_b} \right)^2 \left(\frac{f_{wb}}{f_{wo}} \right)^2 + \frac{2}{\pi} \left(\frac{W_i}{W_b} \right)^2 \left(\frac{f_i}{f_b} \right)^2 \left(\frac{f_{wb}}{f_{wi}} \right)^2 \right]^{-\frac{1}{2}} \quad (3.2.2-72)$$

In making recommendations for reduced waviness limits, the greatest reduction should be made on the part with the highest value of Z .

3.3 OPERATING PARAMETERS

Although the values of these parameters usually cannot be selected arbitrarily, it is often advantageous, in the design stage, to make a brief study of the influence of load and speed on bearing vibration, if the load and speed used in the vibration application are widely different from those normally used in bearing vibration tests.

3.3.1 Effect of Load

The present study covers only bearings under sufficient load to maintain contact at all times between loaded rolling bodies and races. Bearings with lesser load generate "slack" vibrations which have not been discussed.

The only significant effects of radial or axial load above the levels sufficient to maintain contact between loaded elements are:

1. Vibrations induced by rolling body loads. This effect must be considered if the load applied to the bearing is exceptionally heavy.
2. Flexural vibrations induced by low order inner ring waviness. The amplitudes of these vibrations are not significantly affected by small changes in load. If the load in the application is much higher (ten times higher or more) than in the tests, its effect should be considered.
3. Variable compliance vibrations are influenced by the applied radial load. The evaluation of this effect requires a detailed study.
4. Certain bearing types require for satisfactory quiet running operation, a suitable axial preload, to assure that all the rolling bodies are continuously in contact with the race groove while the bearing is rotating. This is particularly the case with ball bearings, where the loose balls in the no-load zone tend to generate high vibration levels. Axial preload should be selected with the knowledge of the radial loads to be expected, and should be heavier for heavier radial loads. A general rule for axial load selection is not available and it appears best to determine the best value by systematic experimentation on a prototype of the machinery.

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The preceding discussions indicate that substantial freedom is available to the designer in selecting axial loads without generating increased bearing vibrations (other than flexural vibration due to ball load, if the axial load is very high).

5. This entire study is limited to the vibration of bearings under constant load. If available, loads, specifically vibratory loads are imposed on a bearing, these will be transmitted by it in accordance with its elastic and damping properties. Deflections will occur in the bearings that vary with load magnitude and direction. The magnitude and nature of these externally induced vibrations are calculable with relative ease, but have not been covered.

3.3.2 Effect of Rotational Speed

1. If the rotational speed of the bearing in the application is different from that in the tests, the octave band spectrum of the bearing at the speed used in the application may be estimated from the spectrum at the test speed by using Equation (1.3.20-1). This equation is only approximate and does not take into account resonant frequencies which are speed independent.

For vibrations generated at discrete frequencies, such as the rotational frequency, twice the rotational frequency or the ball passage frequency, both the frequency and the amplitude of the vibration at the speed used in the application are obtained by multiplying with the ratio of application speed to test speed.

2. The present study covers bearing vibrations under constant or slowly varying speed. Rapid speed variations, particularly angular oscillations imposed on the bearing will cause additional bearing noise which has not been covered in this study.

3.4 MOUNTING PARAMETERS

The attaching of masses and of elastic and damping bodies to the bearing changes the vibration characteristics of the system. These steps affect the resonant and damping characteristics of the system which influence the vibration emitted by the housing. The effects are generally so pronounced that resonant characteristics of the free bearing are no longer recognizable after mounting. However, the discrete frequencies generated by the various bearing vibration sources are not altered by mounting. The following main effects of mounting should be noted:

1. Added mass lowers the natural frequencies of the bearing.
2. A housing which is comparatively free to move angularly gives low amplitudes of the radial vibration at low frequencies.
3. A housing which is comparatively free to move radially gives low amplitudes of the angular vibration at low frequencies.
4. By proper selection of the spring constants of the housing, the amplitudes either in the radial or angular direction can be minimized at any given frequency.
5. Damping members such as laminated elastic structures can be used successfully to attenuate vibrations at comparatively high frequencies, but they are ineffective at frequencies near zero.
6. The housing constrains flexural vibrations of the outer ring, and therefore reduces the low frequency vibrations. The effect of changing the fit between bearing O.D. and housing or between the bearing bore and shaft is rather small and too erratic to be used as a means of controlling the vibration transmission characteristics of the bearing.
7. An out of round or wavy shaft or housing will act as if the bearing ring mounted on to it, had the same imperfections.
8. Misalignment of the bearing caused by mounting conditions causes substantial increases in bearing vibration unless the bearing is self-aligning.

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9. Resonances in machine parts other than the bearing may be excited by bearing vibrations. They should be ascertained and either reduced or the bearing should be selected to have particularly low vibration levels in the band comprising the machine resonance.

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3.5 NUMERICAL EXAMPLE - DESIGN OF A BEARING TO MEET GIVEN VIBRATION LIMITS

The procedure and notation of Section 3.2.2 is followed:

1. Bearing selected as suitable for the application according to general engineering considerations (shaft and housing diameter, width).

6305

This bearing has the following principal dimensions:

Bore :	25 mm	$Z =$	7
O.D. :	62 mm	$D =$	7/16"
Width:	17 mm	$d =$	1.71"

The contact angle α is taken as 15°

2. Vibration Specifications: It is desired that at 1800 RPM under an axial load F_A of 25 lbs, the radial vibrations in the three frequency bands 50-300, 300-1800 and 1800-10000 cps conform to the specifications of MIL-B-17931B (Ships). It is further desired that the RMS vibrational velocity in the 0-50 cps band not exceed 10,000 microinches/second. The limits for each band are summarized below:

Vibration Band, cps	Specification Limit-Microinches/second, RMS
0- 50	10000
50- 300	3760
300- 1800	3010
1800-10000	2260

of these frequency bands, the 50-300 cps band is considered the most critical.

3. The following constants are calculated for the 6305 bearing:

$$\begin{aligned}\bar{A} &= 0.130 \text{ in}^2 \\ \bar{M} &= 0.00067 \text{ lbs inches}^2/\text{in} \\ \bar{R} &= 1.120 \text{ in} \\ \bar{I} &= 5.1 \times 10^{-4} \\ \bar{k} &= 2.21 \times 10^5 \text{ lbs/in}\end{aligned}$$

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4. The waviness levels for the components of a conventionally made 6305 bearing (designated here as reference bearing) are assumed to have been measured and meet these limits:

Race Waviness in Microinches/second, Measured at 1000 RPM

Band, wpc	\bar{w}_o	\bar{w}_i
2		$\bar{w}_{i2} = 1500$
3		$\bar{w}_{i3} = 1000$
3- 6	2000	1500
6- 12	1500	1200
12- 24	1500	1000
24- 48	1500	1000
48- 96	1500	1000
96-192	1500	1000
192-384	1000	800
384-768	1000	800

Inner ring eccentricity $\bar{w}_i = 50$ microinches maximum displacement - 35 microinches, RMS.

Ball Waviness in Microinches/second, Measured at 740 RPM

Band, wpc	\bar{w}_b
2	400
4- 8	100
8- 16	100
16- 32	150
32- 64	200
64-128	200
128-256	300

Ball diameter variation $\bar{D}_v = 15$ microinches maximum variation.

5. The following frequencies are calculated for the 6305 bearing:

$$\begin{aligned} \bar{f}_r &= 30 \text{ cps} \\ \bar{f}_o &= 11 \text{ cps} \\ \bar{f}_i &= 19 \text{ cps} \\ \bar{f}_b &= 54 \text{ cps} \end{aligned}$$

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The waviness bands for each bearing component corresponding to the given vibration frequency bands are tabulated below:

Vibration, cps	50-300 cps	300-1800 cps	1800-10000 cps
Outer ring, wpc	5 - 27	27 - 62	162 - 901
Inner ring, wpc	3 - 16	16 - 96	96 - 532
Ball, wpc	2 - 6	6 - 32	32 - 182

Since the waviness bands used in the measurements do not coincide with the computed bands, estimates of the waviness levels \bar{w}_o , \bar{w}_i , and \bar{w}_b , corresponding to the computed waviness bands were obtained by adding the squared waviness levels in the measured bands contained within any given computed band (taking into account the partial contribution from measured bands only partially within the computed bands).

The waviness values in the computed bands, obtained in this manner are tabulated below for the reference bearing:

Waviness, in microinches/second

	50-300 cps	300-1800 cps	1800-10000 cps
\bar{w}_o	2380	2380	1580
\bar{w}_i	2160	1590	1430
\bar{w}_b	412	206	346

6. The band width amplification factors for the radial vibrations of the reference bearing are:

$$\bar{U}_o = 0.35$$

$$\bar{U}_i = 0.58$$

$$\bar{U}_b = 2.87$$

7. The following natural frequencies were computed for the reference bearing:

$$\bar{f}_r = 5000 \text{ cps}$$

$$\bar{f}_{pi} = 6430 \text{ cps}$$

$$\bar{f}_{FR} = 12000 \text{ cps}$$

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8. The resonance amplification factors \bar{U}_c for the reference bearing are:

Vibration Band, cps	\bar{U}_c
0- 50	1
50- 300	1
300- 1800	1
1800-10000	2.16

9. The following values of \bar{Y}_2 and \bar{T}_{Y2} were computed

$$\bar{Y}_2 = 3.19$$

$$\bar{T}_{Y2} = 0.762$$

10. The following values of \bar{Y}_3 and \bar{T}_{Y3} were computed

$$\bar{Y}_3 = 0.532$$

$$\bar{T}_{Y3} = 0.347$$

11. The following vibration levels were computed for the reference bearing (microinches/second):

	0-50 cps	50-300 cps	300-1800 cps	1800-10000 cps
\bar{V}_o	-	830	830	1200
\bar{V}_i	-	1250	920	1800
\bar{V}_b	-	1180	590	2140
\bar{V}_{Y2}	-	4120	-	-
\bar{V}_{Y3}	-	1880	-	-
\bar{V}_F	-	1170	-	-
\bar{V}_{ti}	6600	-	-	-
\bar{V}_D	300	-	-	-
Total $\bar{V} = \bar{V}_{th}$	6600	5060	1370	3040

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12. The squared vibration level ratios for the reference bearing are:

Squared Vibration Level Ratios

	0-50 cps	50-300 cps	300-1800 cps	1800-10000 cps
R_{0-50}^2	-	0.027	0.366	0.156
R_{50-300}^2	-	0.058	0.448	0.352
$R_{300-1800}^2$	-	0.101	0.186	0.496
$R_{1800-10000}^2$	-	0.660	-	-
R_{0-300}^2	-	0.137	-	-
$R_{300-10000}^2$	-	0.055	-	-
R_{0-1800}^2	0.998	-	-	-
$R_{1800-10000}^2$	0.002	-	-	-

13. The total vibration levels of the reference bearing, compare as follows with the limits.

Frequency Band, cps	Limit Microinches/second	\bar{V} Microinches/second
0- 50	10000	6600
50- 300	3760	5060
300- 1800	3010	1370
1800-10000	2260	3040

It is seen that the limit is exceeded in the 50-300 cps and 1800-10000 cps bands.

$1/R^3$ as a function of D is shown on Enclosure 109, for $\bar{a} = 1.71$ " and boundary dimensions the same as for the reference bearing. Curves showing the squared vibration levels as a function of D are given in Enclosure 110 for the 50-300 cps band, in Enclosure 111 for the 300-1800 cps band and in Enclosure 112 for the 1800-10000 cps band. These curves are based on $\bar{z}D = \bar{z}\bar{D} = 49/16$. It is seen from Enclosure 110 that the $(\bar{V}/\bar{V})^2$ curve in the 50-300 cps band does not have a minimum within the range shown on the graph.

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$(\sqrt{v})^2$ decreases with D over the complete range shown. In the 300-1800 cps band $(\sqrt{v})^2$ has a minimum at $D \approx 1/4"$ and in the 1800-10000 cps band a minimum occurs at $D \approx 5/16"$.

The following value of D was selected as giving an optimum $(\sqrt{v})^2$ value considering all three vibration bands mentioned.

$$D_T = 7/32"$$

The corresponding value for Z is

$$Z_T = 14$$

The selected value of D_T gives a low band (50-300 cps) level within the specified limit. Further decrease in would result in a highly increased vibration level in the 300-1800 cps band, which is not considered desirable, although it would improve the 50-300 cps band. In the high band (1800-10000 cps) the selected value of D_T gives a $(\sqrt{v})^2$ value which is 10% higher than that at optimum. This is not considered significant.

The vibration levels of the bearing with $D_T = 7/32"$ and $Z_T = 14$ are tabulated below:

Frequency Band, cps	Limit Microinches/second	$\sqrt{v_T}$ Microinches/second	$\frac{\sqrt{v_T}}{\sqrt{v}}$
0- 50	10000	6600	1
50- 300	3760	3630	0.72
300- 1800	3010	1160	0.85
1800-10000	2260	2980	0.98

It is seen that the vibration levels in the 0-50, 50-300 and 300-1800 cps are within the specified limits. In the 1800-10000 cps band the limit is exceeded.

- 14-17. Enclosures 113, 114 and 115 show the effect of changes in Z on the vibration levels. It is seen that the squared vibration levels in the three bands increase with decreasing Z , over the range shown although the changes are small. Since the value $Z_T = 14$ is considered the maximum number of balls that can be assembled in the deep-groove bearing, the values $D_T = 7/32"$ and $Z_T = 14$ will be checked for design feasibility and, if found suitable, used in the final design.

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18. Summary of selected parameters for the designed bearing:

Dimensions:

Bore : 25 mm
 O.D. : 62 mm
 Width: 17 mm

$Z_r = 14$
 $D_r = 7/32"$
 $d_r = 1.71$
 $\alpha = 15^\circ$

Vibration Levels:

Frequency Band, cps	Limit-microinches/ second	V_r microinches/ second	$\frac{V_r}{V}$	$\frac{V_r}{V}$ Limit
0- 50	10000	6600	1	0.66
50- 300	3760	3630	0.72	0.96
300- 1800	3010	1160	0.85	0.39
1800-10000	2260	2980	0.98	1.32

It is seen that the bearing is within the specified limits in the 0-50, 50-300 and 300-1800 cps bands, but exceeds the limit by 32% in the 1800-10000 cps band. The vibration level is the same as the reference bearing in the 50-300 cps band, 72% of the reference bearing in the 50-300 cps band, 85% in the 300-1800 cps band and 98% in the 1800-10000 cps band.

Squared Vibration Level Ratios:

0-50	50-300	300-1800	1800-10000 cps
-	0.027	0.368	0.156
-	0.058	0.454	0.352
-	0.101	0.188	0.496
-	0.660	-	-
-	0.137	-	-
-	0.055	-	-
0.998	-	-	-
0.002	-	-	-

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Resonant Frequencies:

$$f_R = 4740 \text{ cps}$$

$$f_{R1} = 8820 \text{ cps}$$

$$f_{R2} = 14400 \text{ cps}$$

Reduction of the high band vibration level to conform to specifications can be accomplished only by reduction of waviness. The squared "contribution factors" in the high band (1800-10000 cps) are:

$$Z_o^2 = 0.11$$

$$Z_i^2 = 0.14$$

$$Z_b^2 = 0.75$$

Since the Z_b^2 factor is the highest, the most effective reduction in vibration level is obtainable by reducing the ball waviness in the waviness bands influencing the high vibration band. These bands are the 16-32 wpc, 32-64 wpc and part of the 64-128 wpc band. The following reduced waviness limits are specified in order to reduce the vibration level in the high band to conform to specifications:

Band, wpc	\bar{W}_b Limit (Microinches/second)
16- 32	100
32- 64	130
64-128	150

Waviness limits of the rings may remain the same as measured on the reference bearing.

A sample of bearings of the dimensions selected and satisfying the waviness limits given must now be made and vibration tested. Corrections to the waviness limits are made on the basis of the test results.

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Progress Reports Nos. 1-17

The following special reports:

Relative Axis Motion Induced by Variable Elastic Compliance in Ball Bearings (SKF Report L60L023).

Harmonic Analysis of the Relative Axis Motion Induced by Variable Elastic Compliance in Ball Bearings (SKF Report AL61L009).

A Study of Elastic Vibrations of the Outer Race of a Rolling Element Bearing (SKF Report AL61L027).

Analytical Study of the Vibration of a Bearing with Flexurally Rigid Races (SKF Report AL61L032).

Flexural Vibrations of a Ball Bearing Outer Ring due to Ball Loads (SKF Report AL61L037).

2. The following reports submitted by SKF Industries, Inc. to the U. S. Department of the Navy, Bureau of Ships, under Contract No. NObs-78593 (Calibration of Anderometers):

Progress Reports Nos. 1-12

Special Report on Low Amplitude Calibration of the Electrical System and of the Pickup of Anderometers (SKF Report L61L001).

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11. Perret, H., Schwingungen in Wälzlager bei statisch bestimmter Abstützung, VDI-Forschungs Heft 434, 1952, pp. 17 to 28.
12. Perret, H., Die Lagerluft als Bestimmungsgrösse für die Beanspruchung eines Wälzlagers, Werkstatt und Betrieb, 1950, Heft 4, pp. 131 to 134.

NOMENCLATURE

- A = Cross-sectional area of a bearing outer ring
 \bar{A}_k = RMS velocity amplitude of the k th harmonic of flexural vibrations
 B = Width of bearing
 C_{ai} = "Amplitude/load parameter" in the direction of the load
 C_{bi} = "Amplitude/load parameter" in the direction perpendicular to the load
 D = Rolling body diameter
 E = Young's modulus of elasticity
 F_A = Axial load
 F_R = Radial load
 G = Load/looseness parameter
 H = Thickness of the outer ring
 I = Second moment of area of the outer ring cross section
 I_M = Moment of inertia of outer ring with respect to a diameter through the center plane of the bearing
 M = outer ring mass
 M_A = Restoring moment due to angular displacement
 N = Rotational speed, in RPM
 P = Rolling body load
 R = Mean radius of outer ring

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R_M = Radius from bearing axis to point of contact between ball and outer ring

T = Displacement amplification factor

T_Y = Displacement amplification factor for flexural outer ring vibrations due to inner ring waviness

U = Velocity band width amplification factor for element waviness

U_c = Band width amplification factor due to resonance

V = RMS vibration level

V_F = Vibration level of flexural outer ring vibrations due to ball loads

V_R = Vibration level of rigid ring vibration

V_{r2}, V_{r3} = Vibration level of flexural vibrations due to two and three wpc inner ring waviness, respectively

W = Element waviness

Z = Contribution factor of element waviness to the bearing vibration level

\bar{a}_i = RMS value of the i th variable compliance vibration harmonic in the direction of load

\bar{b}_i = RMS value of the i th variable compliance vibration harmonic in perpendicular to the load

c_s = Herzian coefficient

d = Pitch diameter of rolling body set

e = Radial looseness

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- f = Frequency in cps
- f_A = Natural frequency of the axial mode of vibration
- f_b = Polar rotational frequency of a rolling body
- f_c = Rotational frequency of the cage with respect to the outer ring.
- f_H = Natural frequency of radial vibrations of a system consisting of the bearing mounted in housing
- f_F = Natural frequency of flexural vibrations
- f_i = Rotational frequency of the cage with respect to the inner ring
- f_m = Resonant frequency
- f_r = Rotational frequency of inner ring
- f_{R1}, f_{R2} = Natural frequencies of radial and angular rigid body modes of vibration
- f_w = Rotational frequency used in measuring element waviness
- k_n = Linearized Hertzian coefficient
- k = Order of element waviness
- n = Any positive integer
- x_v = Vertical displacement of outer ring center
- x_H = Horizontal displacement of outer ring center
- y = Axial displacement of outer ring center
- Z = Number of rolling bodies

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α = Contact angle

β_n = Parameter defined by Equation (1.3.7-5)

γ = Parameter defined by Equations (1.3.18-2) or (3.2.1-1)

δ = Elastic deformation

η = Dimensional parameter defined by Equation (1.3.14-15)

μ = Frequency ratio $\frac{f}{f_n}$

ξ_R, ξ_λ = Elastic parameters defined by Equations (1.3.14-13) and (1.3.14-14)

ρ_o = Mass density of outer ring

ρ = Radius of gyration of outer ring about a diameter

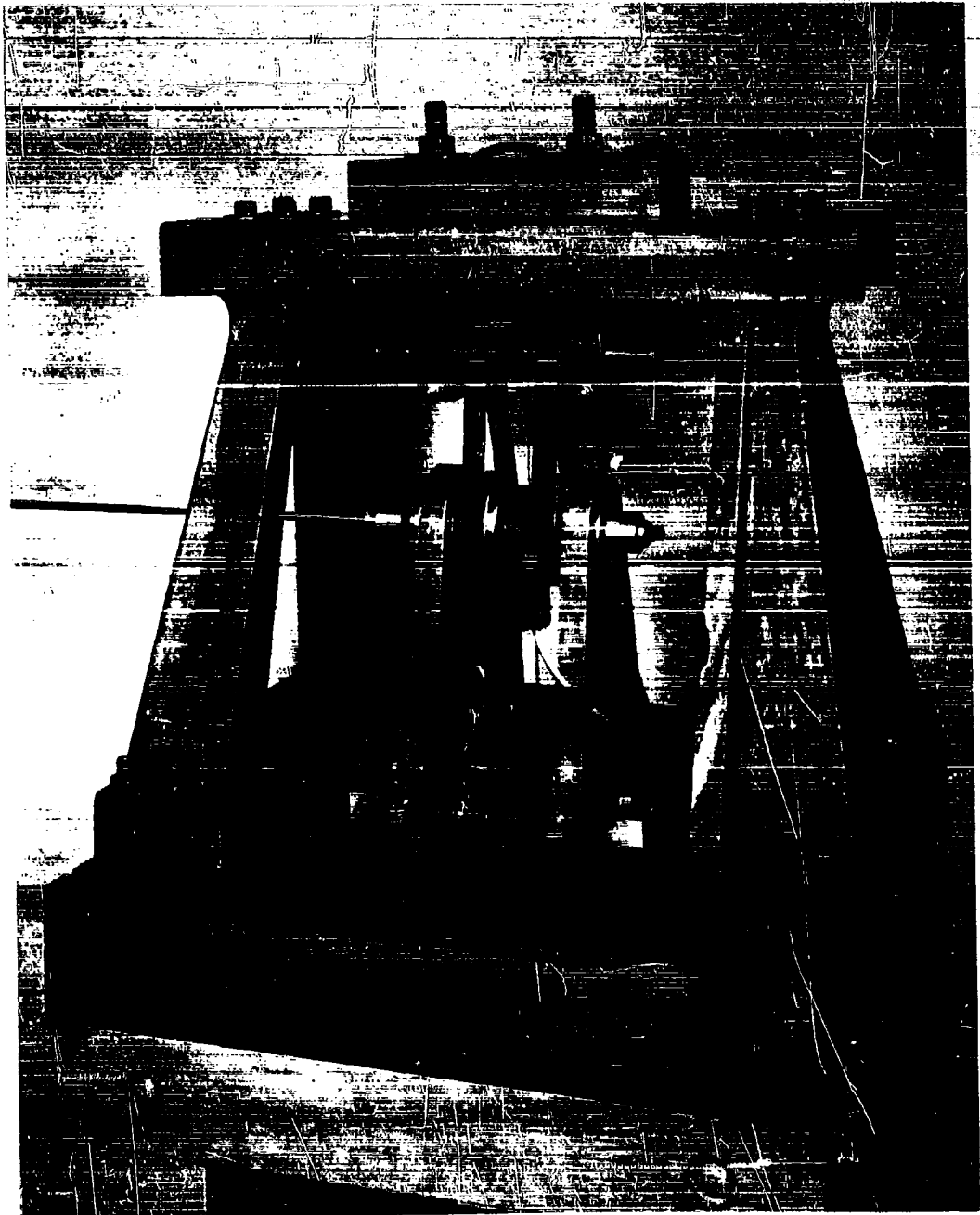
G = Band width, expressed as the number of octaves contained within the band

Subscripts:

b = Refers to rolling body

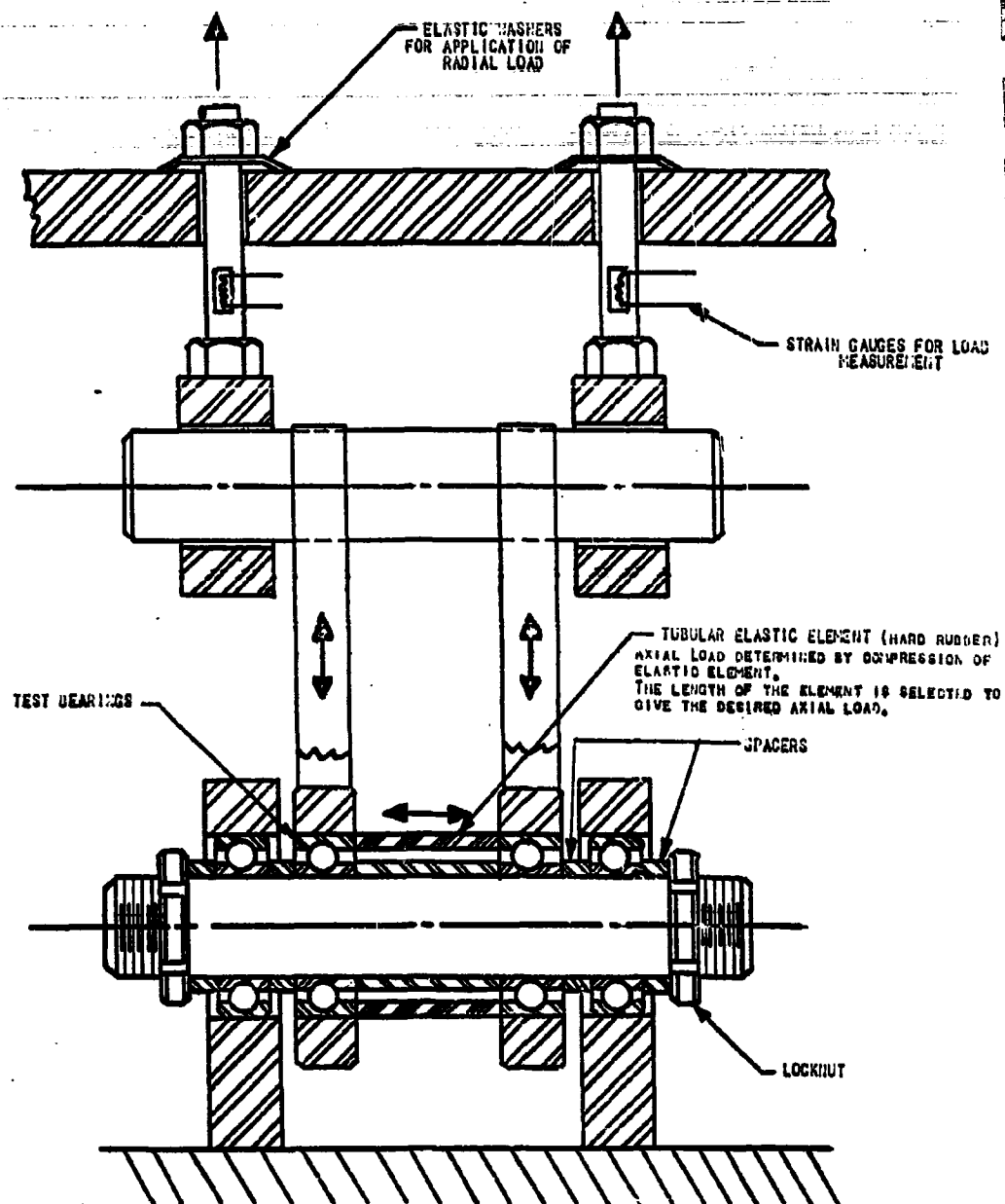
i = Refers to inner race

o = Refers to outer race



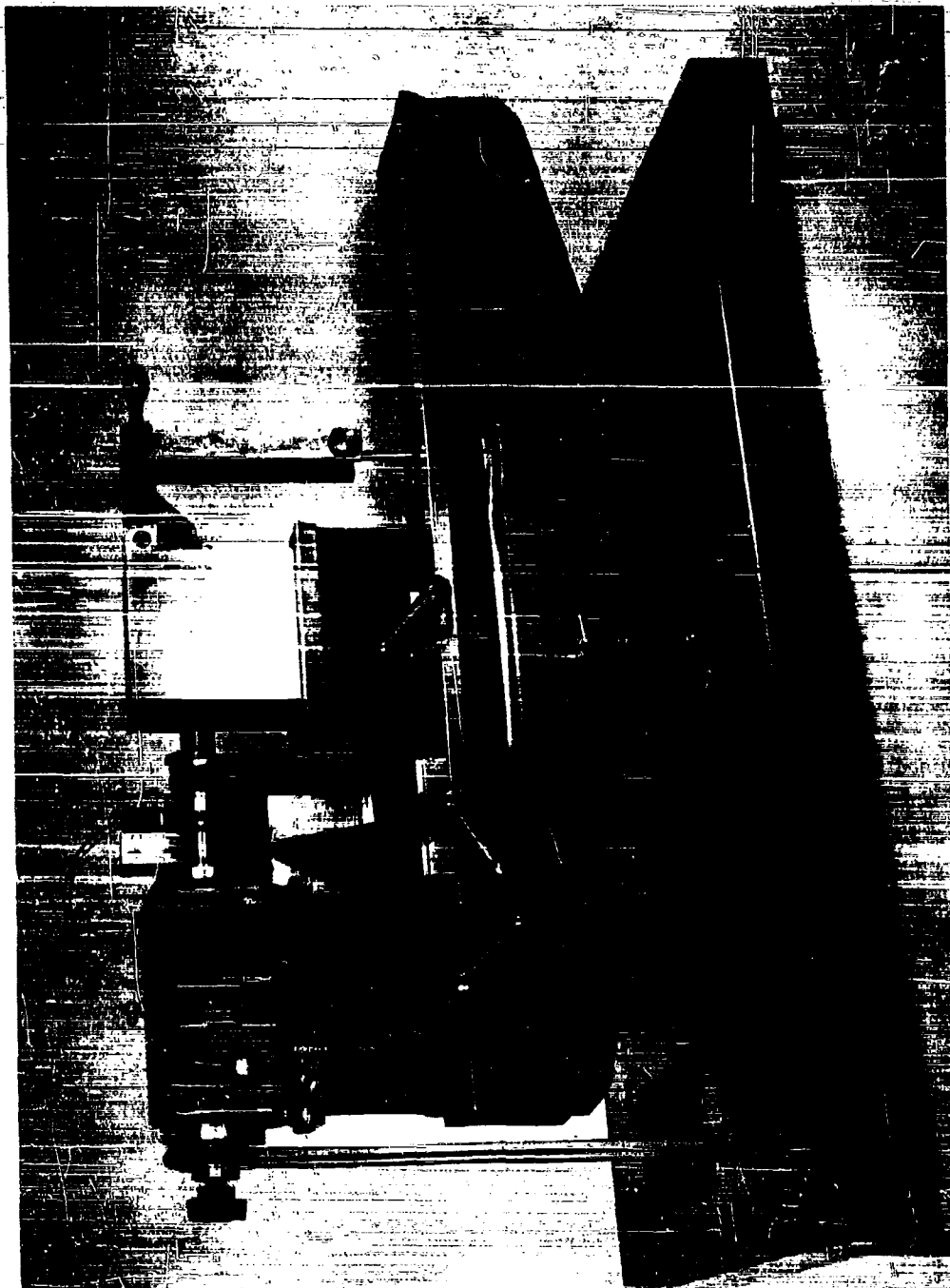
ENCLOSURE 1 VIBRATION TESTER FOR MEASUREMENT OF BEARING
VIBRATION UNDER RADIAL AND AXIAL LOAD

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ENCLOSURE 2 APPLICATION OF RADIAL AND AXIAL LOAD TO TEST BEARINGS

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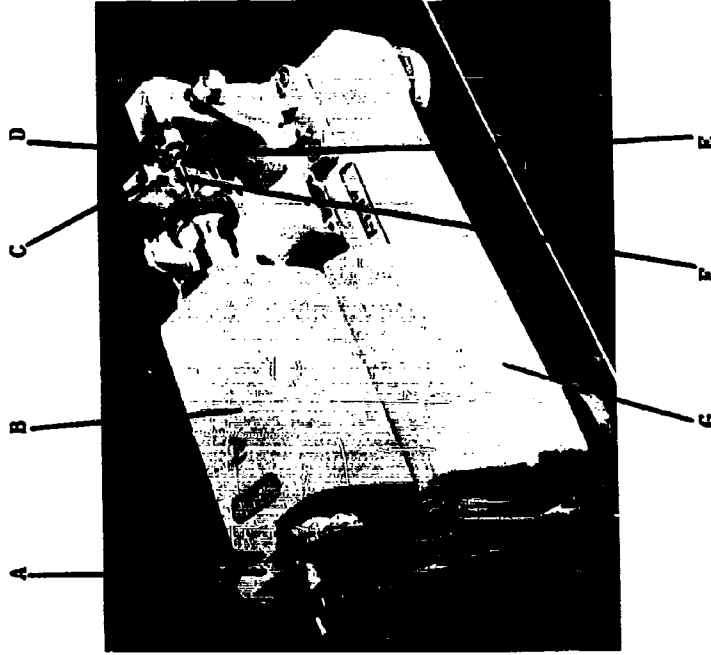
ENCLOSURE 3 MVB-1 VIBRATION TESTER

-160-

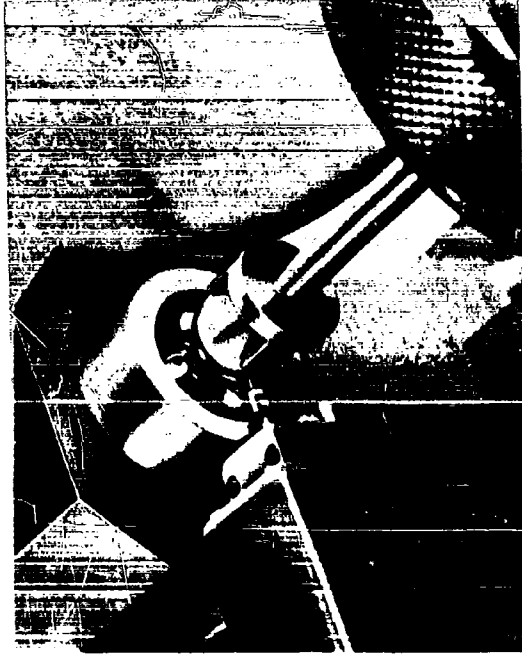
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ENCLOSURE 4 IN LEFT RACE NAVINESS TESTER TYPE NR-1



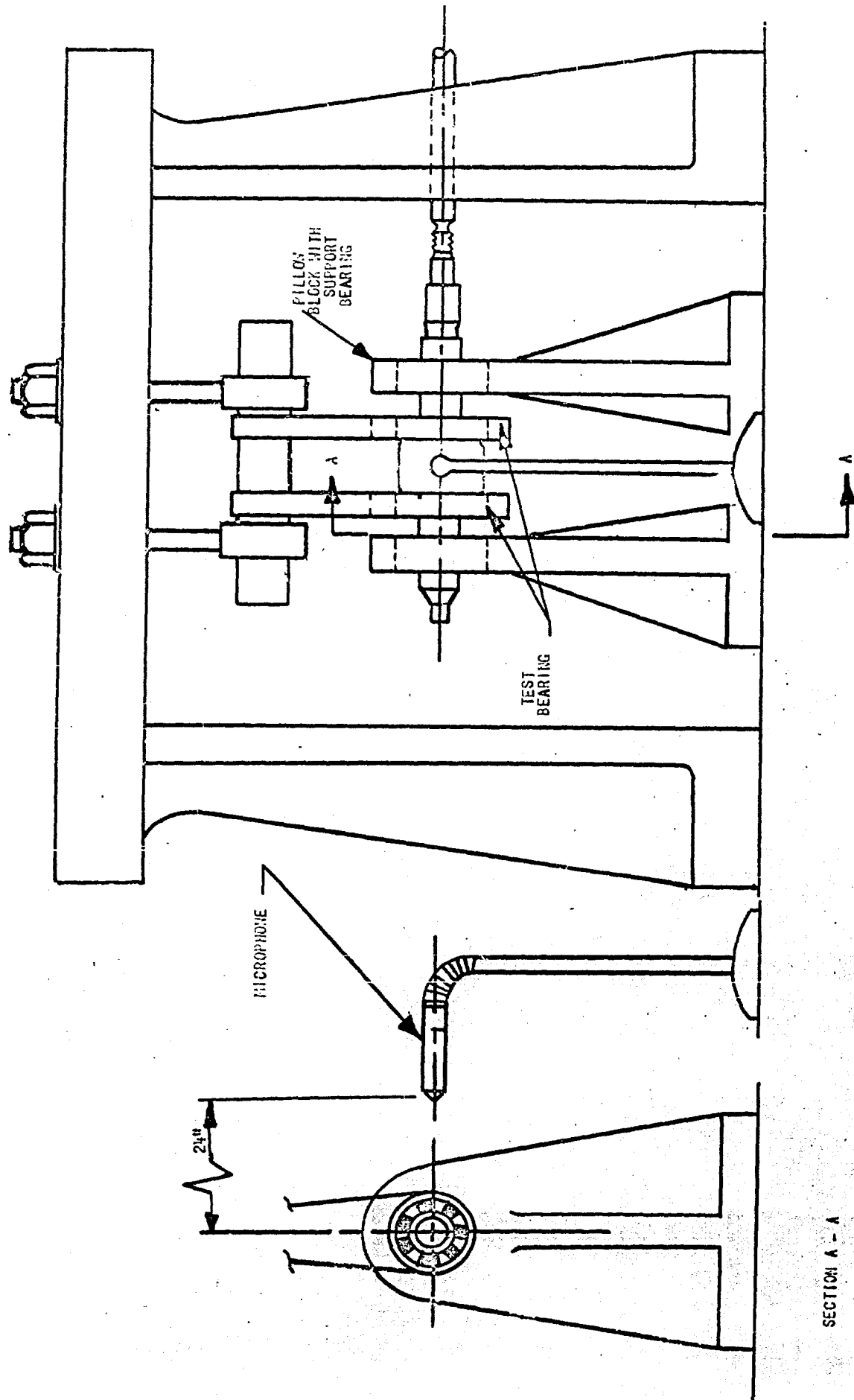
- A - Belt
- B - Spindle Housing
- C - Pickup
- D - Ball Seat
- E - Test Ball
- F - Ball Driver
- G - Base



NEST, BALL and PICKUP

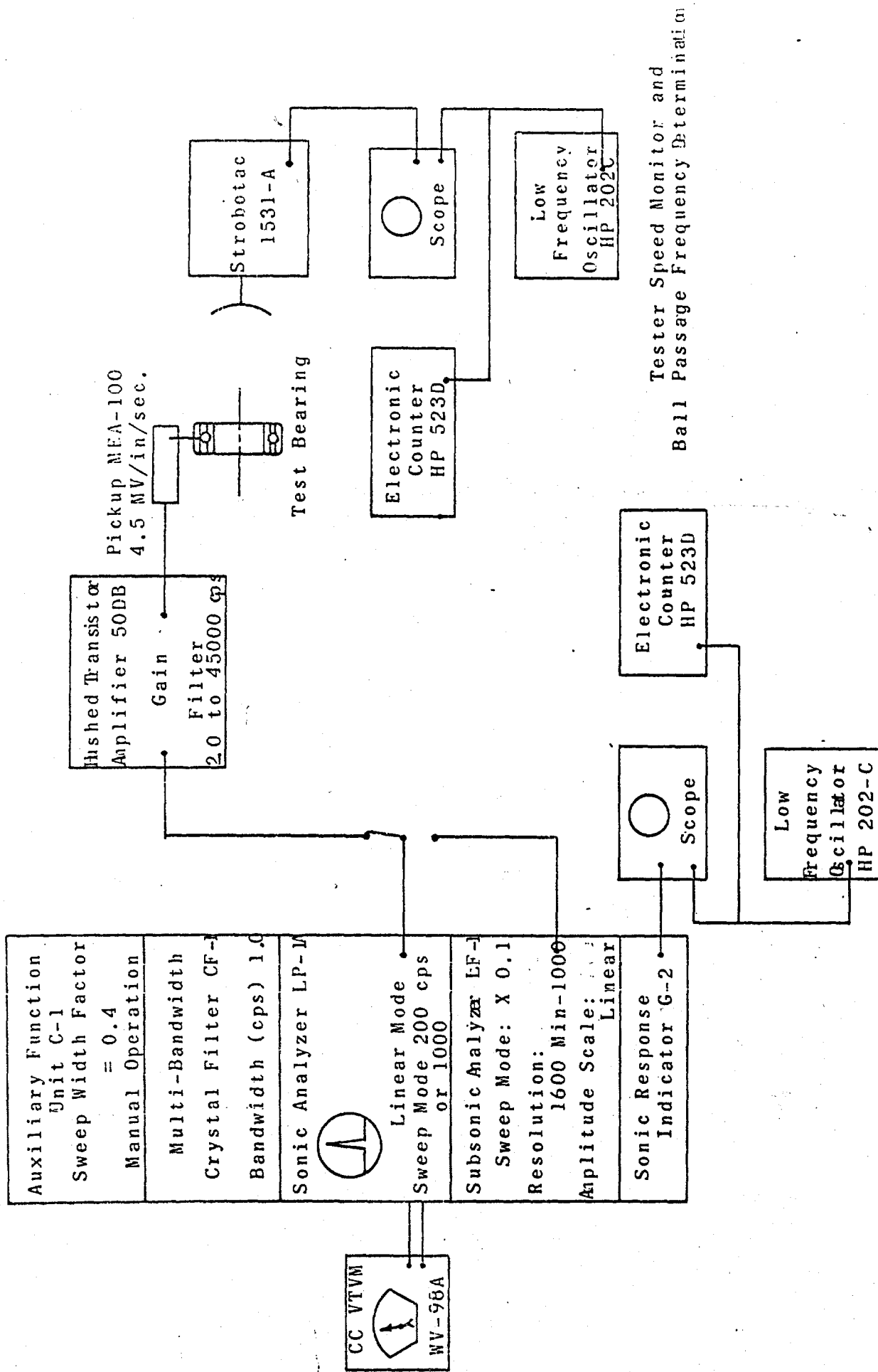
ENCLOSURE 5 MECHANICAL UNIT OF BALL MAYNESS TESTER TYPE MB-1

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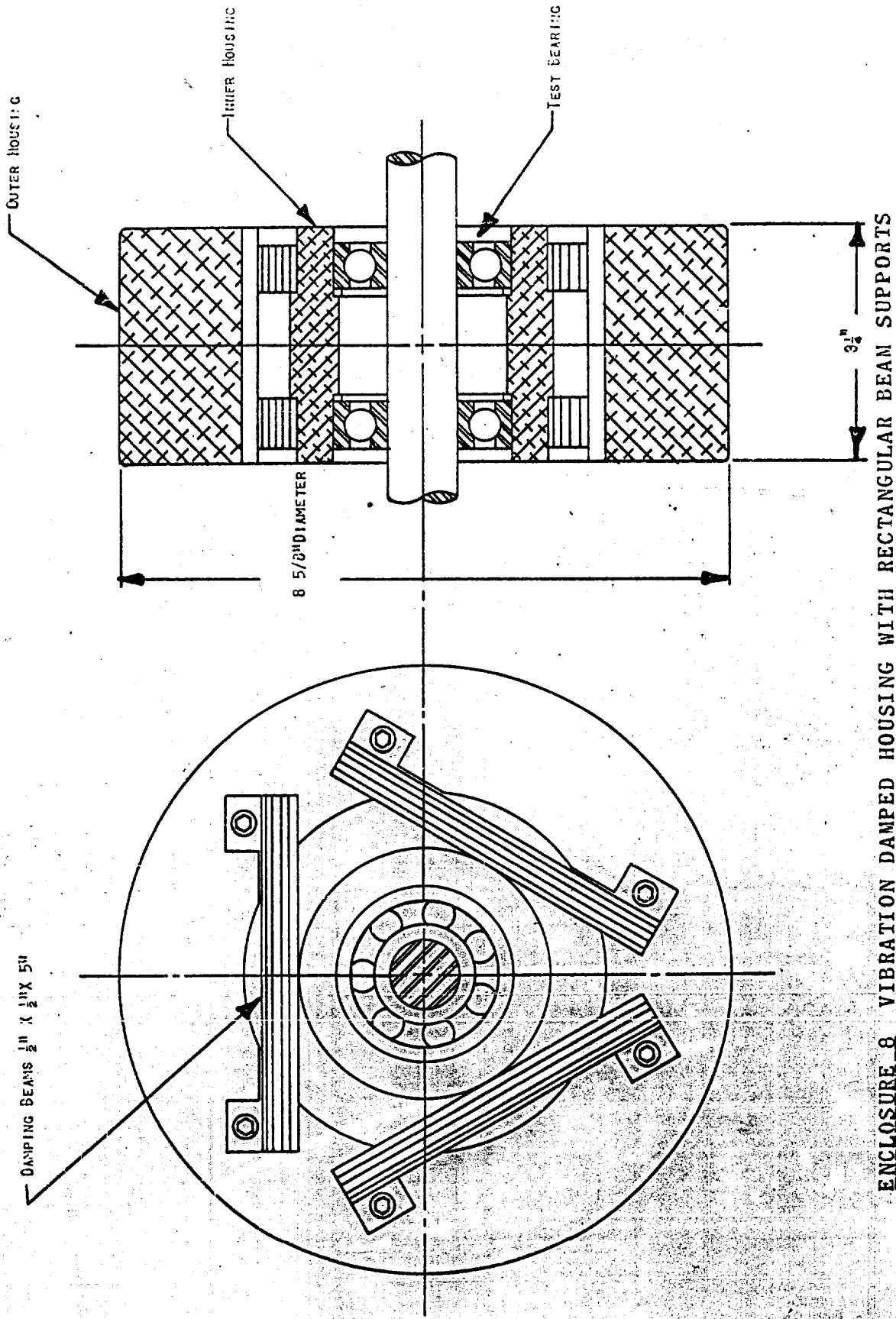
ENCLOSURE 6 LOCATION OF MICROPHONE WITH RESPECT TO TESTER

Panoramic Radio Products



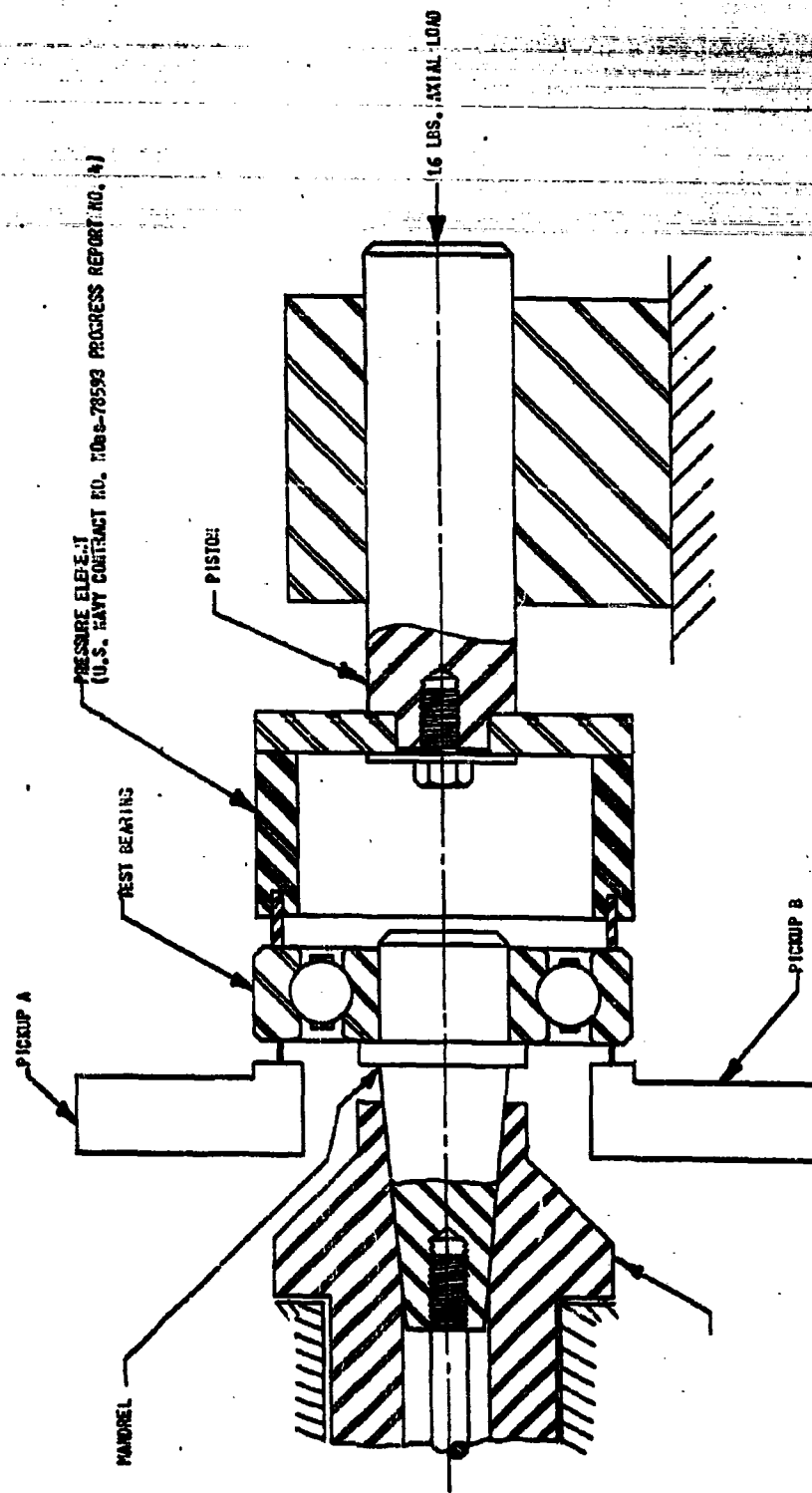
ENCLOSURE 7 INSTRUMENTATION LAYOUT FOR NARROW BAND ANALYSIS OF BEARING VIBRATION

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ENCLOSURE 8 VIBRATION DAMPED HOUSING WITH RECTANGULAR BEAM SUPPORTS

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ENCLOSURE 9 MECHANICAL SET UP FOR MEASURING AXIAL AND ANGULAR VIBRATION

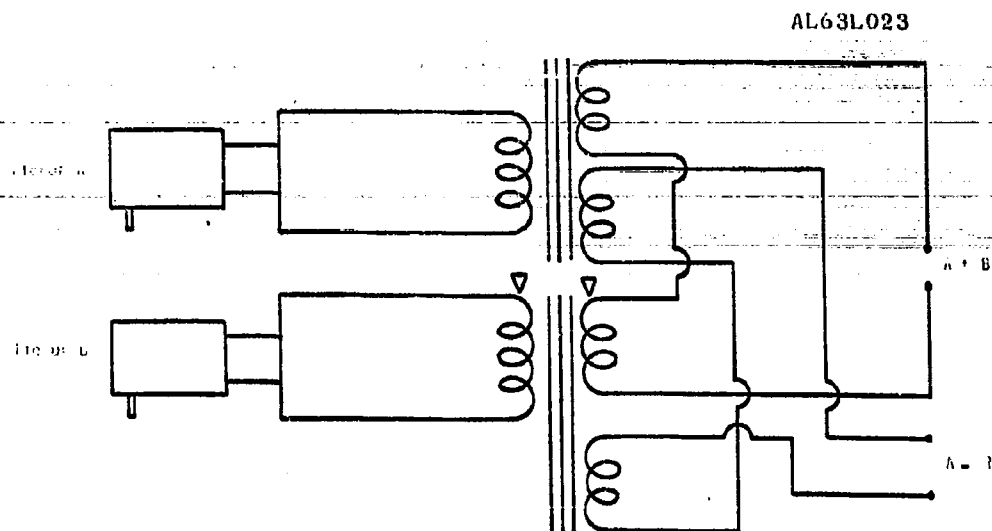
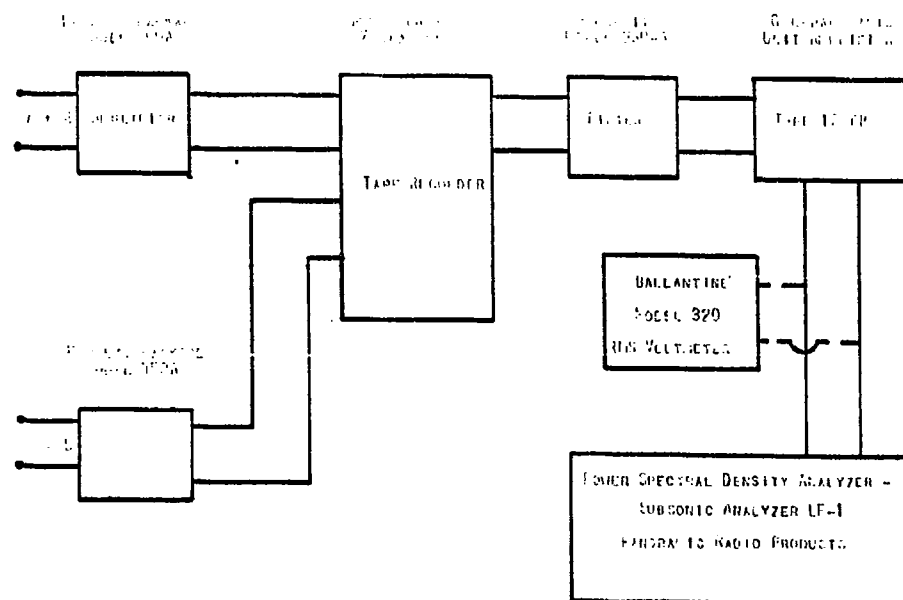


Fig. 10-6-12



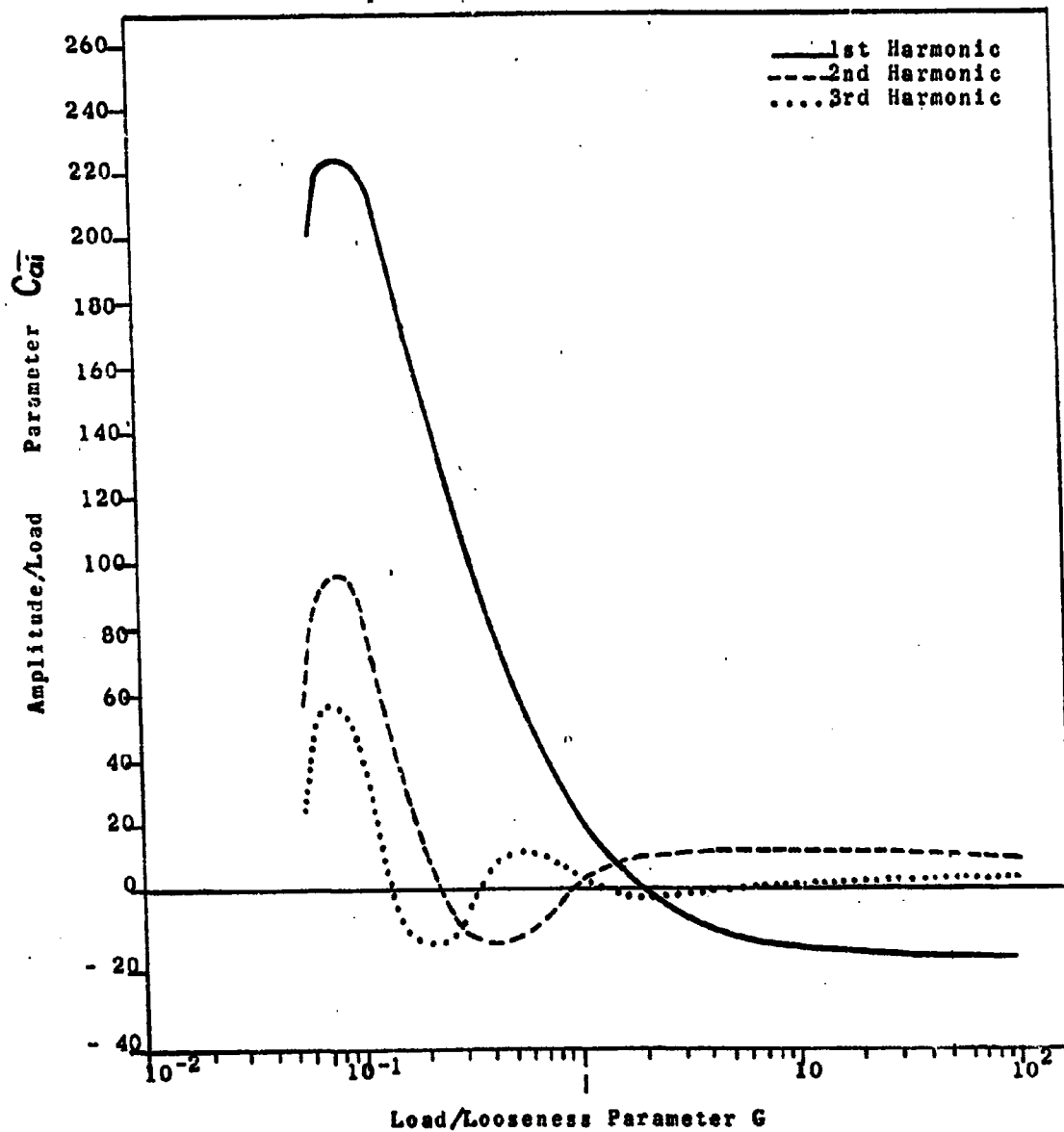
ENCLOSURE 10 BLOCK DIAGRAM OF EQUIPMENT FOR MEASURING AXIAL AND ANGULAR VIBRATIONS

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ENCLOSURE 11

VARIABLE COMPLIANCE VIBRATION IN A BEARING WITH SEVEN BALLS

COMPONENT COLLINEAR WITH LOAD

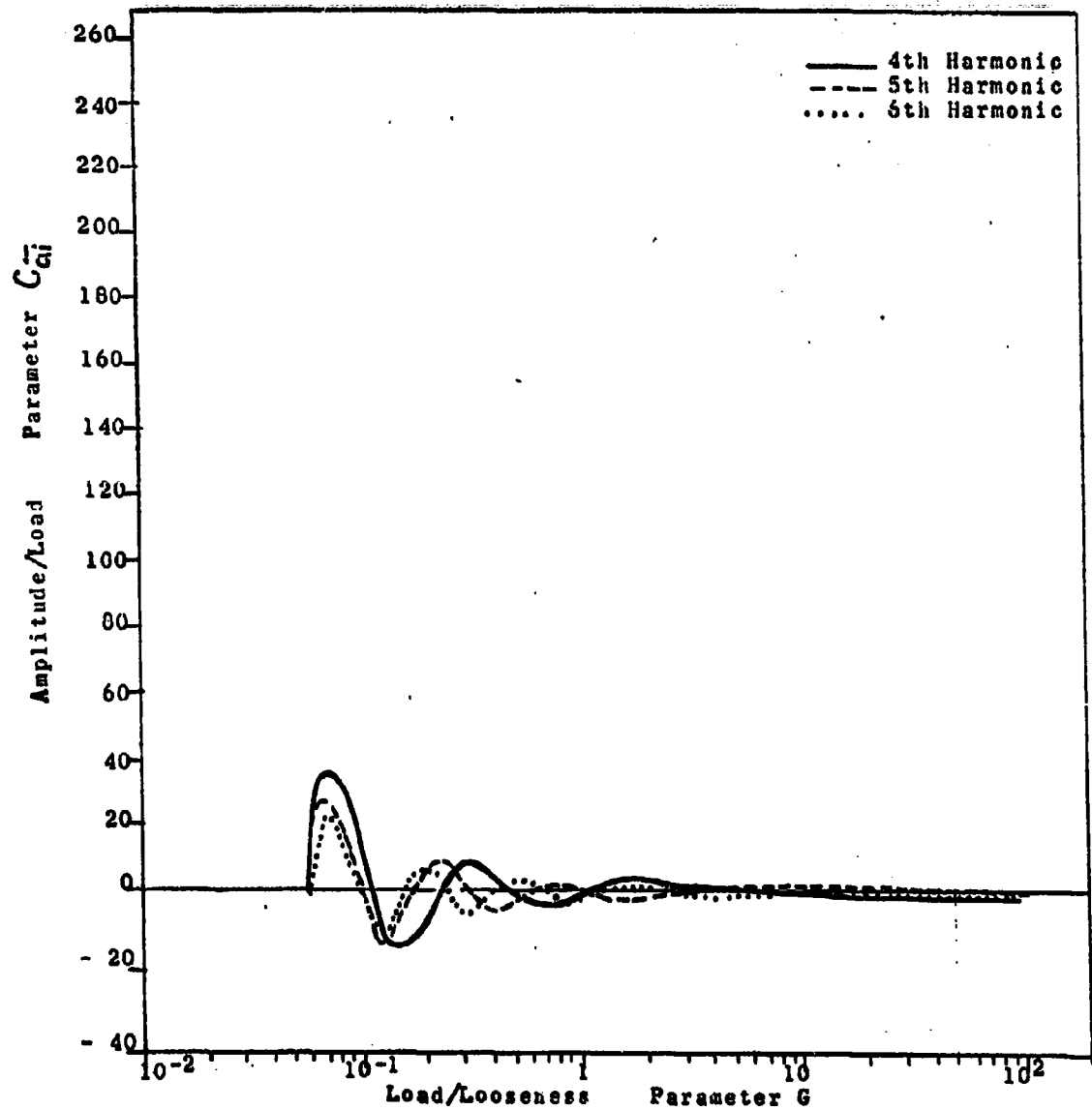


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ENCLOSURE 12

VARIABLE COMPLIANCE VIBRATION IN A BEARING WITH SEVEN BALLS

COMPONENT COLLINEAR WITH LOAD

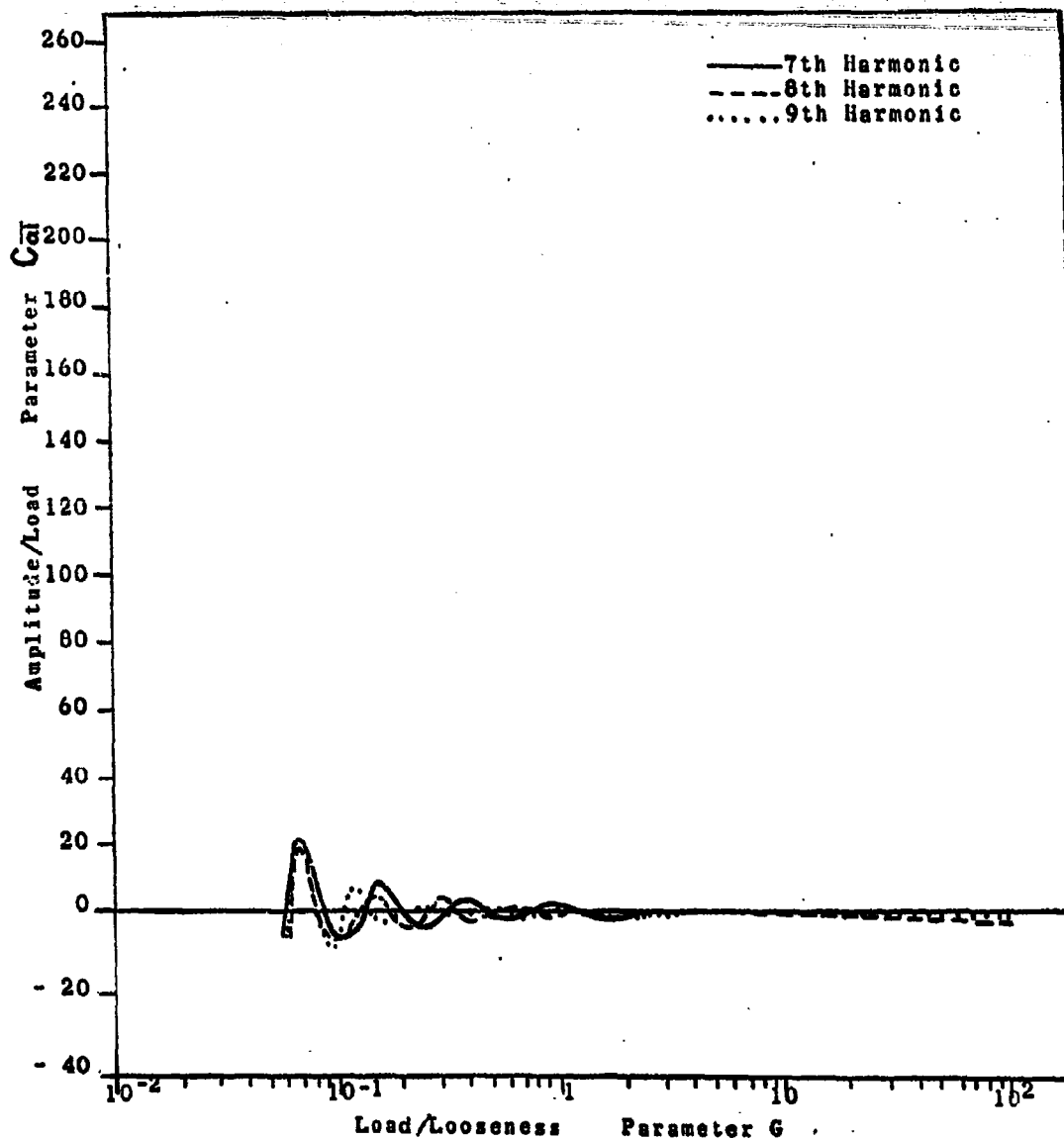


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ENCLOSURE 13

VARIABLE COMPLIANCE VIBRATION IN A BEARING WITH SEVEN BALLS

COMPONENT COLLINEAR WITH LOAD

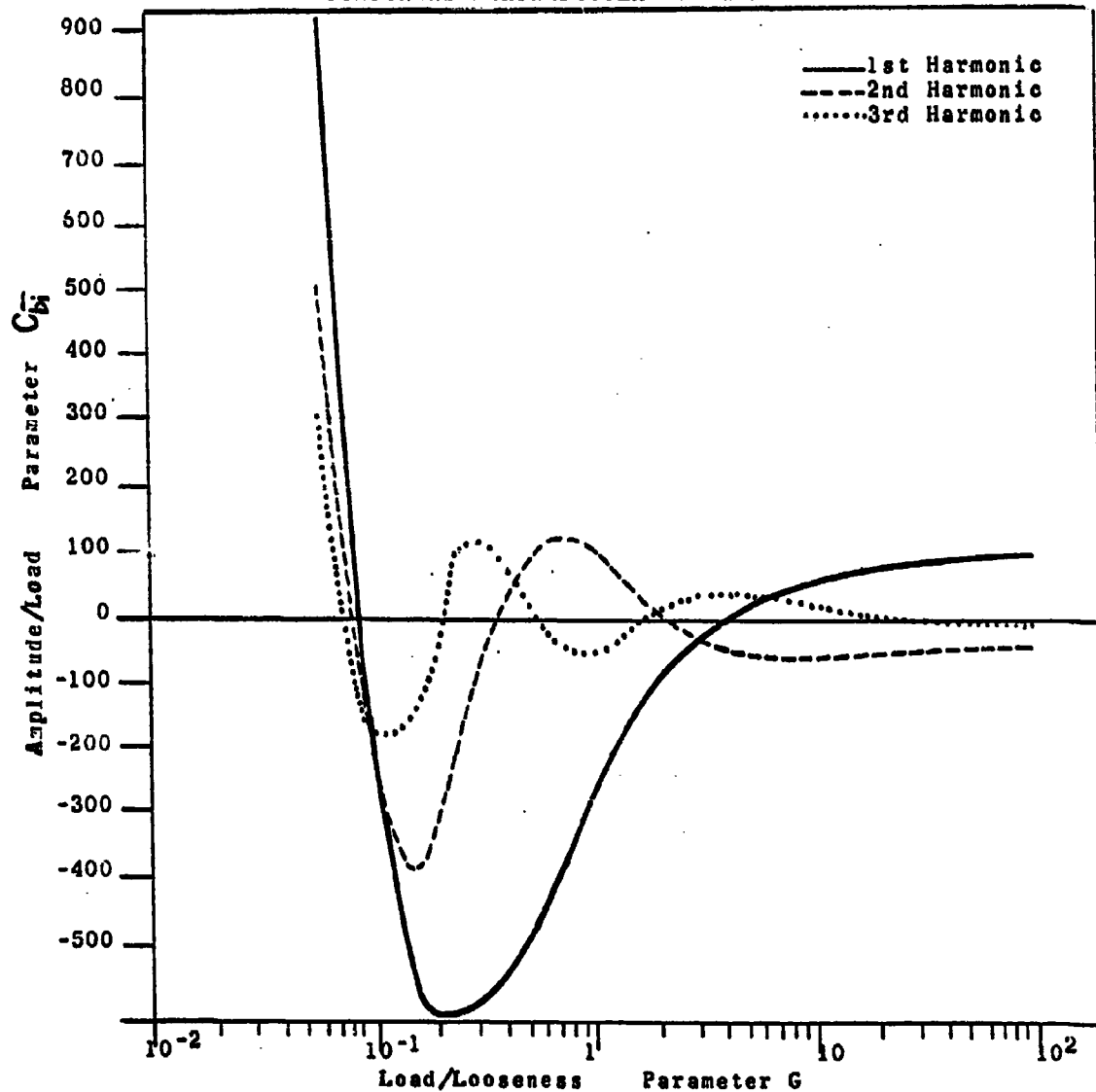


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ENCLOSURE 14

VARIABLE COMPLIANCE VIBRATION IN A BEARING WITH SEVEN BALLS

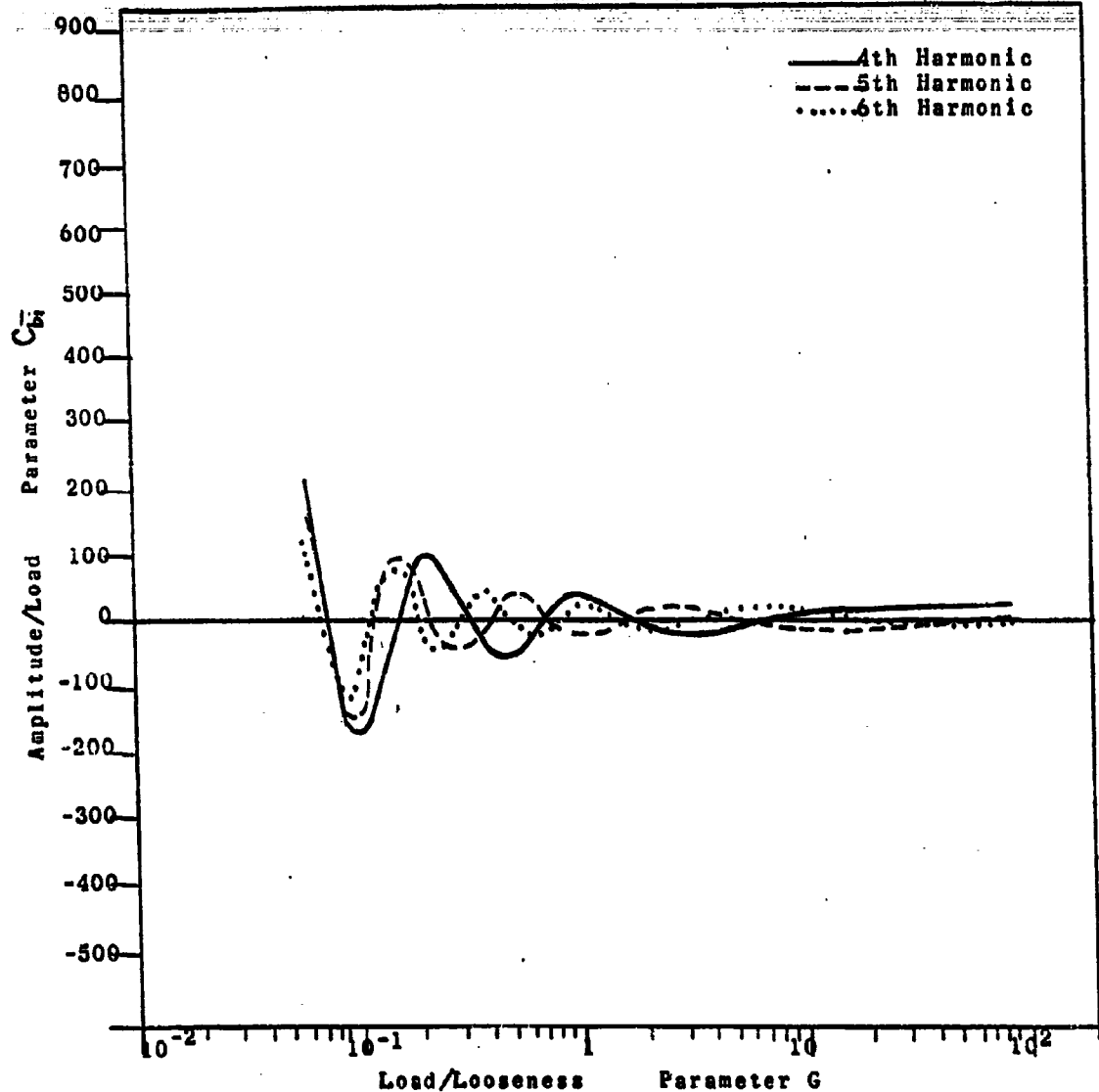
COMPONENT PERPENDICULAR TO LOAD



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ENCLOSURE 15

VARIABLE COMPLIANCE VIBRATION IN A BEARING WITH SEVEN BALLS
COMPONENT PERPENDICULAR TO LOAD

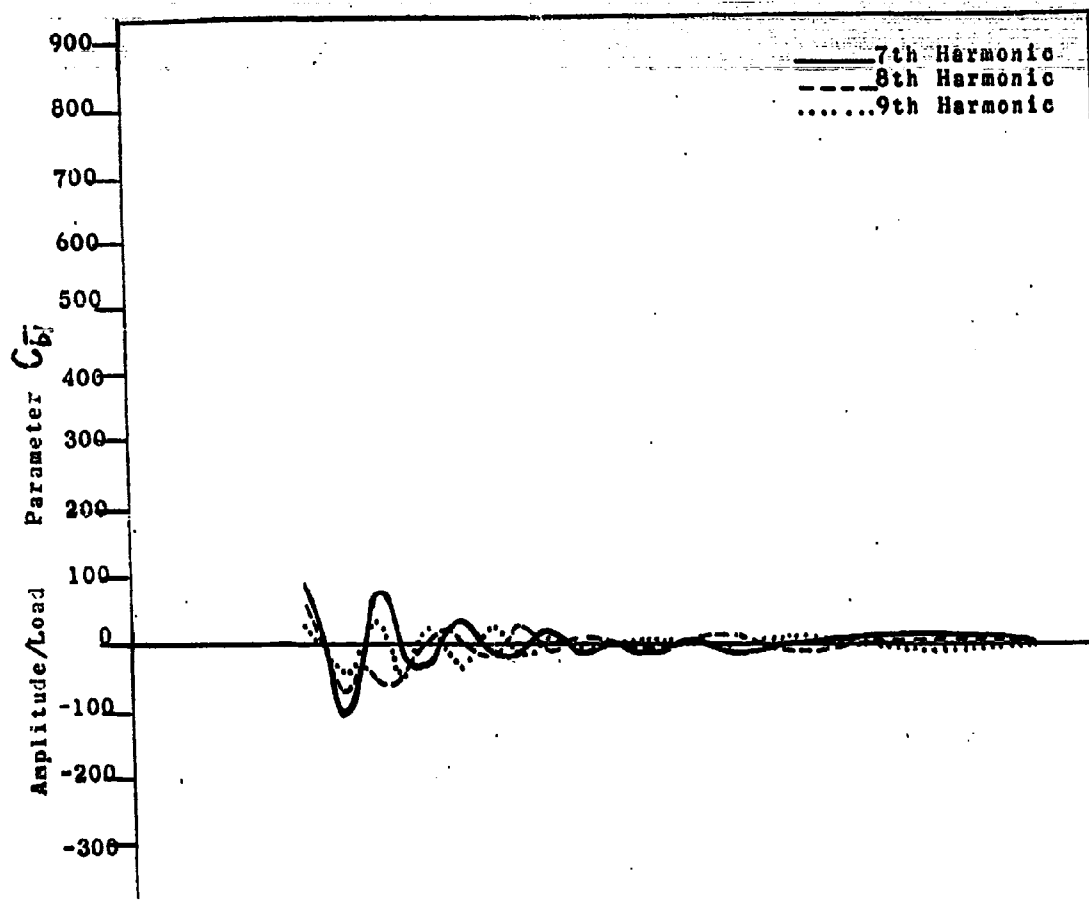


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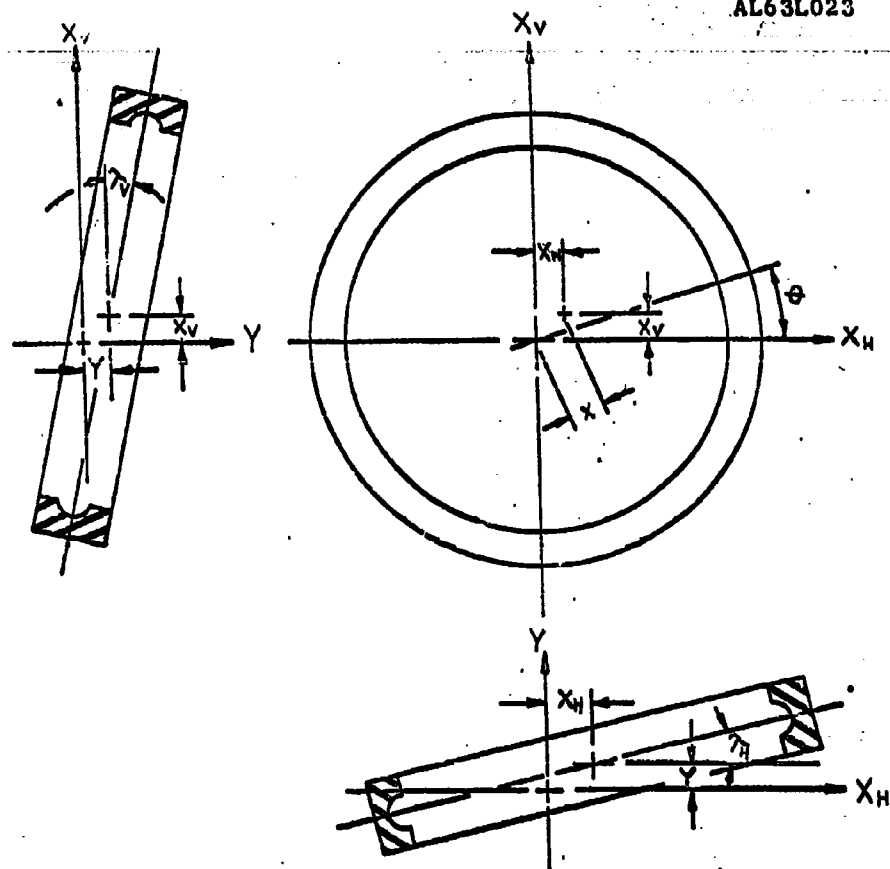
ENCLOSURE 16

VARIABLE COMPLIANCE VIBRATION IN A BEARING WITH SEVEN BALLS

COMPONENT PERPENDICULAR TO LOAD



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- X_v - VERTICAL DISPLACEMENT COMPONENT
- X_h - HORIZONTAL DISPLACEMENT COMPONENT
- Y - AXIAL DISPLACEMENT COMPONENT
- λ_h - ANGULAR DISPLACEMENT COMPONENT IN THE HORIZONTAL DIRECTION
- λ_v - ANGULAR DISPLACEMENT COMPONENT IN THE VERTICAL DIRECTION

ENCLOSURE 17 OUTER RING DISPLACEMENT COMPONENTS

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ENCLOSURE 10COMPUTED VALUES OF $k_N \times F_A^{-1/3}$ FOR BALL BEARINGS

Inner and Outer Ring Groove Conformity = 51.75%

 k_N = Linearized Hertzian Coefficient of Contact Deformation Lb/In. F_A = Axial Load Lbs.

Bearing No.	Ball Size (in)	No. of Balls	Pitch Circle Diameter (in)	Contact Angle (Degree)	$k_N F_A^{-1/3}$ Lb/In
6203	17/64	8	1.122	5	8630
				15	6000
				25	5100
6205	5/16	9	1.516	5	8830
				15	6150
				25	5220
6207	7/16	9	2.106	5	9980
				15	6950
				25	5910
6305	7/16	7	1.713	5	16460
				15	7550
				25	6410
6310	3/4	8	3.150	5	12420
				15	8640
				25	7340
6312	7/8	8	3.740	5	13190
				15	9170
				25	7790
6322	1 5/8	8	6.889	5	16200
				15	11270
				25	9570

$$\beta_N = k_N \left[\frac{z \sin \alpha}{F_A} \right]^{\frac{1}{3}}$$

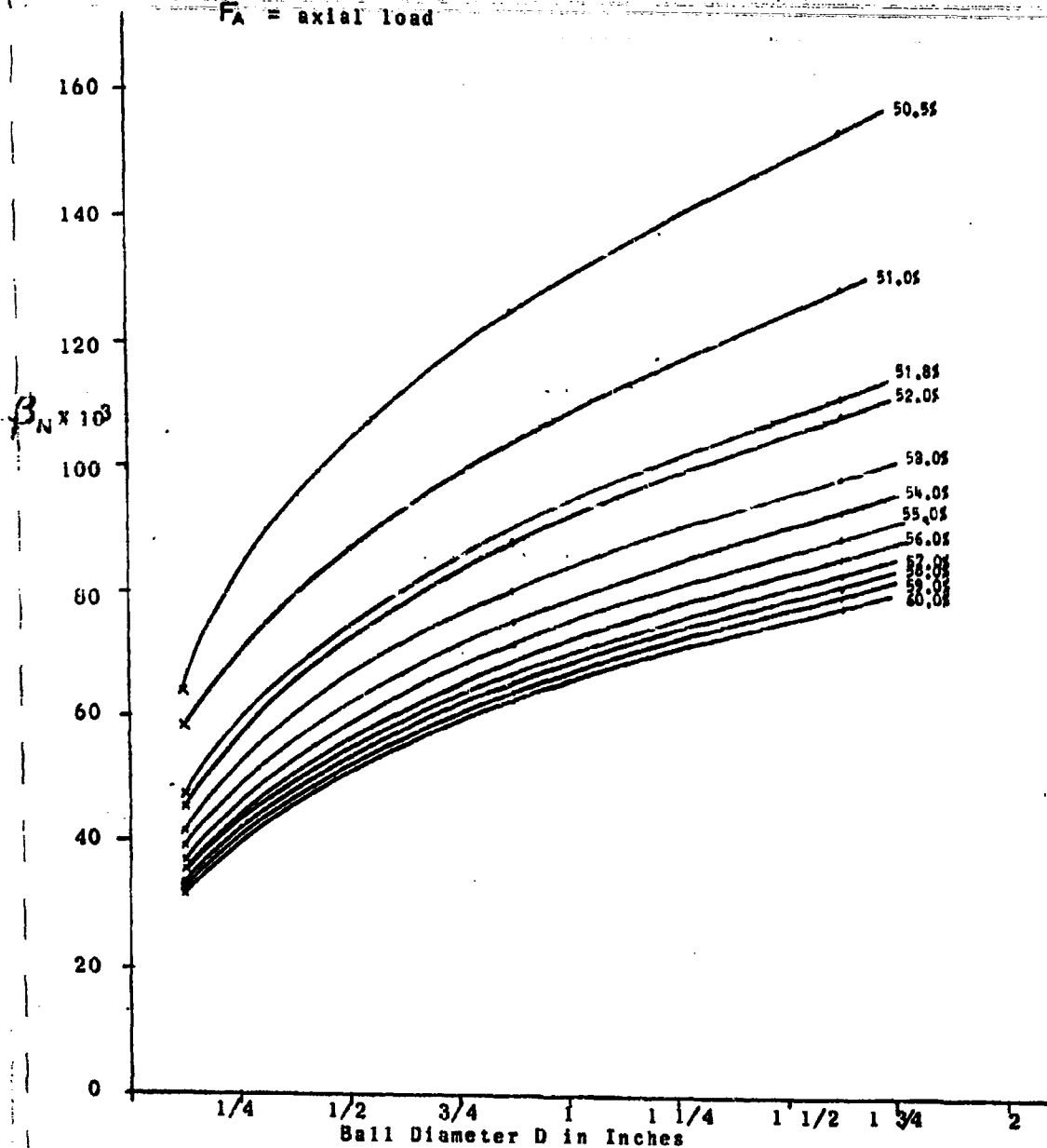
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k_N = linearized Hertzian Coefficient

z = number of balls

α = contact angle = 15°

F_A = axial load



ENCLOSURE 19 β_N AS A FUNCTION OF BALL DIAMETER FOR VARIOUS CONFORMITIES (THE CONFORMITY IS THE SAME FOR THE INNER AND OUTER RING GROOVES.)

ENCLOSURE 20

VIBRATION FREQUENCIES GENERATED BY VARIOUS IMPERFECTIONS OF THE ROLLING SURFACES ACCORDING TO RIGID RING THEORY

LINEAR THEORY

TYPE OF SURFACE IMPERFECTION	ORDER OF WAVINESS	VIBRATION FREQUENCIES	
		RADIAL AND ANGULAR VIBRATION	AXIAL VIBRATION
INNER RING ECCENTRICITY	$k = 1$	f_r	-
INNER RING WAVINESS	$k = nZ - 1$	$k f_i - f_c = n F_i - f_r$	-
	$k = nZ$	-	$k f_i = n F_i$
	$k = nZ + 1$	$k f_i + f_c = n F_i + f_r$	-
OUTER RING WAVINESS	$k = nZ - 1$	$(k+1) f_c = n F_o$	-
	$k = nZ$	-	$k f_c = n F_o$
	$k = nZ + 1$	$(k-1) f_c = n F_o$	-
BALL WAVINESS	$k = 2n$	$\begin{cases} k f_b - f_c \\ k f_b + f_c \end{cases}$	$k f_b$
BALL DIAMETER VARIATION		f_c	-

NON-LINEAR THEORY

INNER RING WAVINESS	$k \neq nZ \pm 1$	$k f_i \pm f_c$	-
OUTER RING WAVINESS	$k \neq nZ \pm 1$	$(k \pm 1) f_c$	-

$f_r = \frac{1}{2} \omega$ = ROTATIONAL FREQUENCY OF INNER RING
 $f_i = \frac{1}{2} \omega \left(1 + \frac{D}{d} \cos \alpha \right)$ = FREQUENCY OF THE ROTATING BALL SET WITH RESPECT TO THE INNER RING
 $f_c = \frac{1}{2} \omega \left(1 - \frac{D}{d} \cos \alpha \right)$ = FREQUENCY OF THE ROTATING BALL SET WITH RESPECT TO THE OUTER RING (CASE FREQUENCY)
 $f_b = \frac{1}{2} \omega \left(1 - \frac{D^2}{d^2} \cos^2 \alpha \right)$ = FREQUENCY OF THE POLAR ROTATION OF THE BALL IN A SYSTEM ATTACHED TO THE CASE
 $F_i = Z f_i$ = BALL PASSAGE FREQUENCY OVER THE INNER RING
 $F_o = Z f_c$ = BALL PASSAGE FREQUENCY OVER THE OUTER RING

D = BALL DIAMETER

d = BEARING PITCH DIAMETER

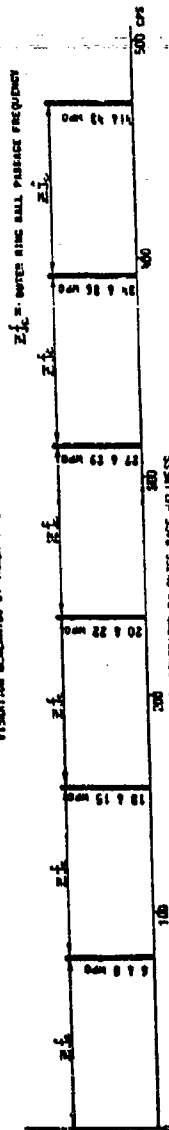
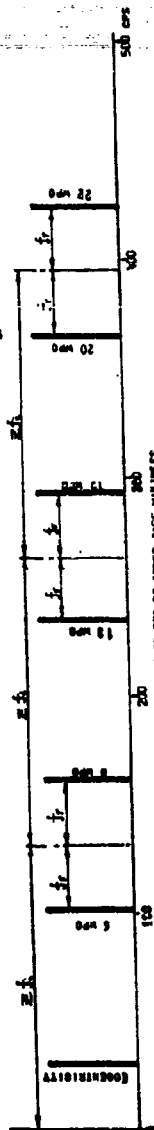
α = CONTACT ANGLE

$n = 1, 2, 3, \dots$

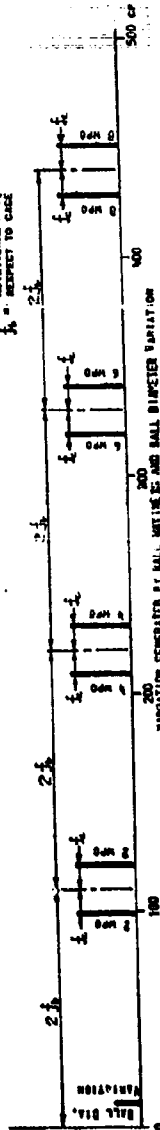
Z = NUMBER OF BALLS

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$Z_{f_r} =$ INNER RING BALL PASSAGE FREQUENCY
 $f_r =$ ROTATIONAL FREQUENCY OF INNER RING

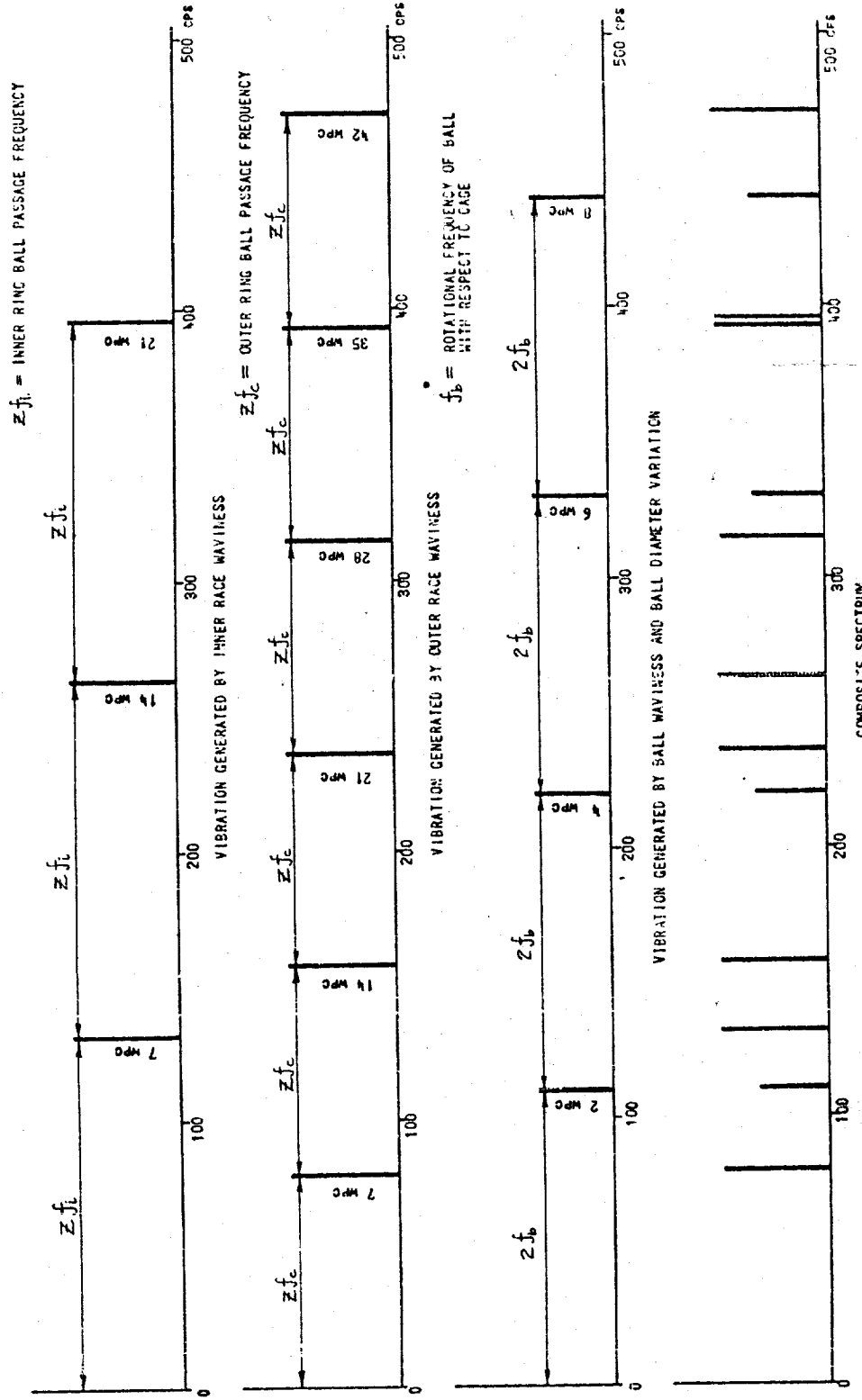


$f_r =$ ROTATIONAL FREQUENCY OF BALL WITH
 RESPECT TO CASE



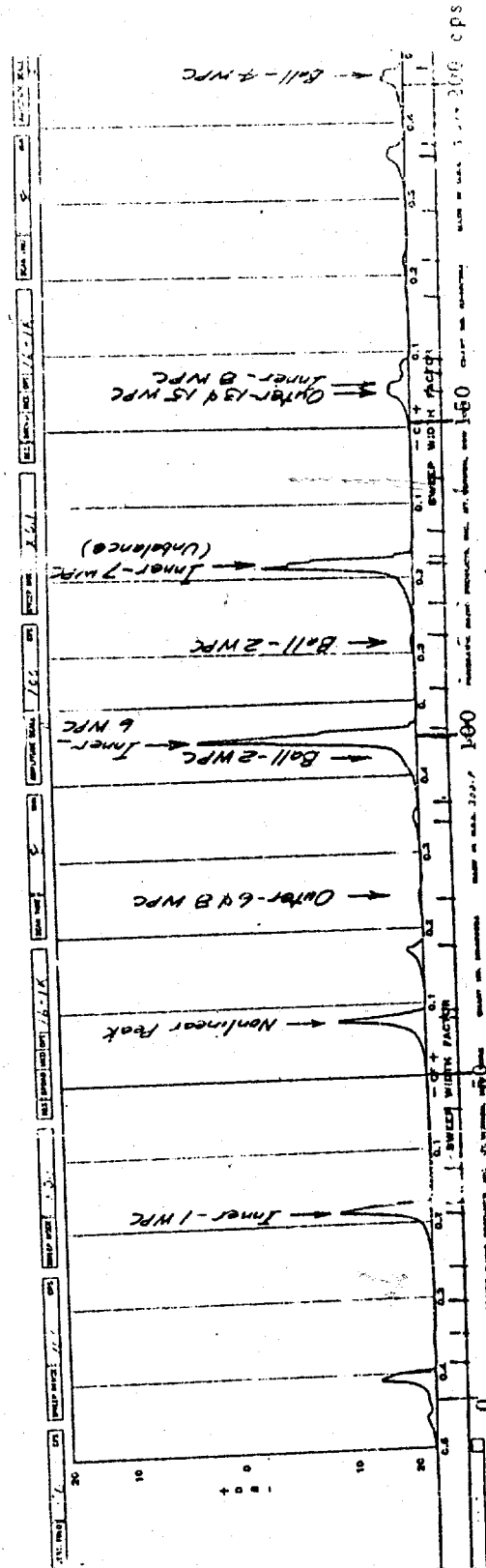
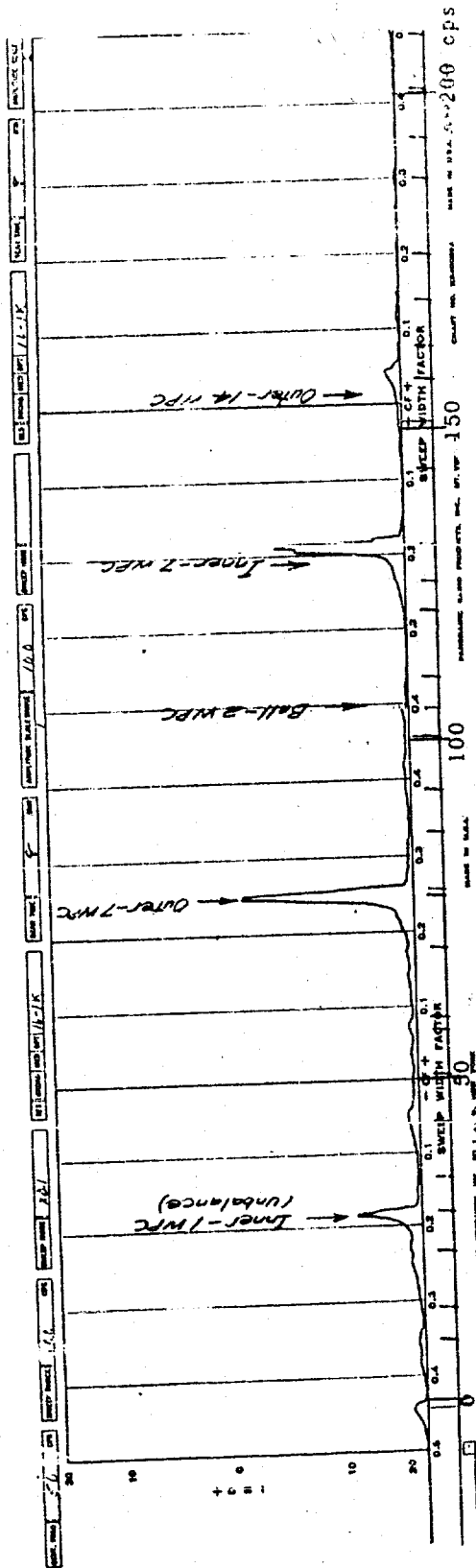
ENCLOSURE 21 SYNTHETIC SPECTRUM OF RADIAL AND ANGULAR VIBRATION OF 6305 BEARING
 ROTATING AT 1800 RPM (LINEAR RIGID RING THEORY)

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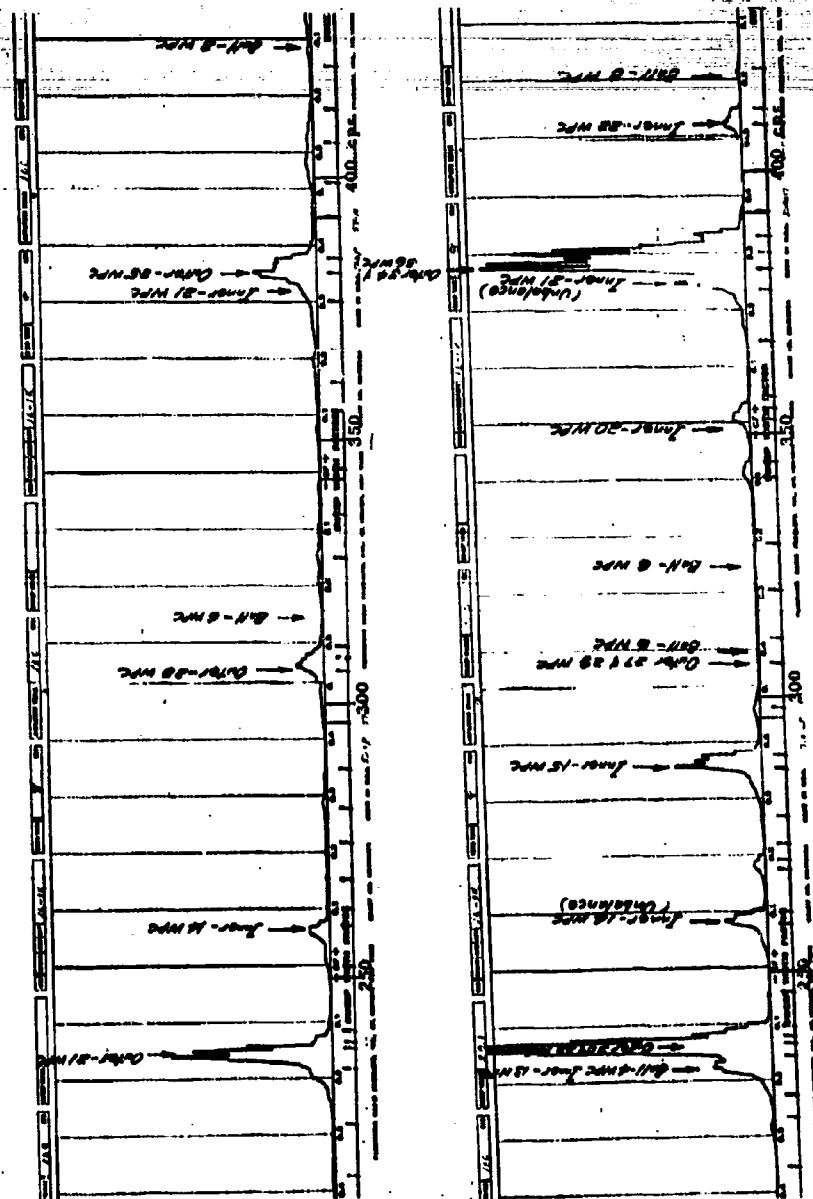
ENCLOSURE 22 SYNTHETIC SPECTRUM OF AXIAL VIBRATION OF 6305 BEARING ROTATING AT 1800 RPM (LINEAR RIGID RING THEORY)

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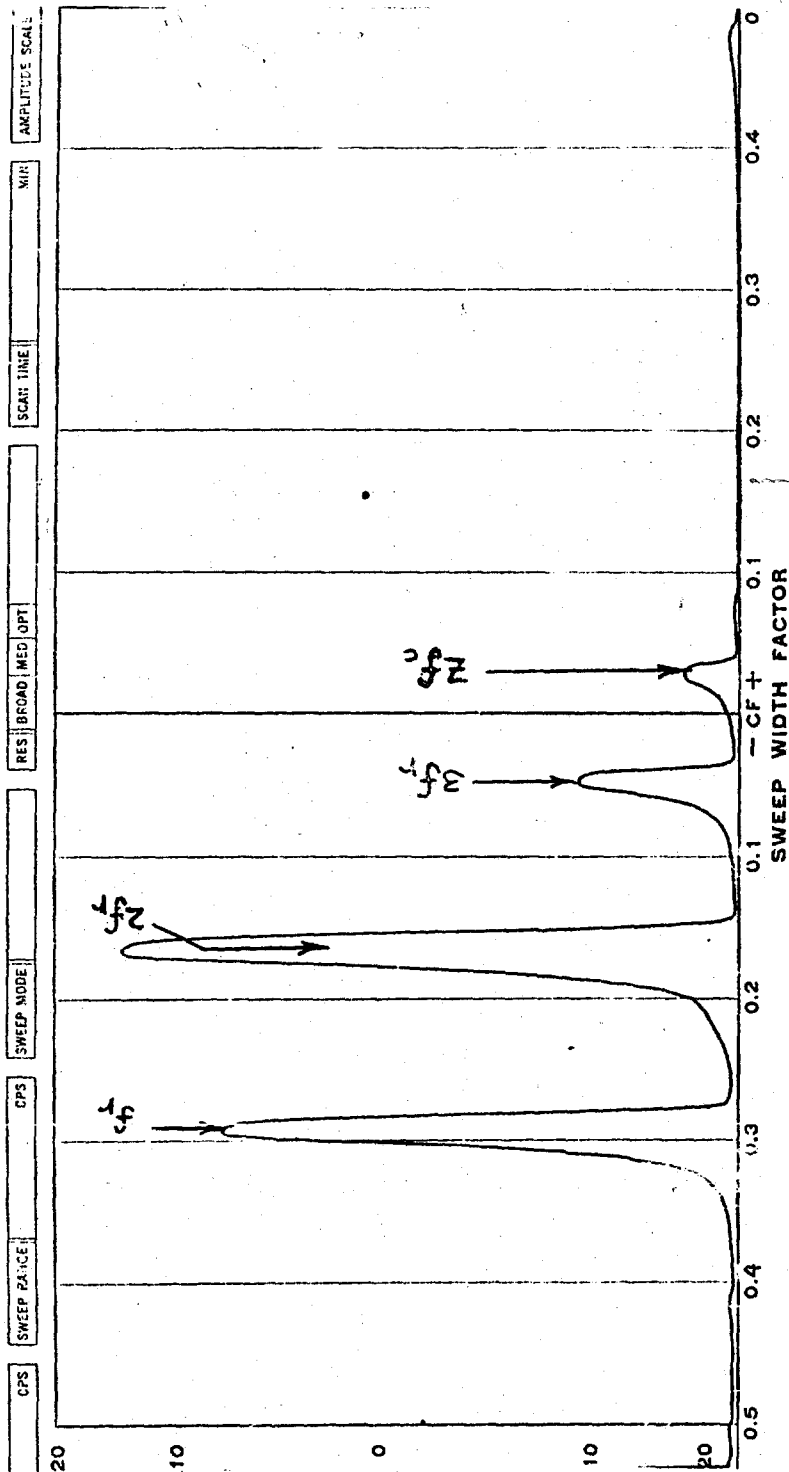
ENCLOSURE 24 AXIAL (TOP) AND ANGULAR (BOTTOM) VIBRATION SPECTRUM OF 6305 BEARING 0-200 CPS

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f_r = Rotational Frequency
 $z f_c$ = Ball Pass Frequency

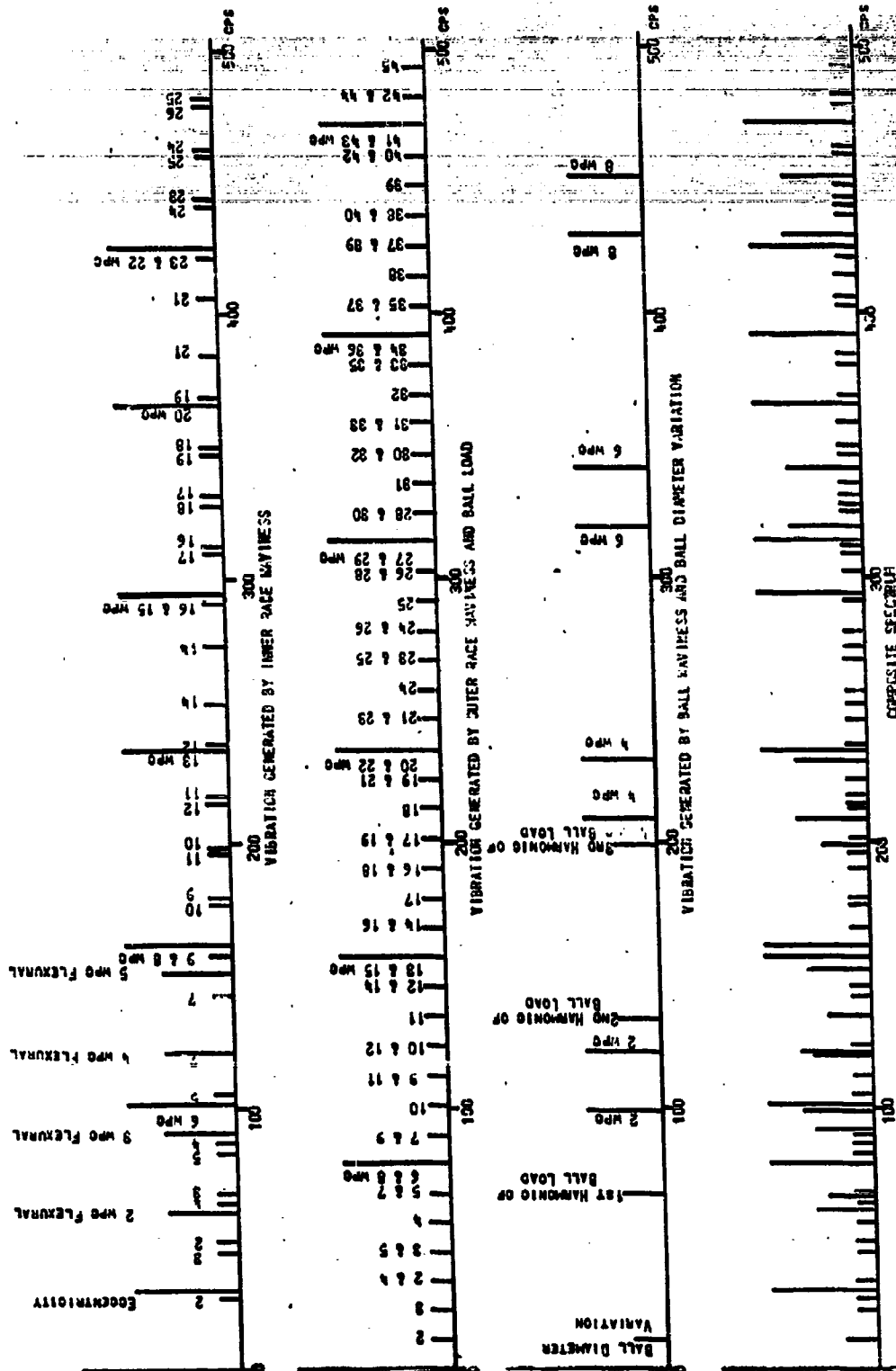


PANORAMIC RADIO PRODUCTS, INC., MT. VERNON, NEW YORK CHART NO. NR-10056A MADE IN U.S.A.

0 50 100 150 200

ENCLOSURE 26 NARROW BAND SPECTRUM OF A 6205 BALL BEARING WITH HIGH
 INNER RING 2 WPC AND 3 WPC WAVINESS AT A ROTATIONAL
 SPEED OF 1800 RPM IN THE 0-200 CPS FREQUENCY RANGE.

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ENCLOSURE 27 SYNTHETIC VIBRATION SPECTRUM OF 6305 BEARING WITH LARGE BALL DIAMETER VARIATION, ROTATING AT 1800 RPM (NON-LINEAR THEORY)

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ENCLOSURE 28

NUMBER OF WAVES PER CIRCUMFERENCE CORRESPONDING TO VIBRATION FREQUENCIES ACCORDING TO RIGID RING THEORY
FOR THE 6200 BEARING SERIES ROTATING AT 1000 RPM

BEARING SIZE	BALL SIZE (IN)	NO. OF BALLS	PITCH DIAMETER (IN)	BEARING PART	WAVINESS RANGES (WPO) VIBRATION FREQUENCY BANDS (OPS)		
					50-300	300-1800	1800-10000
6200	3/16	7	0.787	INNER	3-16	16-96	96-534
				OUTER	4-27	27-159	159-887
				BALL	2-6	6-40	40-222
6201	15/64	7	0.867	INNER	3-15	16-94	94-520
				OUTER	5-28	28-166	166-925
				BALL	2-6	6-36	36-210
6202	15/64	8	0.984	INNER	3-16	16-97	97-538
				OUTER	4-26	26-158	158-876
				BALL	2-6	6-36	36-210
6203	17/64	8	1.122	INNER	3-16	16-96	96-535
				OUTER	4-26	26-159	159-882
				BALL	2-6	6-36	36-210
6204	5/16	8	1.319	INNER	3-16	16-97	97-538
				OUTER	4-26	24-158	158-878
				BALL	2-6	6-38	38-214
6205	5/16	9	1.516	INNER	3-17	17-99	99-552
				OUTER	4-25	25-151	151-842
				BALL	2-4	4-32	32-180
6206	8/8	9	1.811	INNER	3-17	17-99	99-552
				OUTER	4-25	25-152	152-844
				BALL	2-4	4-26	26-148
6207	7/16	9	2.106	INNER	3-17	17-99	99-548
				OUTER	4-25	24-158	153-850
				BALL	2-6	6-34	34-188
6208	15/82	9	2.362	INNER	3-17	17-99	99-552
				OUTER	4-25	25-151	151-841
				BALL	2-4	4-32	32-180
6209	1/2	9	2.559	INNER	3-17	17-100	100-556
				OUTER	4-25	25-150	150-834
				BALL	2-4	4-32	32-170
6210	1/2	10	2.756	INNER	3-17	17-101	101-564
				OUTER	4-24	24-147	147-817
				BALL	2-4	4-28	28-156

1. VIBRATIONS IN THE 0-50 OPS RANGE ARE INDUCED BY ECCENTRICITY AND BALL DIAMETER VARIATIONS FOR ALL BEARING SIZES.

2. FLEXURAL BENDING OF THE OUTER RING DUE TO 2-5 WPO INNER RING WAVINESS INDUCES ADDITIONAL VIBRATIONS IN THE 50-300 OPS BAND FOR ALL BEARING SIZES. IN THE 300-1800 OPS BANDS, THE EFFECT OF INNER RING WAVINESS ON FORCED FLEXURAL VIBRATION IS NEGLIGIBLE.

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ENCLOSURE 29

NUMBER OF WAVES PER CIRCUMFERENCE CORRESPONDING TO VIBRATION FREQUENCIES ACCORDING TO RIGID RING THEORY
FOR THE 6300 BEARING SERIES ROTATING AT 1800 RPM

BEARING SIZE	BALL SIZE (IN)	NO. OF BALLS	PITCH DIAMETER (IN)	BEARING PART	WAVINESS RANGES (WPC)		
					VIBRATION FREQUENCY BANDS (CPS)		
					50-300	300-1800	1800-10000
6305	7/16	7	1.713	INNER	3-16	16- 96	96-532
				OUTER	5-27	27-162	162-901
				RAIL	2- 6	6- 32	32-182
6306	81/64	8	2.008	INNER	3-16	16- 97	97-538
				OUTER	4-26	26-158	158-877
				BALL	2- 4	4- 30	30-166
6307	17/32	8	2.263	INNER	3-16	16- 97	97-538
				OUTER	4-26	26-158	158-877
				BALL	2- 4	4- 30	30-164
6308	19/32	8	2.559	INNER	3-17	17- 99	99-546
				OUTER	3-25	25-154	154-855
				BALL	2- 4	4- 30	30-162
6309	11/16	8	2.854	INNER	3-16	16- 97	97-540
				OUTER	5-26	26-156	156-870
				BALL	2- 6	6- 30	30-170
6310	3/4	8	3.150	INNER	3-16	16- 97	97-540
				OUTER	5-26	26-156	156-870
				BALL	2- 6	6- 30	30-170
6311	13/16	8	3.445	INNER	3-16	16- 98	98-549
				OUTER	5-26	26-155	155-860
				BALL	2- 6	6- 30	30-166
6312	7/8	8	3.740	INNER	3-16	16- 98	98-549
				OUTER	5-26	26-155	155-860
				BALL	2- 6	6- 30	30-166
6313	15/16	8	4.035	INNER	3-16	16- 98	98-549
				OUTER	5-26	26-155	155-860
				BALL	2- 6	6- 30	30-164

1. VIBRATIONS IN THE 0-50 CPS RANGE ARE INDUCED BY ECCENTRICITY AND BALL DIAMETER VARIATIONS FOR ALL BEARING SIZES.

2. FLEXURAL BENDING OF THE OUTER RING DUE TO 2-5 WPC INNER RING WAVINESS INDUCES ADDITIONAL VIBRATIONS IN THE 50-300 CPS BAND FOR ALL BEARING SIZES. IN THE 300-1800 CPS BAND, THE EFFECT OF INNER RING WAVINESS ON FORCED FLEXURAL VIBRATION IS NEGLIGIBLE.

ENCLOSURE 30

TABULATION OF NATURAL FREQUENCIES OF THE BEARING OUTER RING WITHOUT HOUSING
FOR BALL BEARINGS WITH 15° CONTACT ANGLE AND 25 LBS AXIAL LOAD

BEARING	DESIGN PARAMETERS					NATURAL FREQUENCIES (CPS)			AXIAL VIBRATIONS	
	Z	A(in. ²)	R(in.)	I(in. ⁴)	f _R	f _{F₁}	f _{F₂}	f _{F₃}	f _A	f _°
6203	8	0.0595	0.773	.000010	995	1460	2500	-	250	-
6205	9	0.1344	0.903	.00013	304	471	852	1520	79	-
6207	9	0.1774	1.278	.00031	234	352	605	1060	61	-
6305	7	0.1293	1.116	.00040	5000	6430	12000	-	1300	-
6312	8	0.4186	2.378	.00428	2490	3510	5370	-	647	-
6322	8	1.247	4.442	.04900	1180	1730	2930	-	307	-

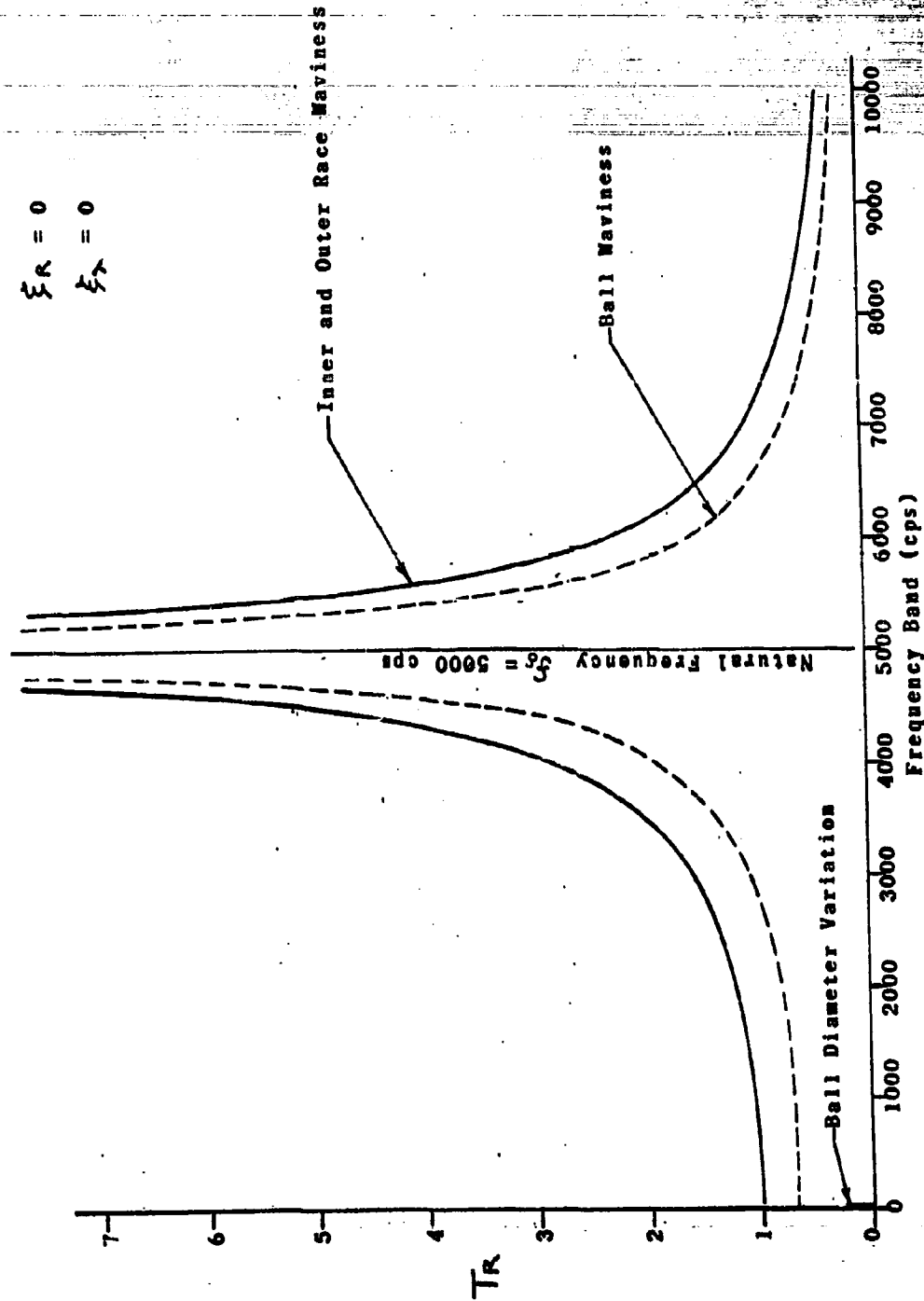
f_R AND f_A ARE THE NATURAL FREQUENCIES OF RIGID BODY VIBRATION
IN THE RADIAL AND IN THE AXIAL DIRECTION, RESPECTIVELY

f_{F₁} · f_{F₂} · f_{F₃} ARE THE NATURAL FREQUENCIES OF THE FLEXURAL
VIBRATION

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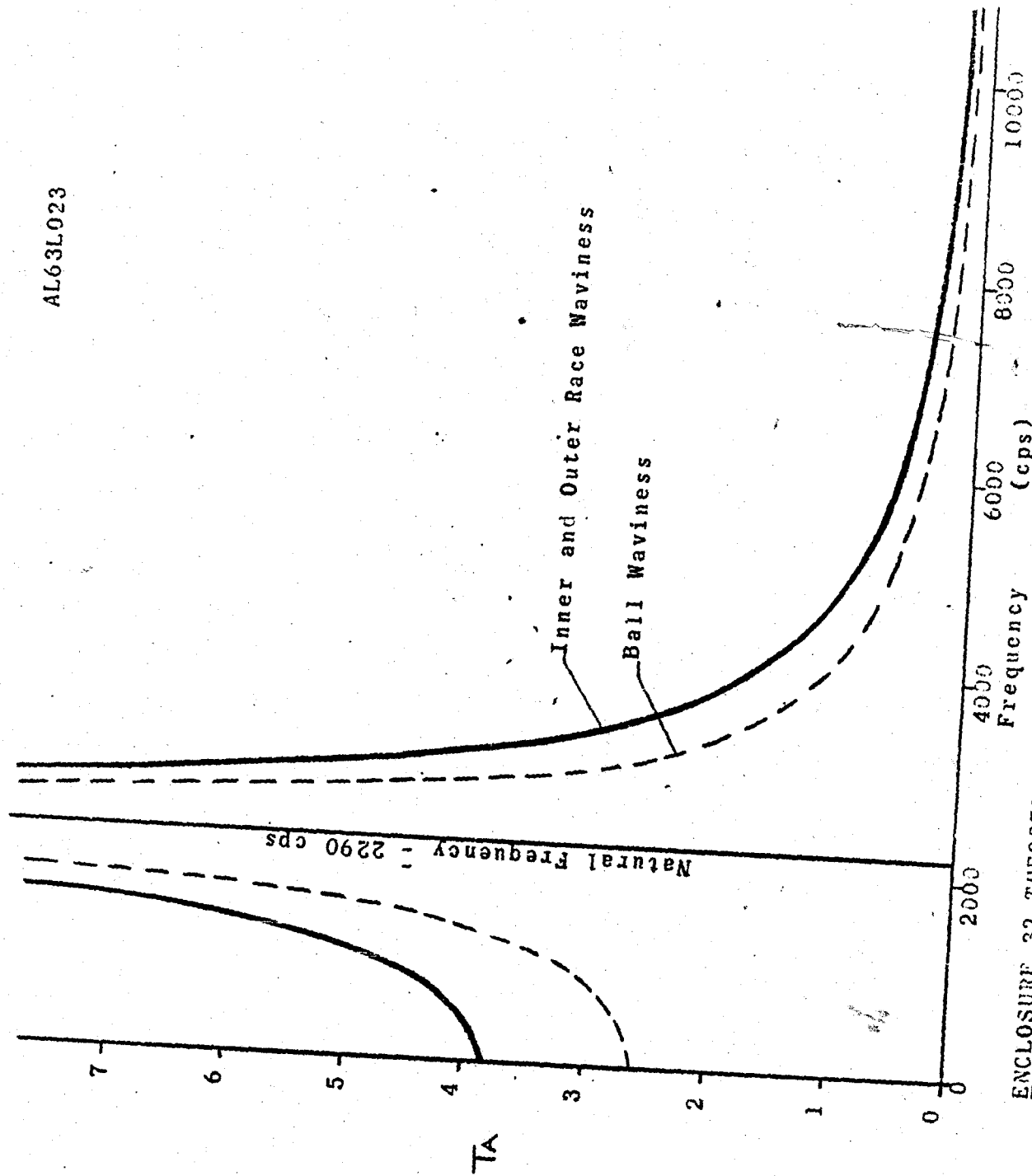
$\epsilon_R = 0$

$\epsilon_A = 0$



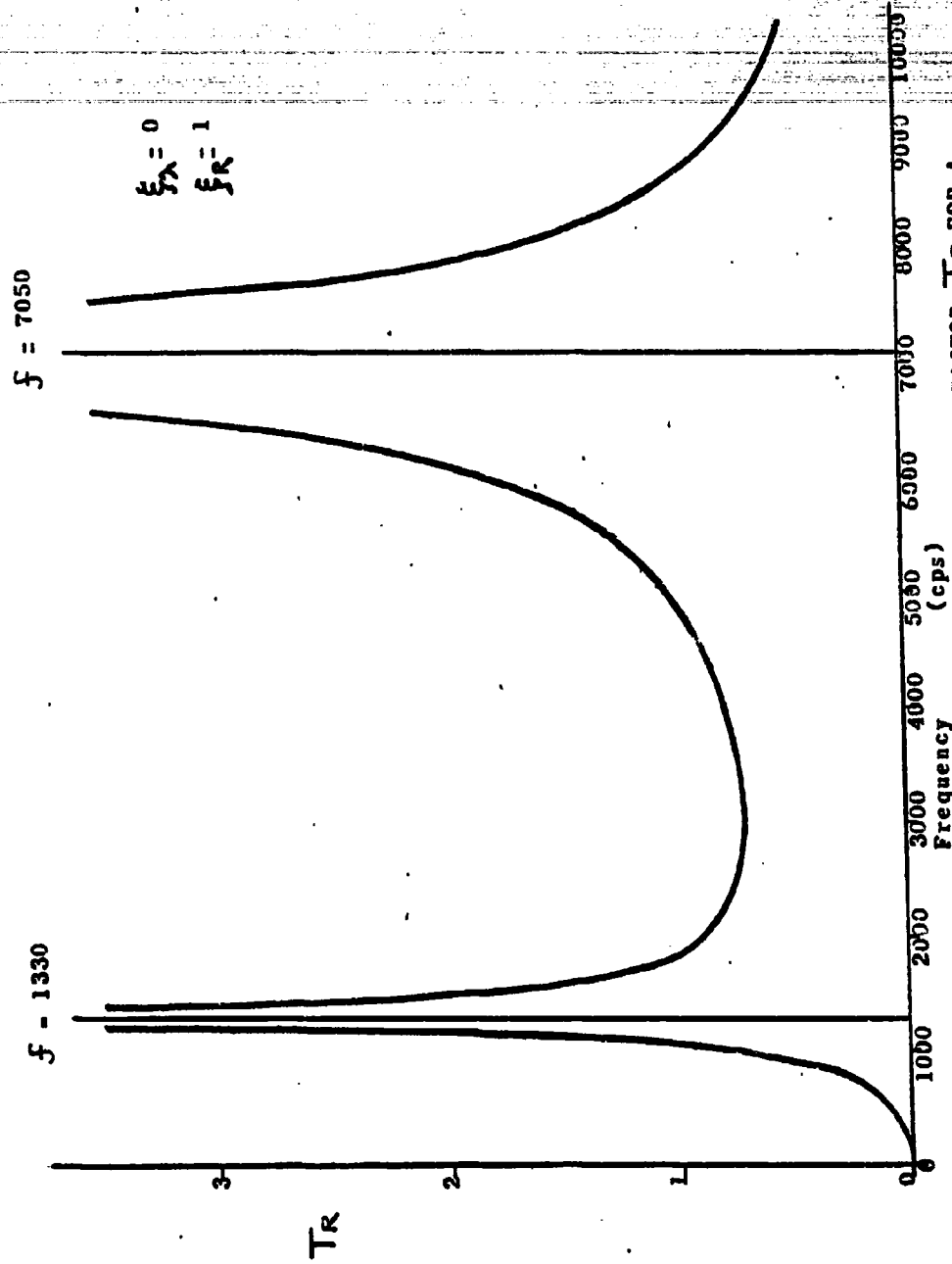
ENCLOSURE 31 DISPLACEMENT AMPLIFICATION FACTORS T_R AS FUNCTION OF FREQUENCY FOR A 6305 BEARING ROTATING AT 1800 RPM.

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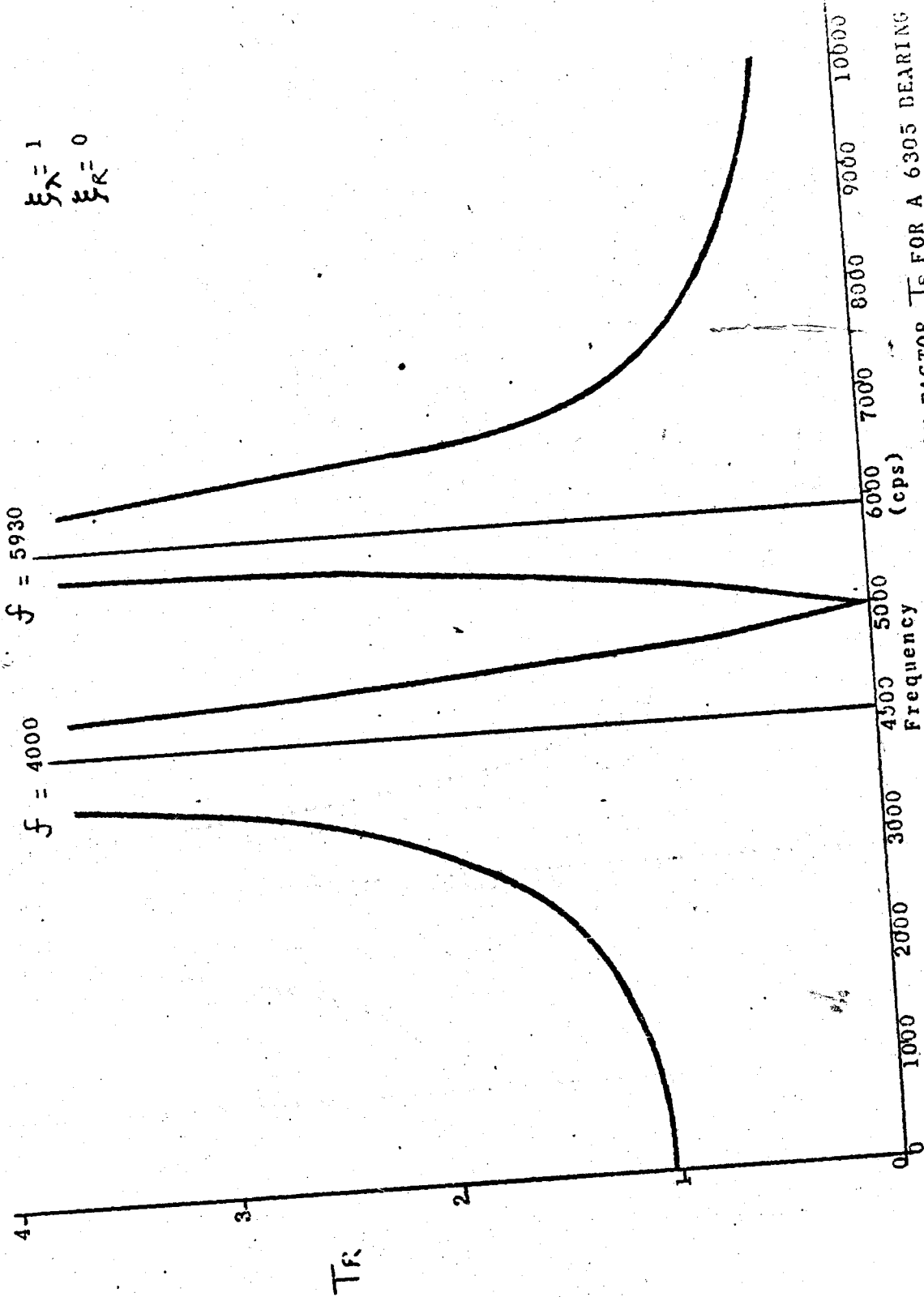
ENCLOSURE 32 THEORETICAL DISPLACEMENT FACTORS T_A OF THE AXIAL VIBRATION OF 6305 BEARING AS FUNCTION OF FREQUENCY. THE BEARING IS ROTATING AT 1800 RPM $\propto = 16$

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ENCLOSURE 33 THEORETICAL DISPLACEMENT AMPLIFICATION FACTOR T_R FOR A 6305 BEARING ROTATING AT 1800 RPM $\alpha = 18$

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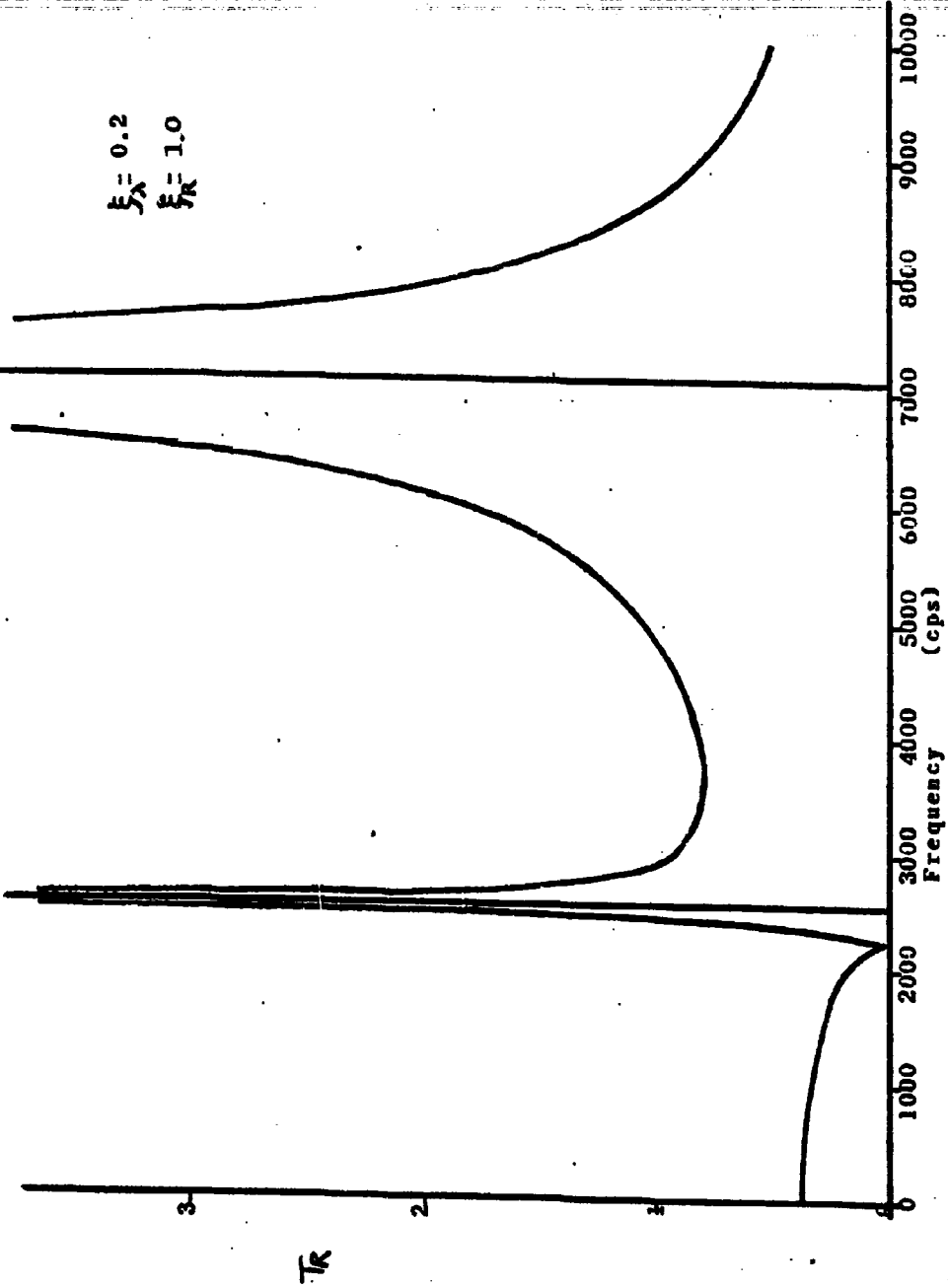
ENCLOSURE 34 THEORETICAL DISPLACEMENT AMPLIFICATION FACTOR T_R FOR A 6305 BEARING
ROTATING AT 1800 RPM $\alpha = 18^\circ$

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$f = 7050$

$f = 2560$

$\xi_A = 0.2$
 $\xi_R = 1.0$



ENCLOSURE 35 THEORETICAL DISPLACEMENT AMPLIFICATION FACTOR T_r FOR A
 6305 BEARING ROTATING AT 1800 RPM $\alpha = 18^\circ$

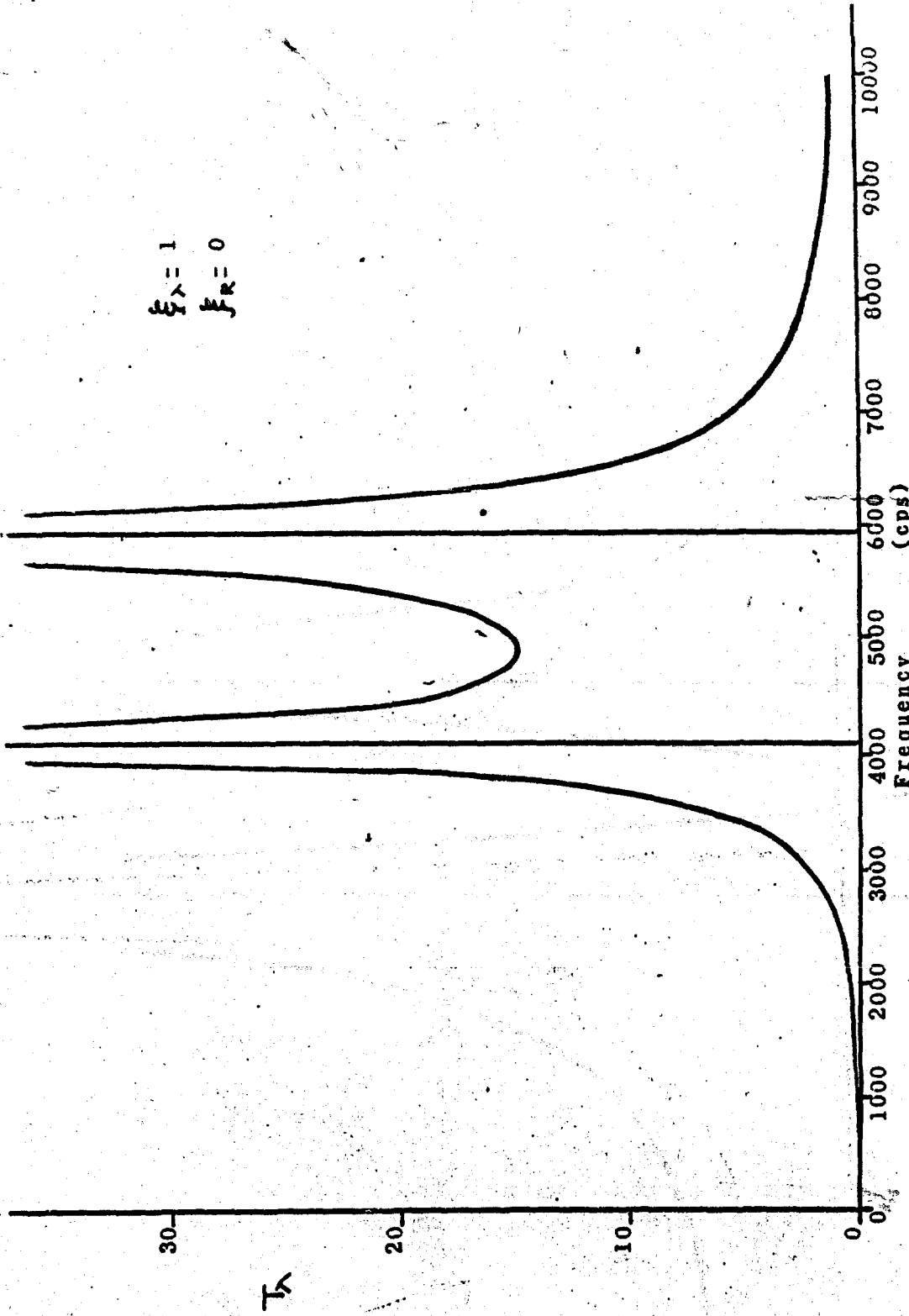
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$f = 5930$

$f = 4000$

$\xi_L = 1$

$\xi_R = 0$



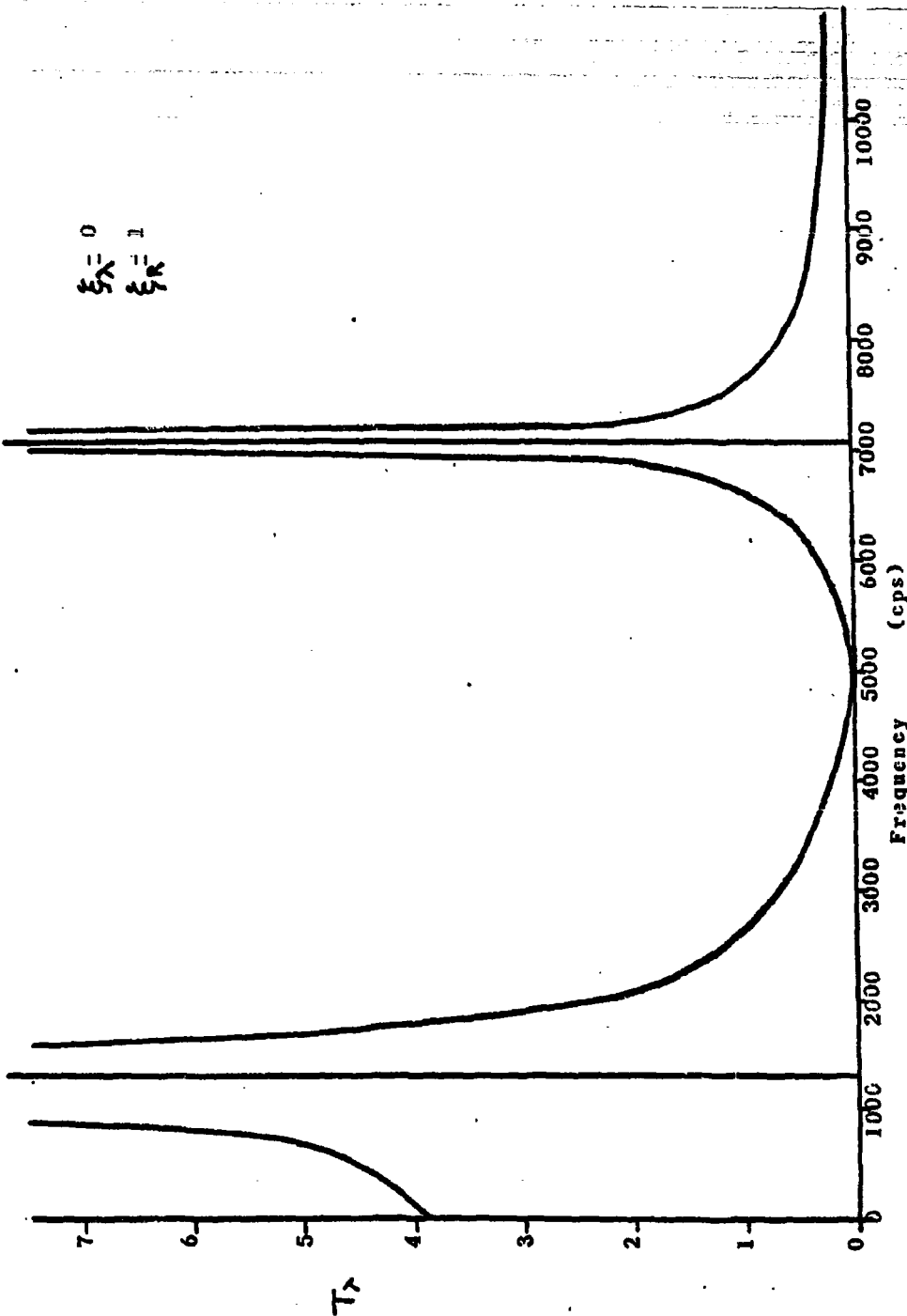
ENCLOSURE 36 THEORETICAL DISPLACEMENT AMPLIFICATION FACTOR T_L FOR A
6305 BEARING ROTATING AT 1800 RPM $\alpha = 18$

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$f = 7050$

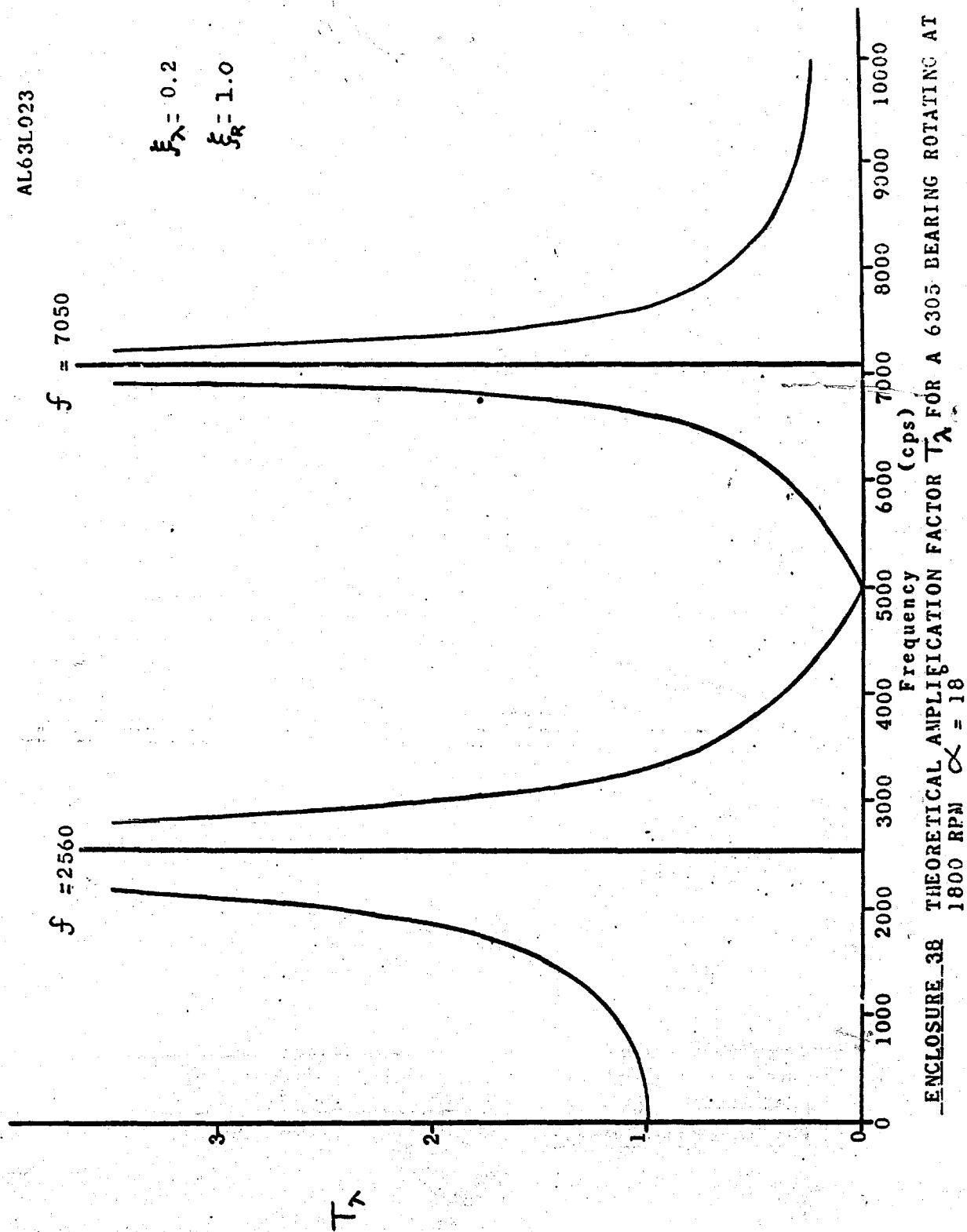
$f = 1330$

$\xi_A = 0$
 $\xi_R = 1$



ENCLOSURE 37 THEORETICAL DISPLACEMENT AMPLIFICATION FACTOR T_d FOR A
 6305 BEARING ROTATING AT 1800 RPM $\alpha = 18^\circ$

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ENCLOSURE 38 THEORETICAL AMPLIFICATION FACTOR T_λ FOR A 6305 BEARING ROTATING AT 1800 RPM $\alpha = 18$

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ENCLOSURE 39

VELOCITY BANDWIDTH AMPLIFICATION FACTORS: U_o FOR OUTER RING WAVINESS, U_i FOR INNER RING WAVINESS,
AND U_b BALL WAVINESS FOR A BEARING ROTATING AT 1800 RPM
CONTACT ANGLE $\alpha = 15^\circ$

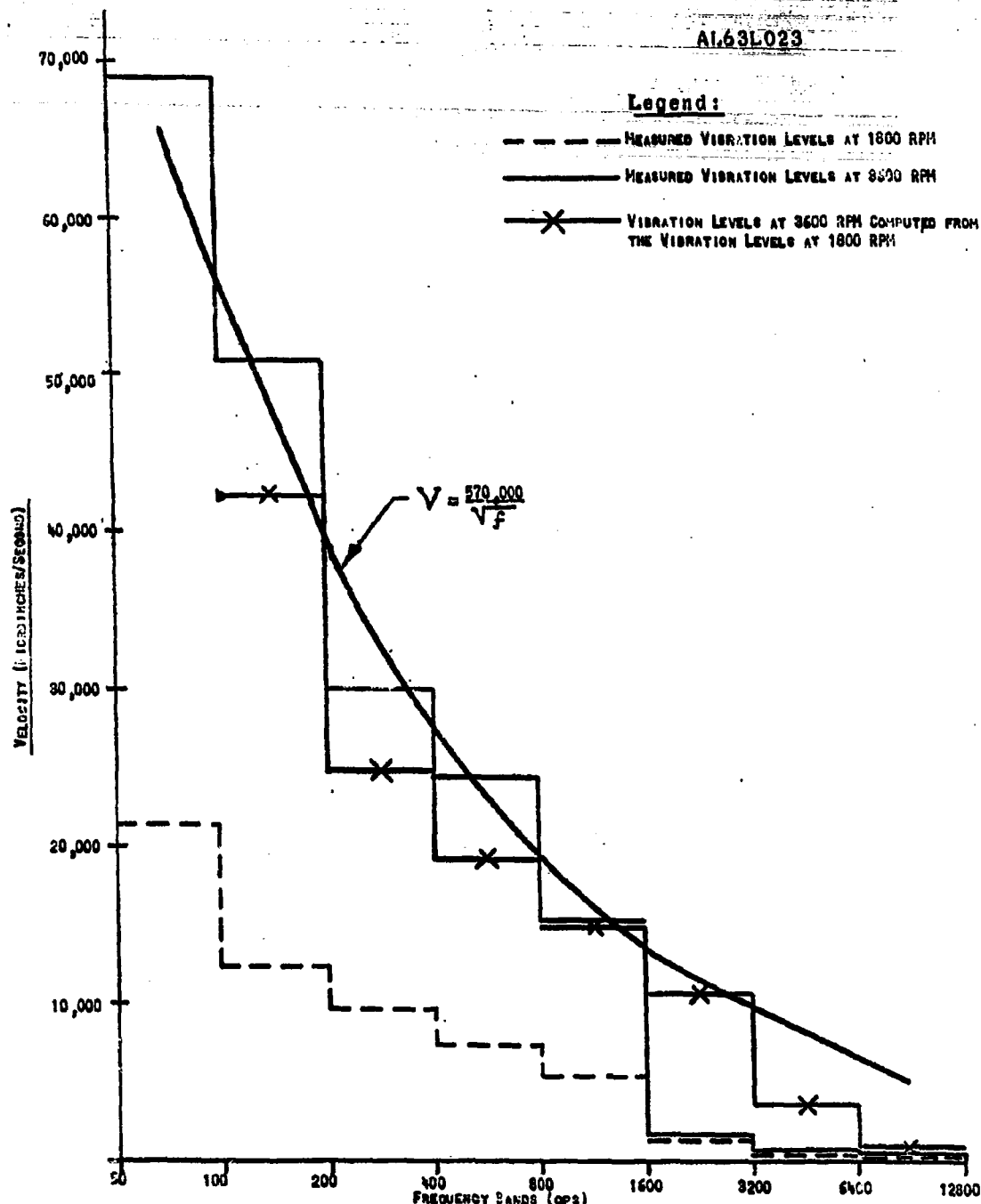
BEARING SIZE	BALL SIZE (IN)	No. OF BALLS	PITCH CIRCLE DIAMETER (IN)	RADIAL VIBRATIONS			AXIAL VIBRATIONS		
				U_o	U_i	U_b	U_o	U_i	U_b
6203	17/64	8	1.122	0.33	0.55	2.92	1.27	2.18	11.24
6205	5/16	9	1.516	0.33	0.49	3.11	1.27	1.89	11.97
6207	7/17	9	2.015	0.33	0.49	3.17	1.27	1.89	12.20
6305	7/17	7	1.713	0.35	0.58	2.87	1.35	2.23	11.05
6310	3/4	8	3.150	0.33	0.54	2.89	1.27	2.08	11.13
6312	7/8	8	3.750	0.33	0.54	2.97	1.27	2.08	11.42
6322	1 5/8	8	6.889	0.32	0.54	2.96	1.23	2.08	11.40

WAVINESS TESTER SPEEDS:

1000 RPM FOR INNER AND OUTER RINGS

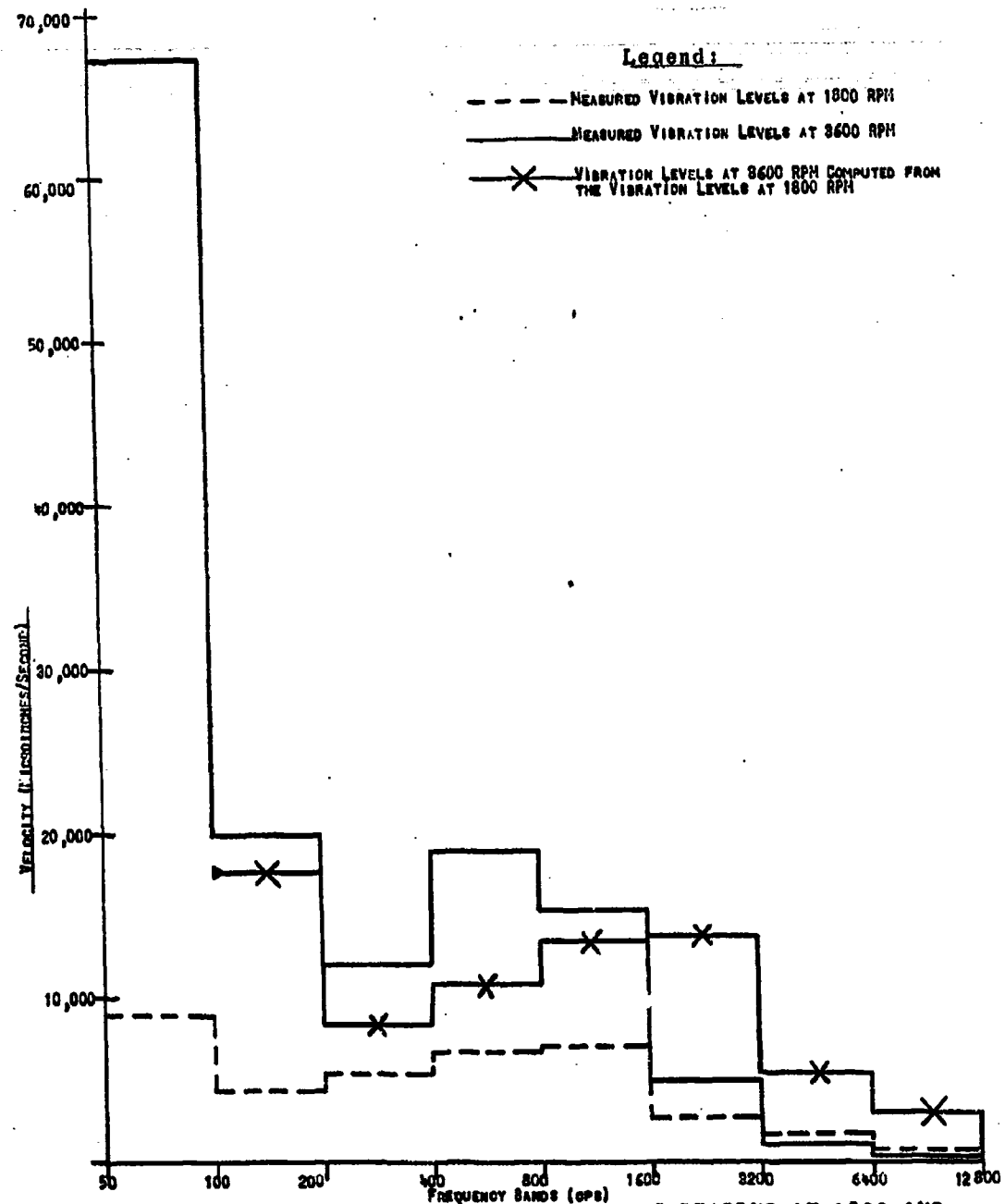
740 RPM FOR BALLS

AI.63L023



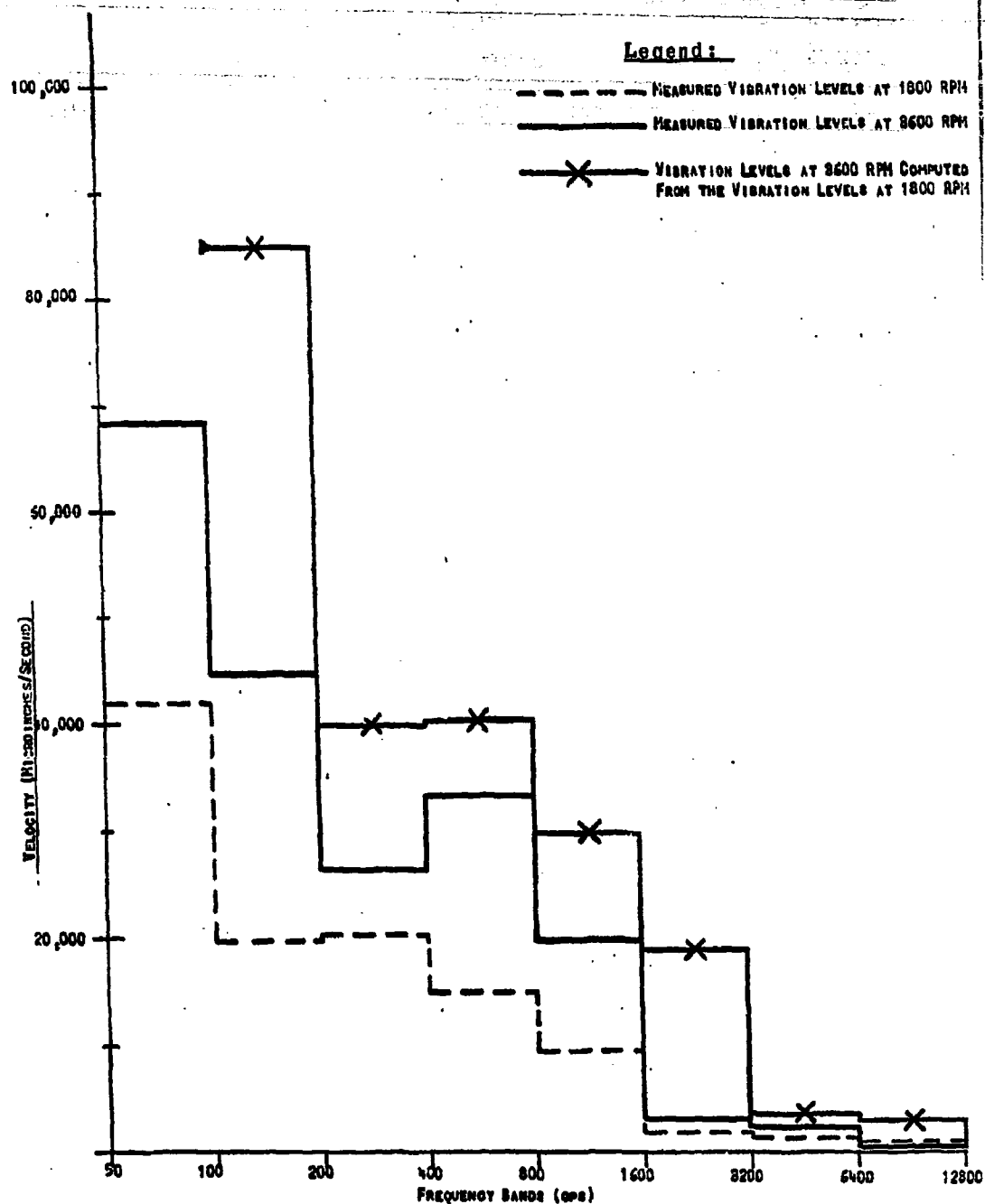
ENCLOSURE 40 OCTAVE BAND SPECTRA OF 6305 BEARING AT 1800 AND 3600 RPM IN THE DIRECTION NORMAL TO LOAD

AL63L023



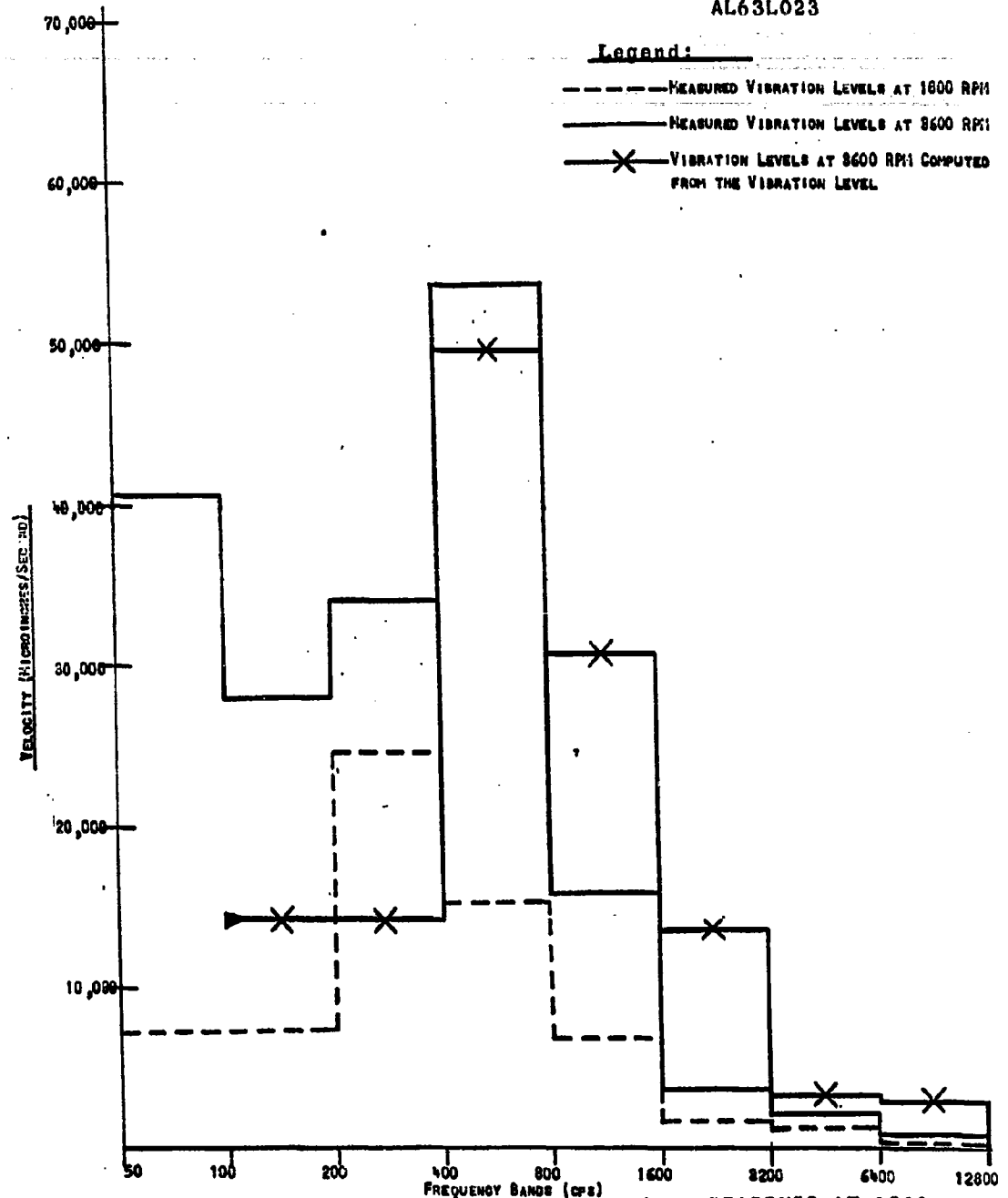
ENCLOSURE 41 OCTAVE BAND SPECTRA OF 6305 BEARING AT 1800 AND 3600 RPM IN THE DIRECTION OF LOAD

AL631023



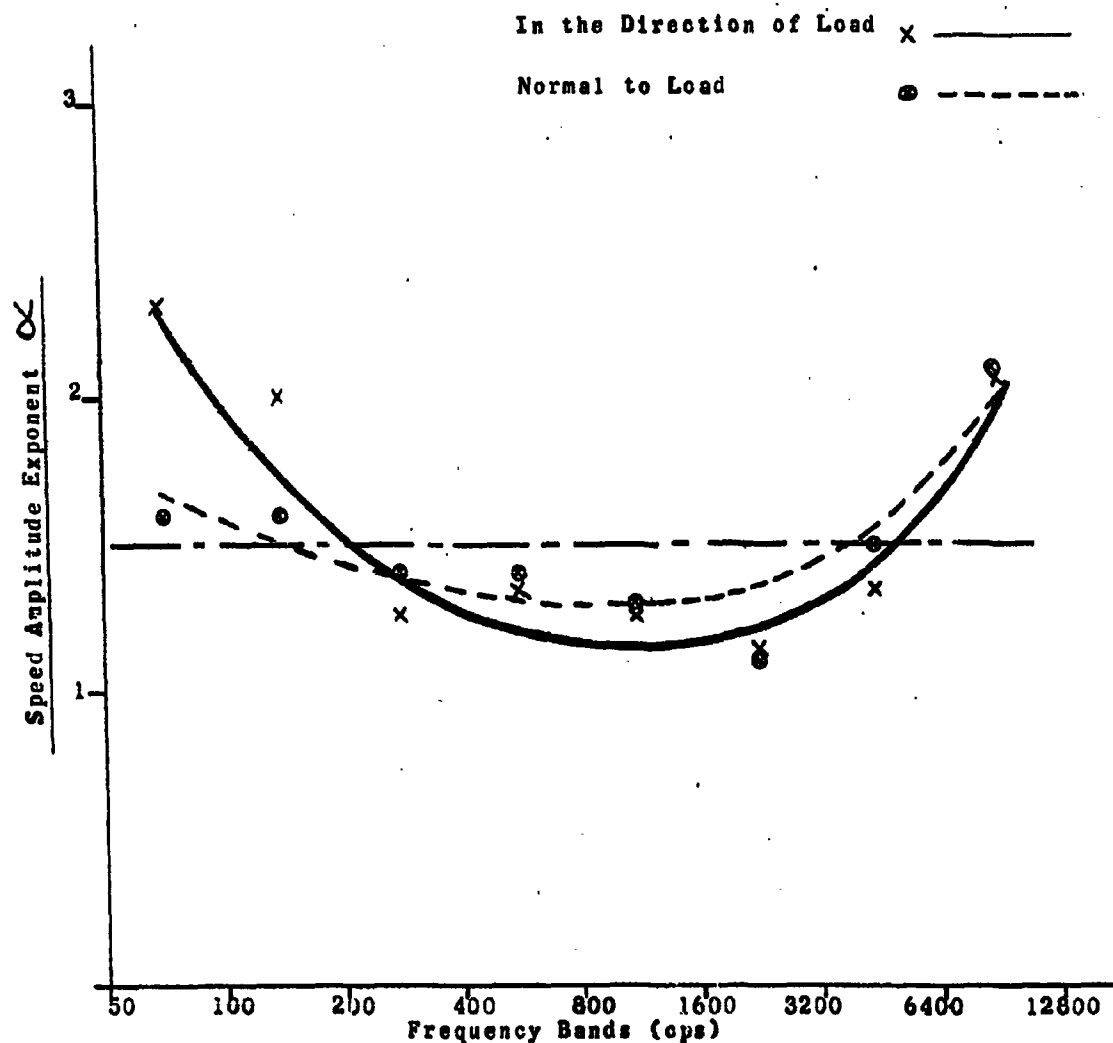
ENCLOSURE 42 OCTAVE BAND SPECTRA OF 6310 BEARING AT 1800 RPM AND 3600 RPM IN THE DIRECTION NORMAL TO LOAD.

AL63L023



ENCLOSURE 43 OCTAVE BAND SPECTRA OF 6310 BEARINGS AT 1800 AND 3600 RPM IN THE DIRECTION OF LOAD

AL63L023

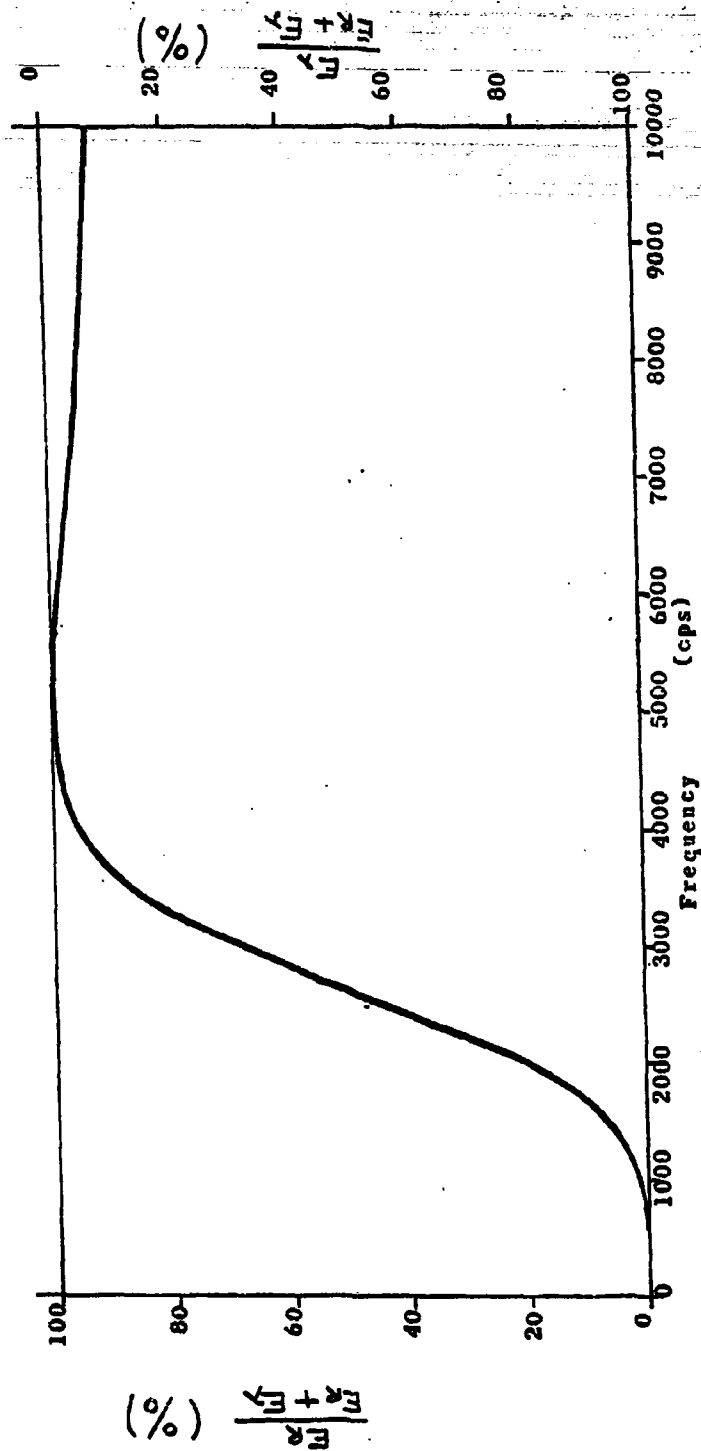


ENCLOSURE 44 SPEED AMPLITUDE EXPONENT AS A FUNCTION OF FREQUENCY FOR BALL BEARINGS

AL63L023

$$\xi_{\lambda} = 0$$

$$\xi_{\kappa} = 1$$

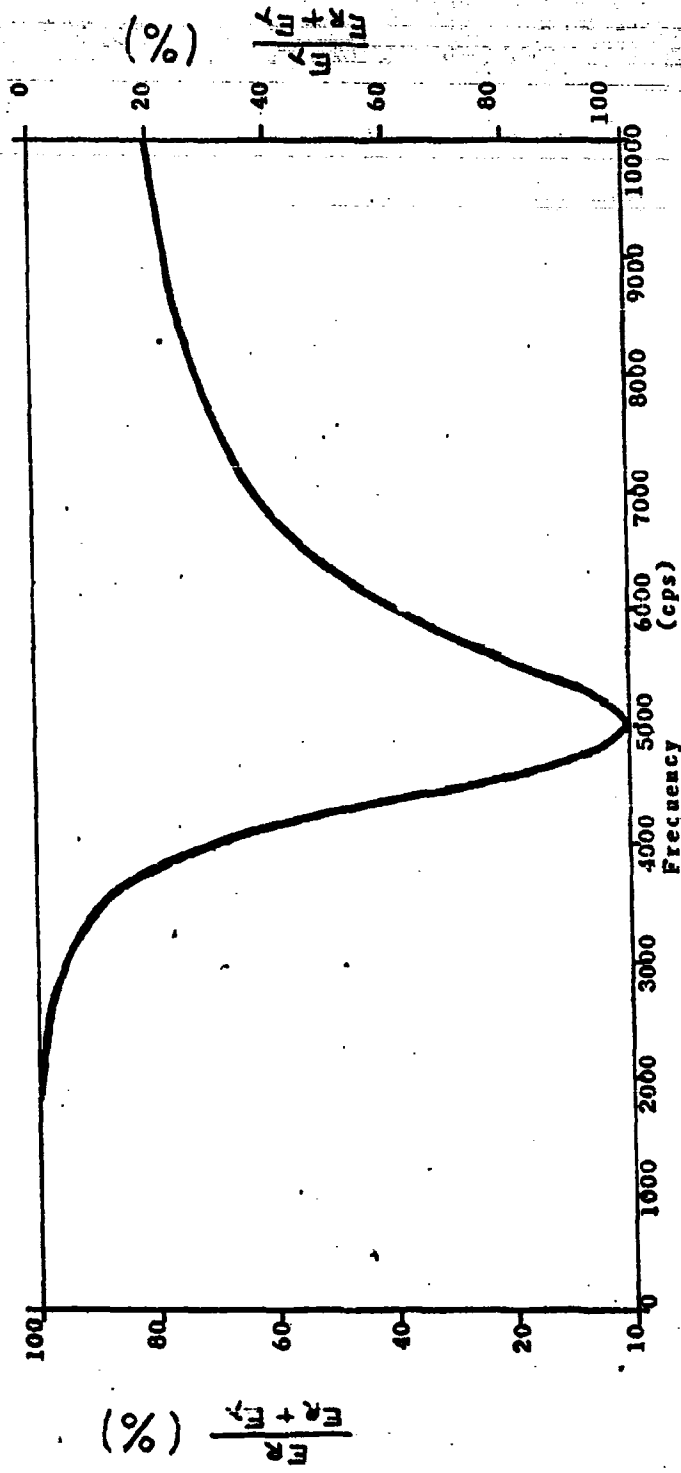


ENCLOSURE 45 THEORETICAL DISTRIBUTION OF VIBRATORY ENERGY OF A 6305 BEARING BETWEEN THE RADIAL AND THE ANGULAR MODE

AL63L023

$$\xi_r = 1$$

$$\xi_k = 0$$

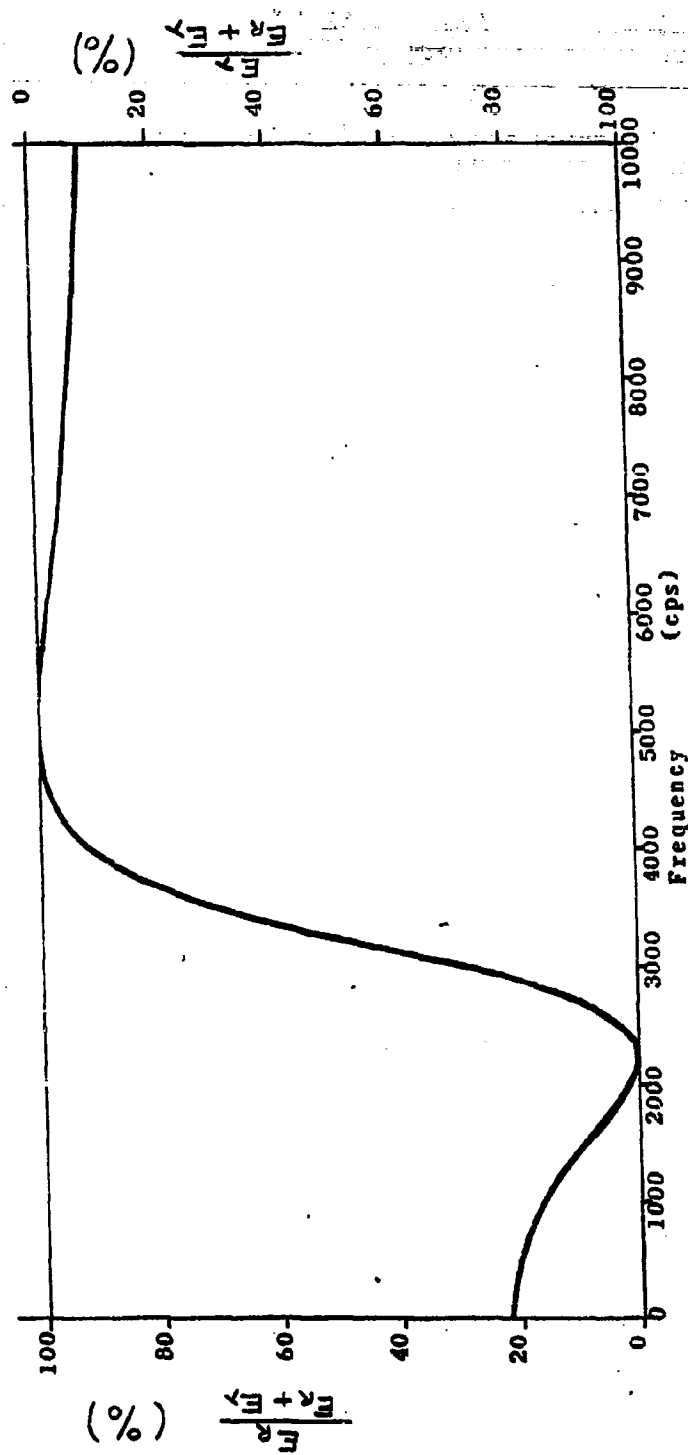


ENCLOSURE 46 THEORETICAL DISTRIBUTION OF VIBRATORY ENERGY OF A 6305 BEARING BETWEEN THE RADIAL AND THE ANGULAR MODE

AL63L023

$\xi_r = 0.2$

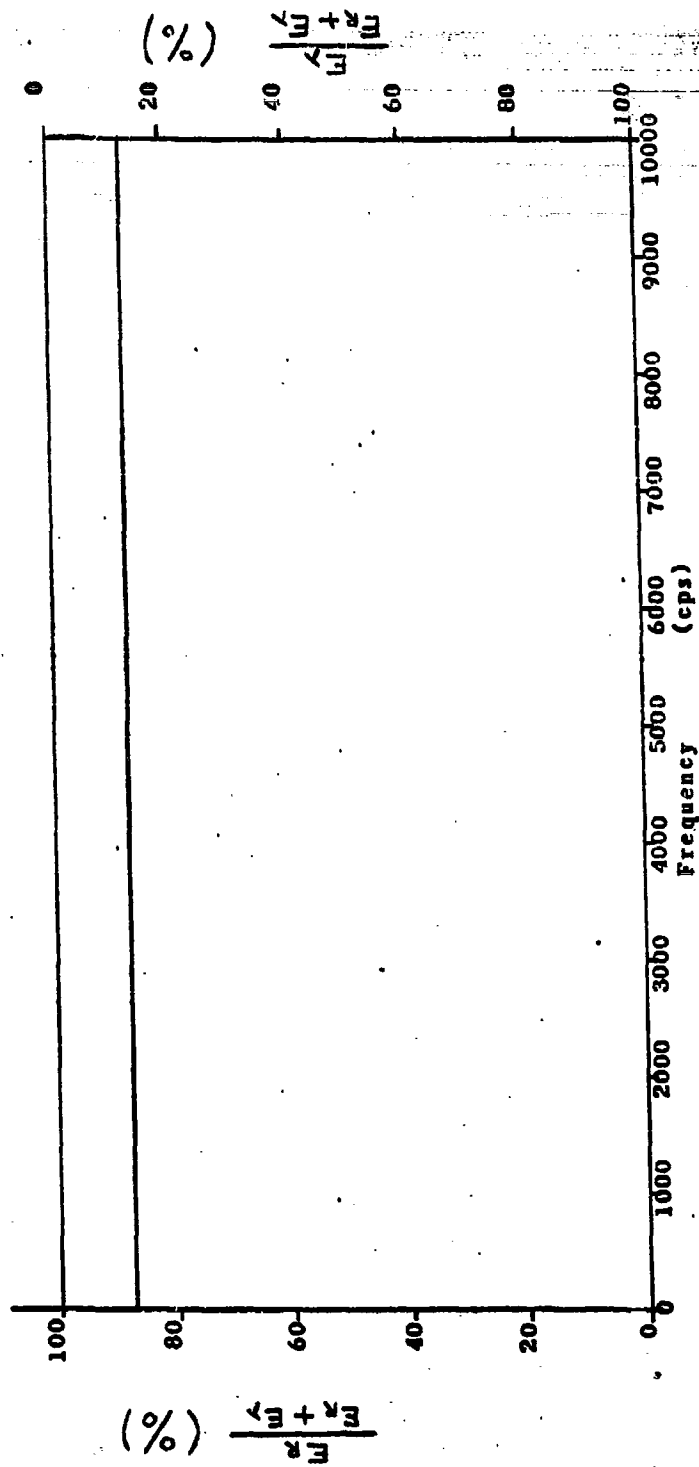
$\xi_R = 1.0$



ENCLOSURE 47 THEORETICAL DISTRIBUTION OF VIBRATORY ENERGY OF A 6305 BEARING BETWEEN THE RADIAL AND THE ANGULAR MODE

AL63L023

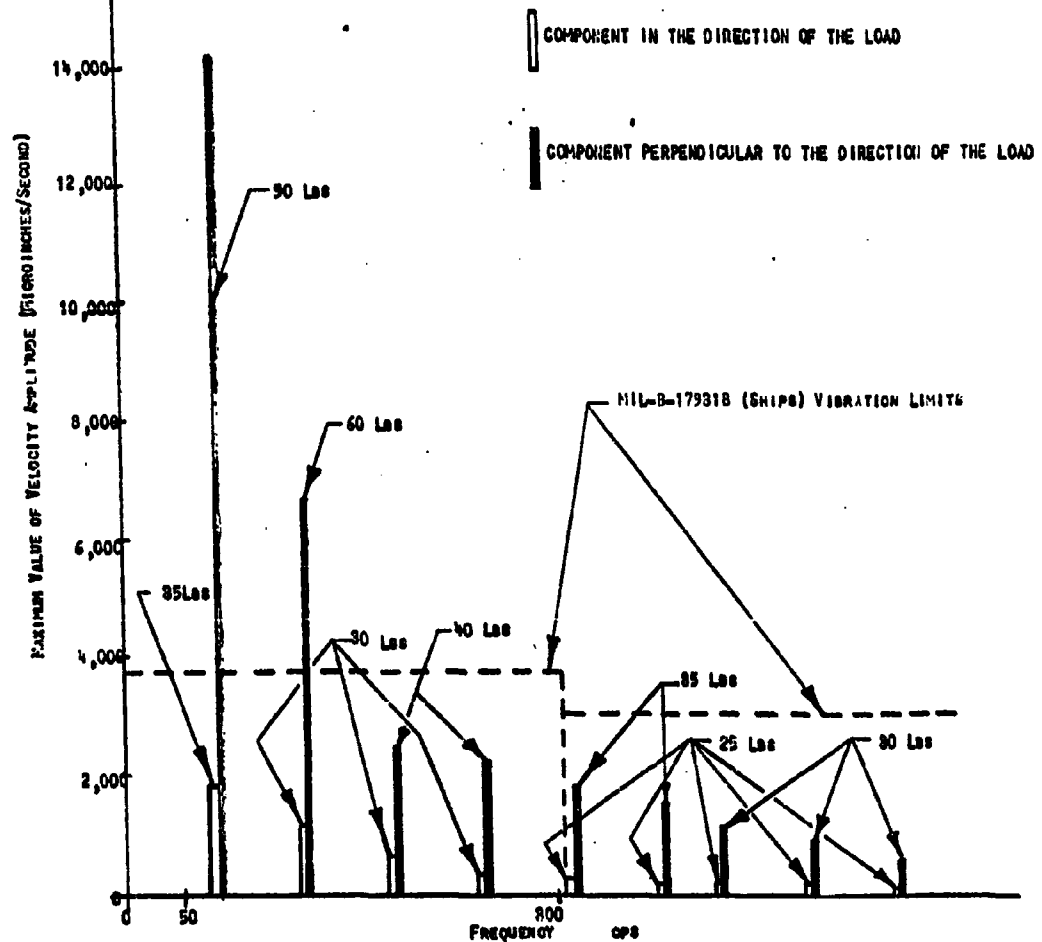
$$\xi_r = \xi_R$$



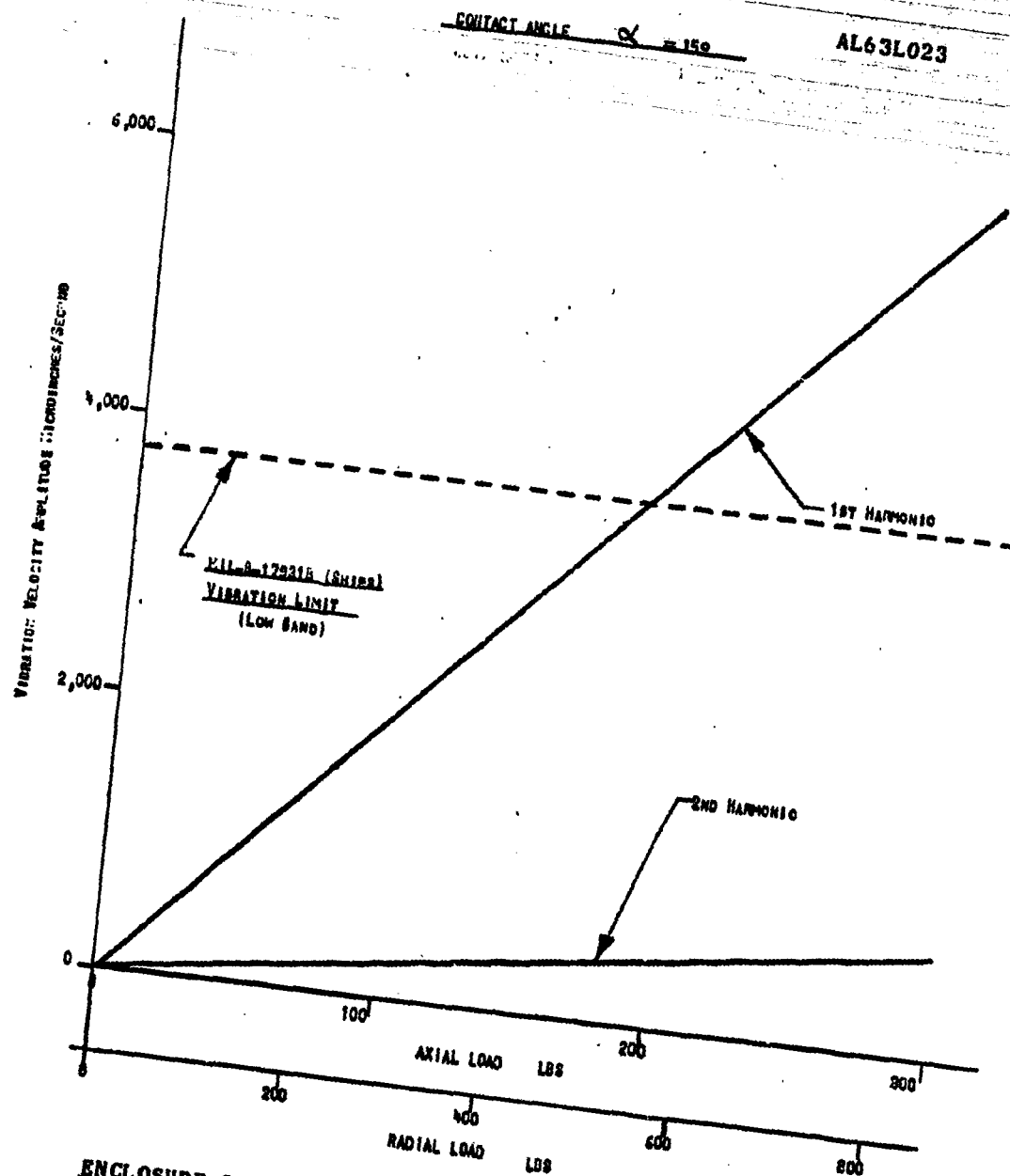
ENCLOSURE 48 THEORETICAL DISTRIBUTION OF VIBRATORY ENERGY OF A 6305 BEARING BETWEEN THE RADIAL AND THE ANGULAR MODE

AL63L023

RADIAL LOOSENESS = 0.015 MM
LOAD RANGE = 20 - 2000 LBS



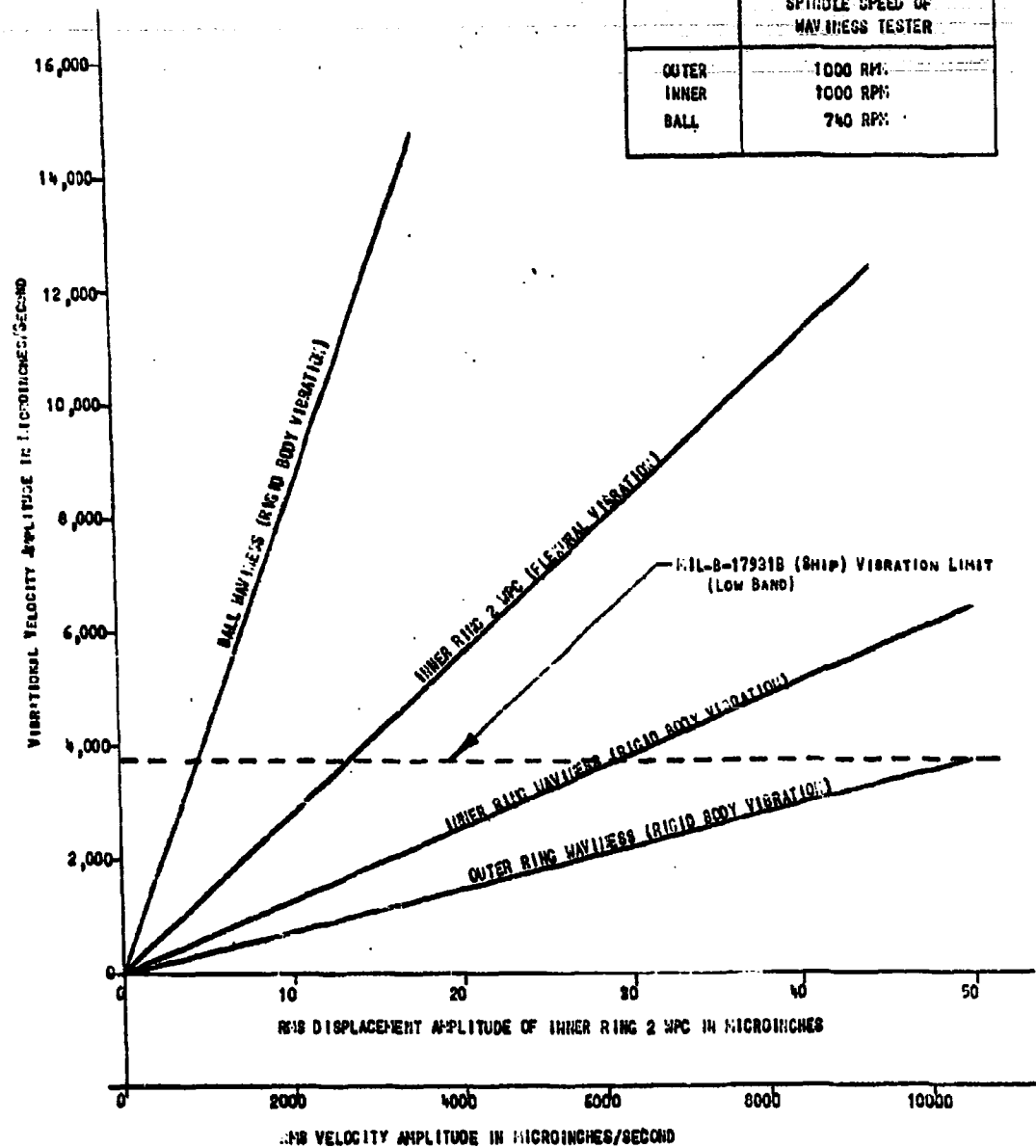
ENCLOSURE 49 MAXIMUM VALUES OF VELOCITY AMPLITUDES OF THE HARMONICS OF THE VARIABLE COMPLIANCE VIBRATIONS FOR A 6305 BEARING ROTATING AT 1800 RPM



ENCLOSURE 50 VIBRATION INDUCED BY BALL LOADS IN A 6305 BEARING ROTATING AT 1800 RPM.

AL63LQ23

SPINDLE SPEED OF WAVINESS TESTER	
OUTER	1000 RPM
INNER	1000 RPM
BALL	740 RPM

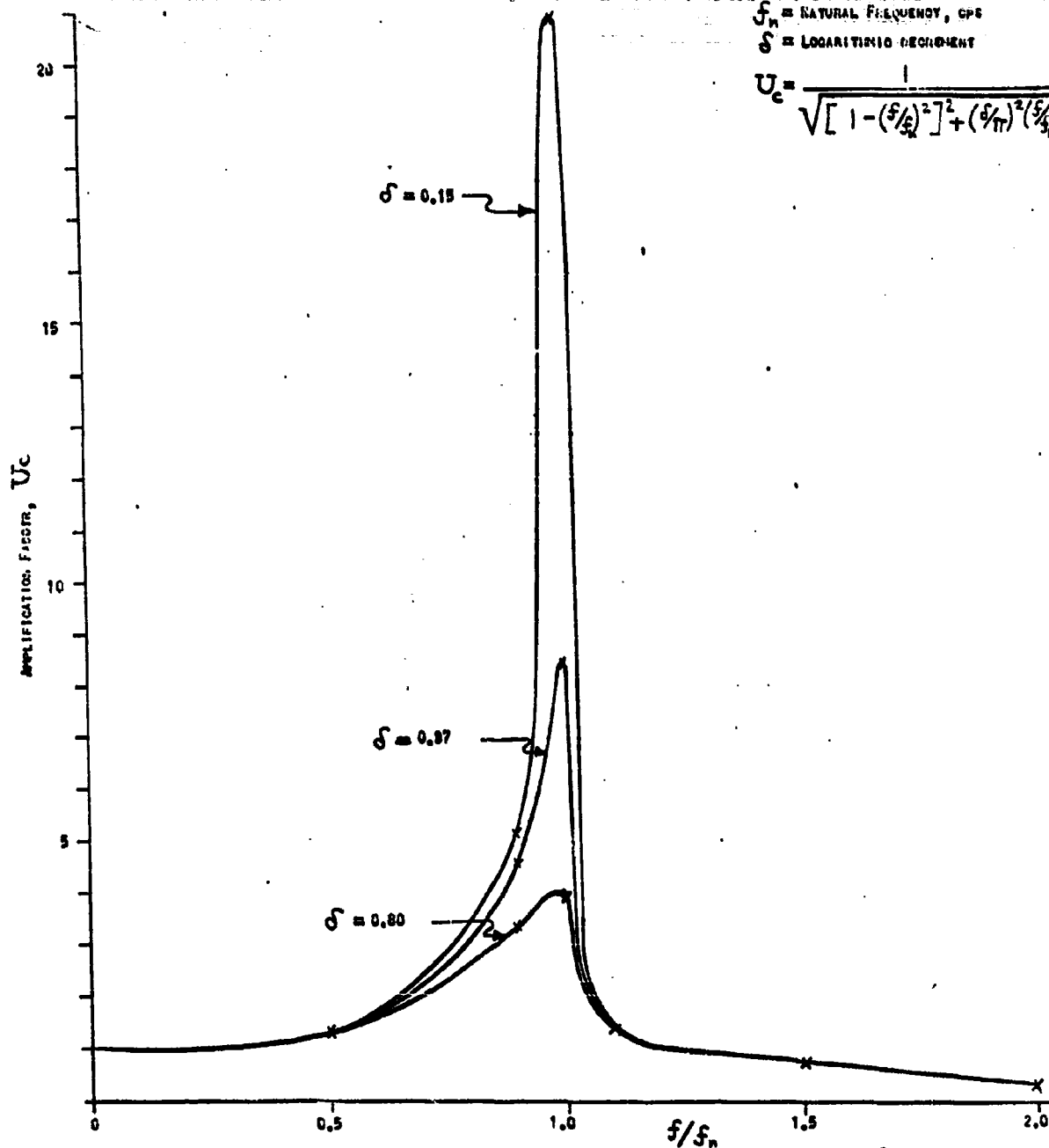


ENCLOSURE 51 FLEXURAL AND RIGID BODY VIBRATIONS IN THE 50-300 CPS FREQUENCY BAND FOR A 6305 BEARING ROTATING AT 1800 RPM.

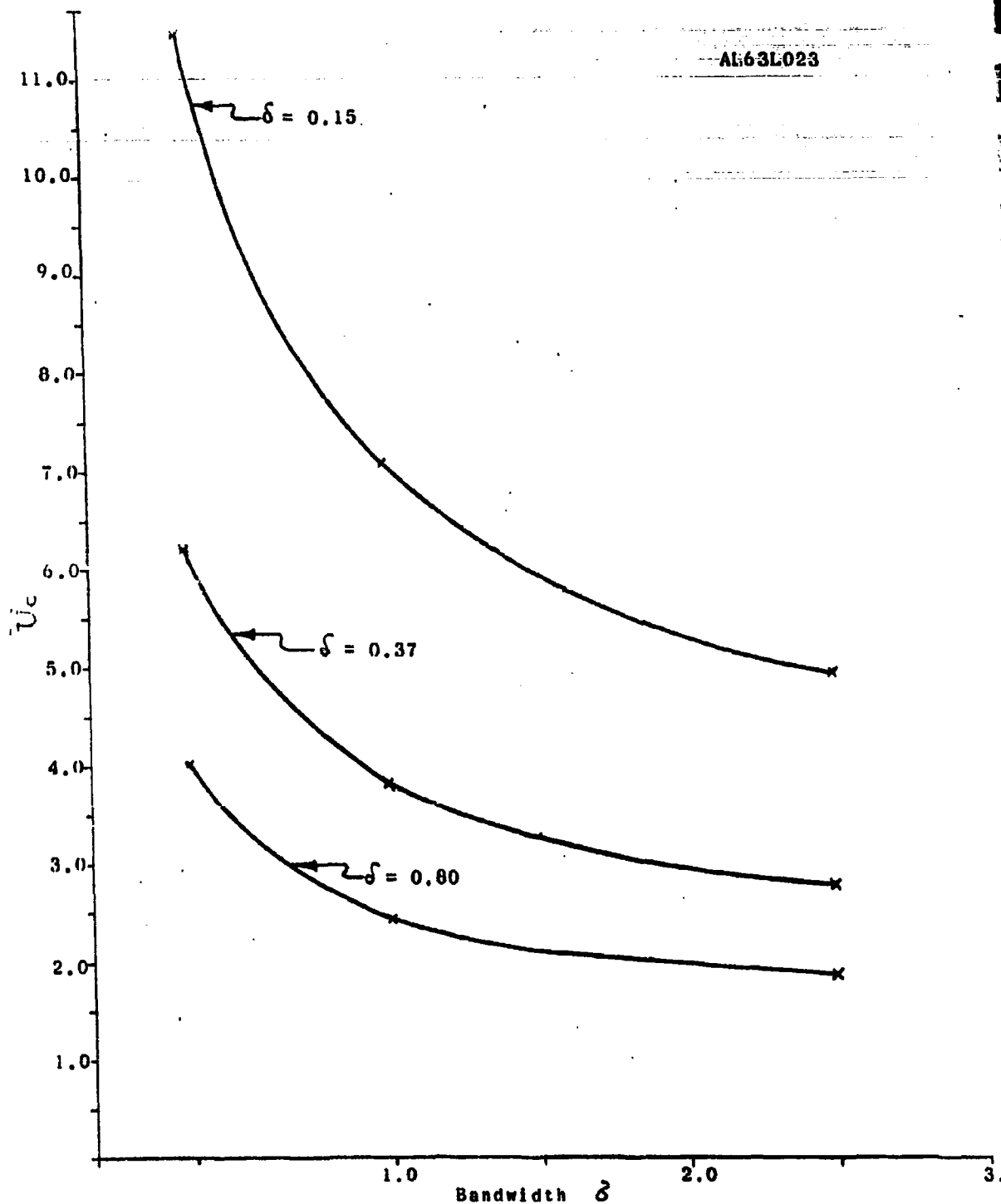
AL63L023

f = FREQUENCY, CPS
 f_n = NATURAL FREQUENCY, CPS
 δ = LOGARITHMIC DECREMENT

$$U_c = \frac{1}{\sqrt{[1 - (f/f_n)^2]^2 + (\delta/\pi)^2 (f/f_n)^4}}$$



ENCLOSURE 52 AMPLIFICATION FACTOR U_c AS A FUNCTION OF f/f_n

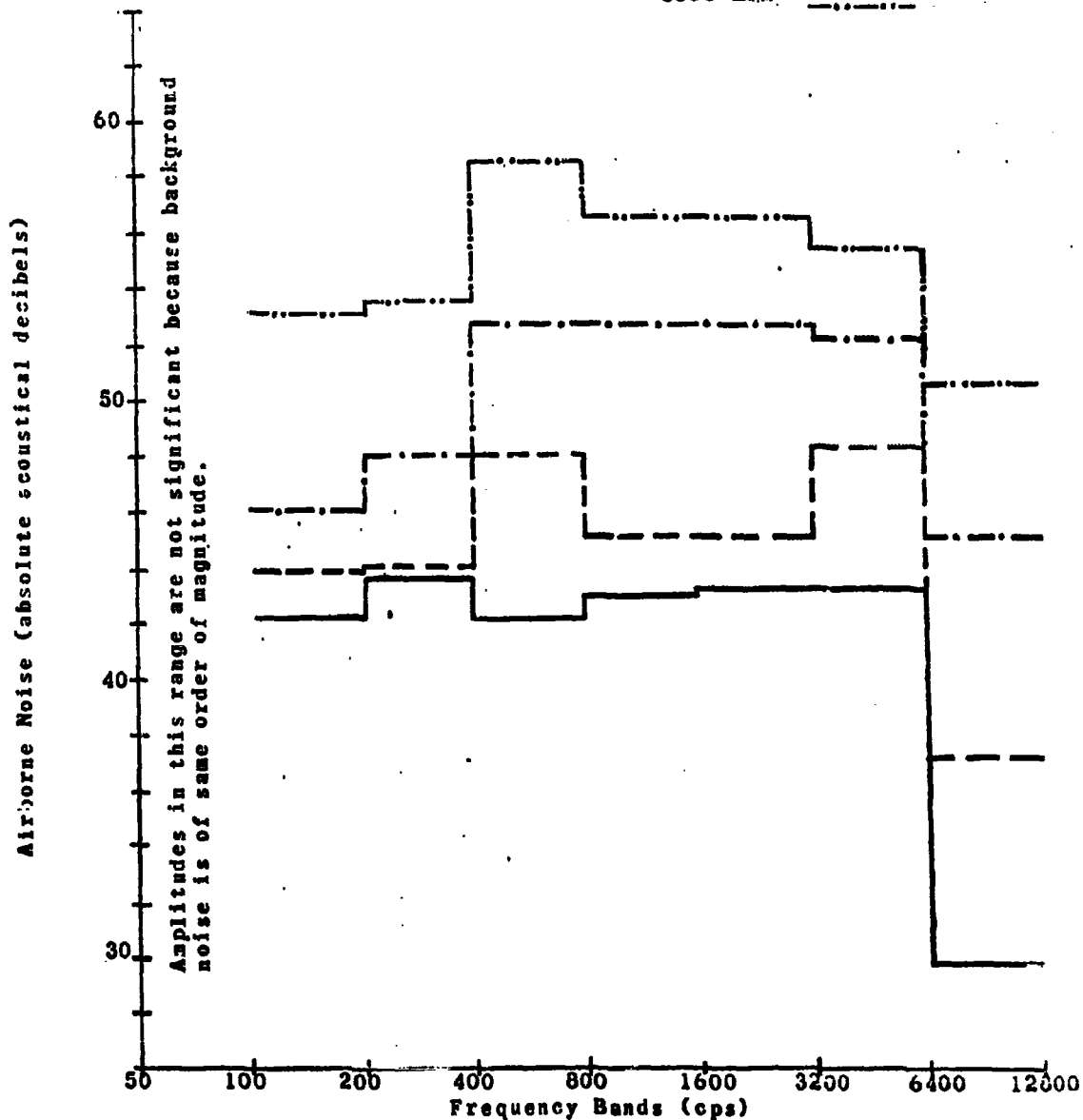


ENCLOSURE 53 BAND WIDTH AMPLIFICATION FACTOR \bar{U}_c AS A FUNCTION OF BANDWIDTH δ (IN OCTAVES).

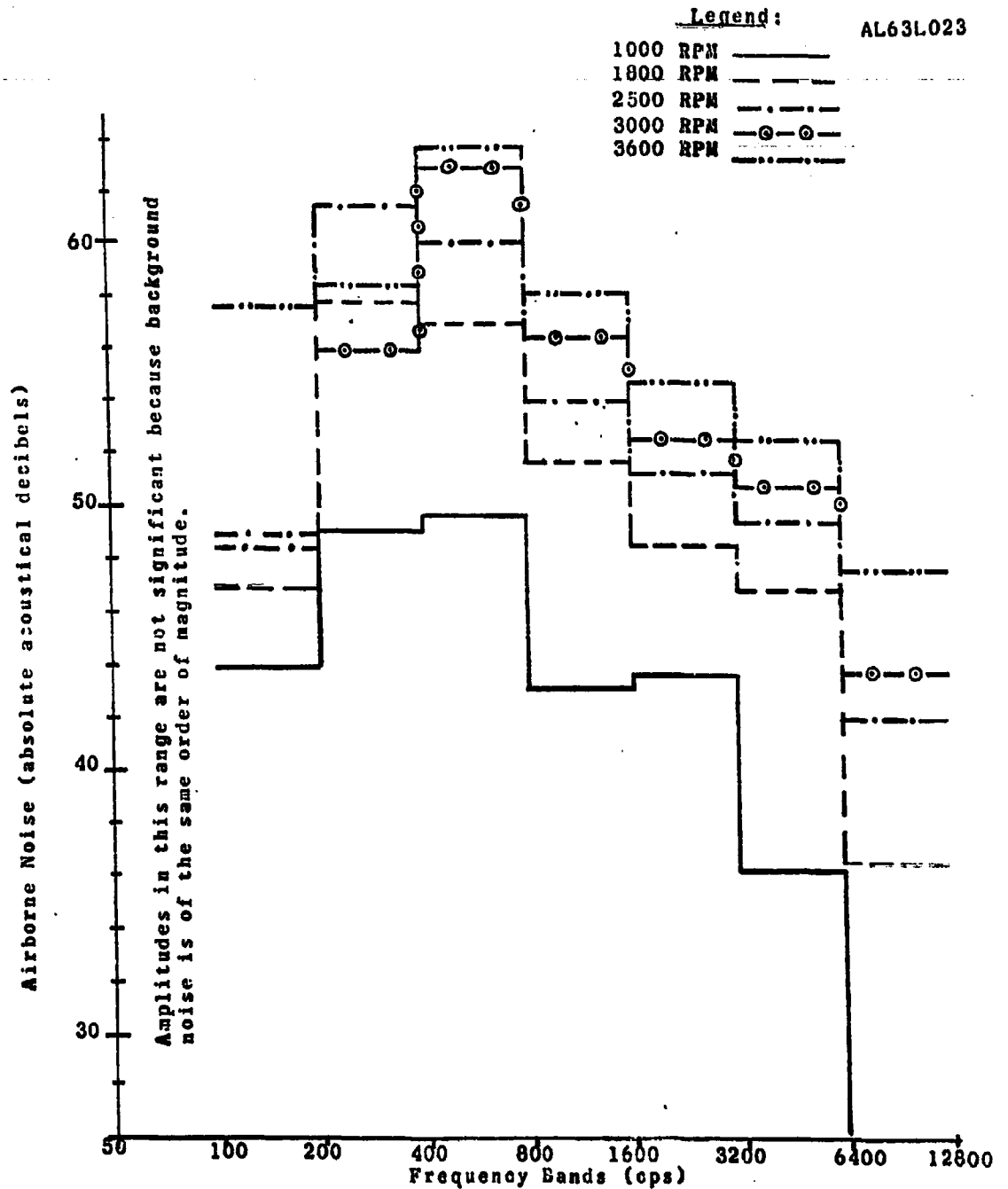
AL63L023

Legend:

1000 RPM	_____
1800 RPM	-----
2500 RPM	-.-.-.-
3600 RPM



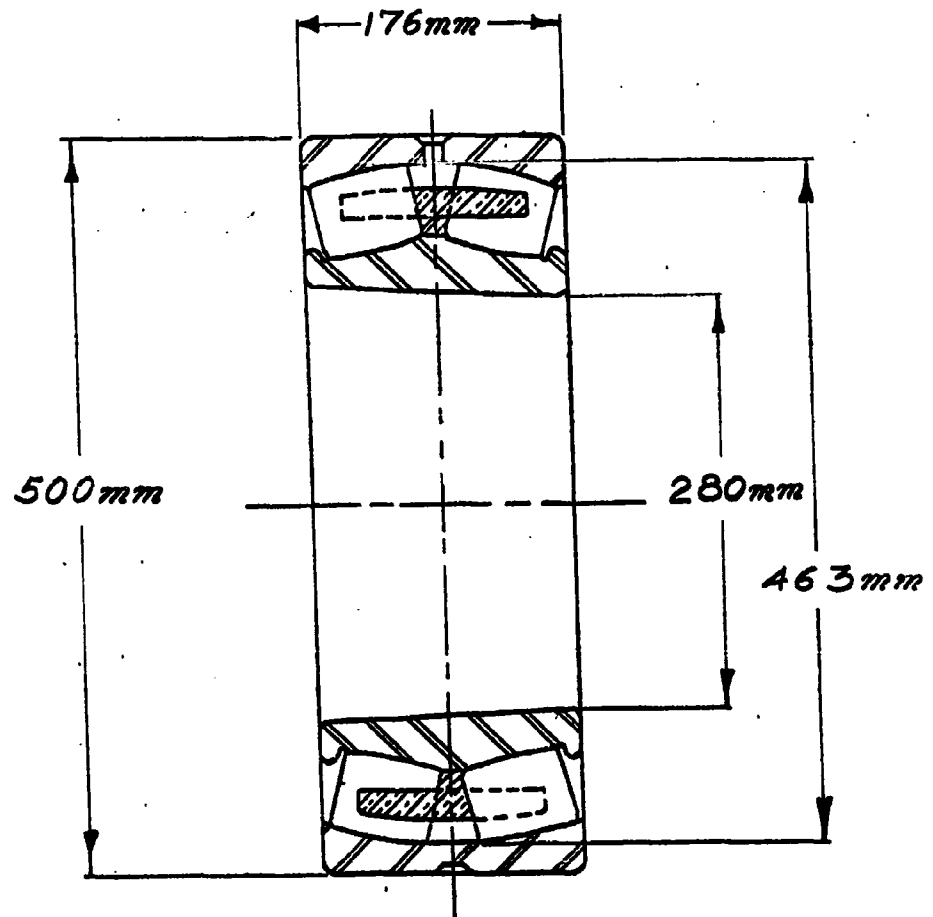
ENCLOSURE 5A OCTAVE BAND ANALYSIS OF AIRBORNE NOISE GENERATED BY FOUR 6305 BEARINGS (IN ABSOLUTE UNITS, AVERAGE OF FIVE SETS AND SIX LOADS)



ENCLOSURE 55 OCTAVE BAND ANALYSIS OF AIRBORNE NOISE GENERATED BY FOUR 6310 BEARINGS (IN ABSOLUTE UNITS, AVERAGE OF FIVE SETS AND SIX LOADS)

AL63L023

ROLLER PITCH CIRCLE DIAMETER	390 mm	CONTACT ANGLE	13°
ROLLER	56 X 70 mm	BASIC DYNAMIC LOAD RATING C = 520,000 LBS.	
NO. OF ROLLERS PER ROW	18		

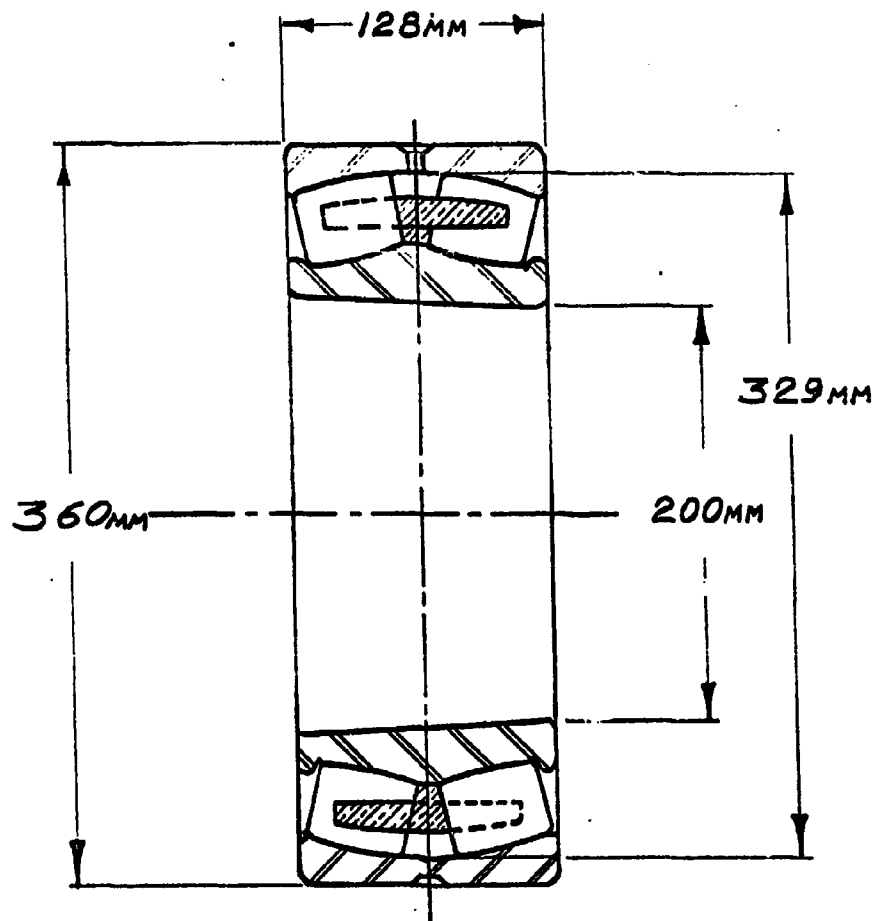


ENCLOSURE 56

23256CAK SPHERICAL ROLLER BEARING

AL63L023

ROLLER PITCH CIRCLE DIAMETER	202 MM	CONTACT ANGLE	130
ROLLER	39 X 52 MM	BASIC DYNAMIC LOAD RATING	
NO. OF ROLLERS PER ROW	19	C = 202,000 LBS.	

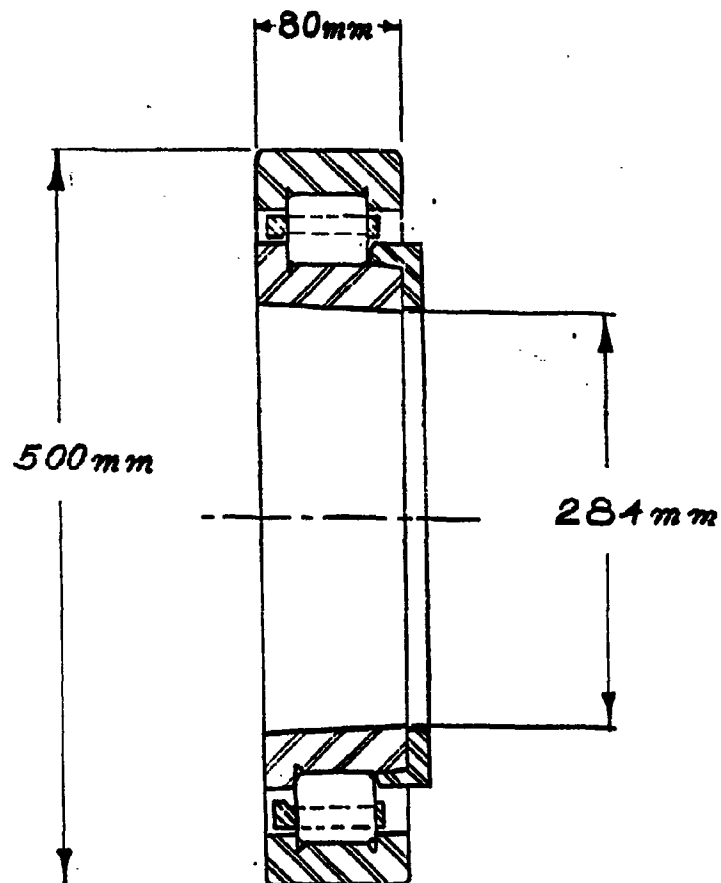


ENCLOSURE 57

23240CK SPHERICAL ROLLER BEARING

ROLLER PITCH CIRCLE DIAMETER	820 mm	CONTACT ANGLE	0°
ROLLER	50 X 50 mm	BASIC DYNAMIC LOAD RATING	
NO. OF ROLLERS	19	C	= 225,000 LBS.

AL63L023

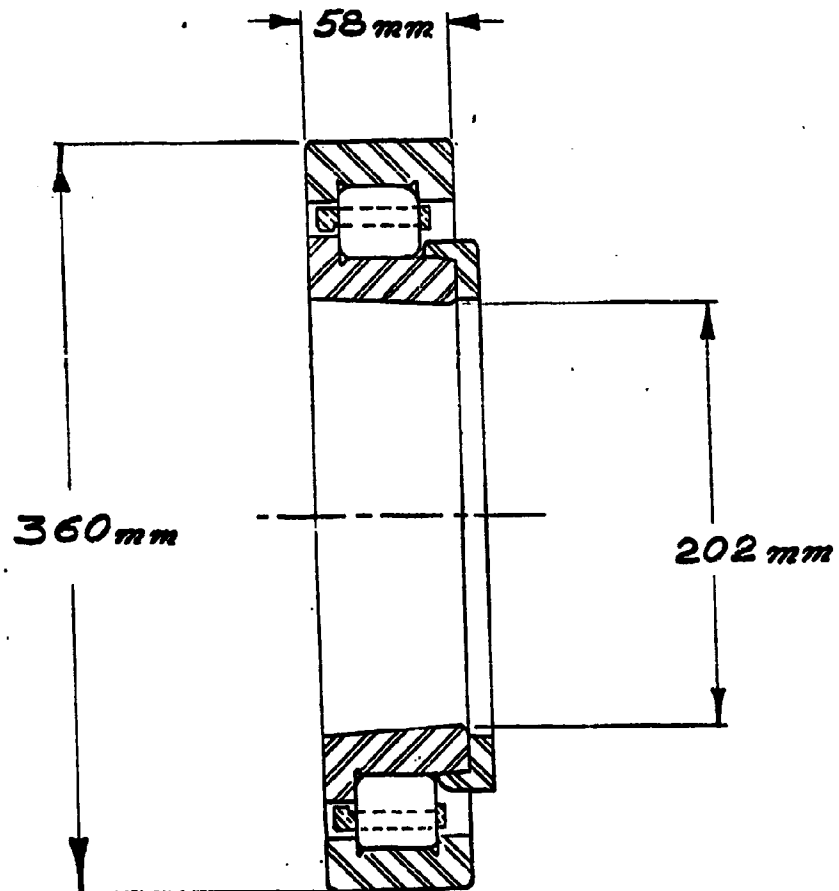


ENCLOSURE 58

NJ256K CYLINDRICAL ROLLER BEARING

AL63L023

ROLLER PITCH CIRCLE DIAMETER	280 mm	CONTACT ANGLE	0°
ROLLER	36 X 36 mm	BASIC DYNAMIC LOAD RATING	
NO. OF ROLLERS	19	C	125,000 LBS

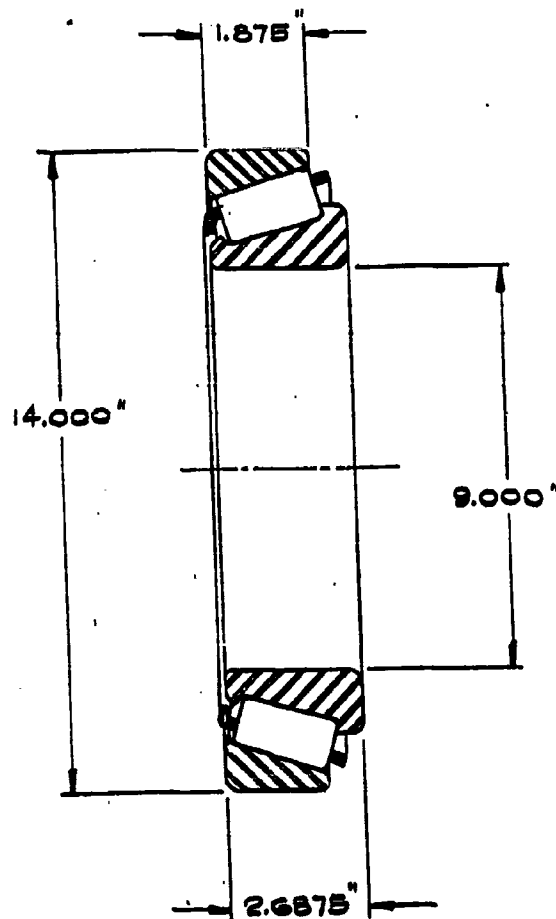


ENCLOSURE 59

NJ240K CYLINDRICAL ROLLER BEARING

AL63E023

ROLLER PITCH CIRCLE DIAMETER	11.31"	CONC. ANGLE	$\approx 21.5^\circ$
ROLLER MEAN DIAMETER	0.938"	BASIC DYNAMIC LOAD RATING	
ROLLER LENGTH	0.855"		
NO. OF ROLLERS	32	C	$\approx 97,800 \text{ Lbs.}$

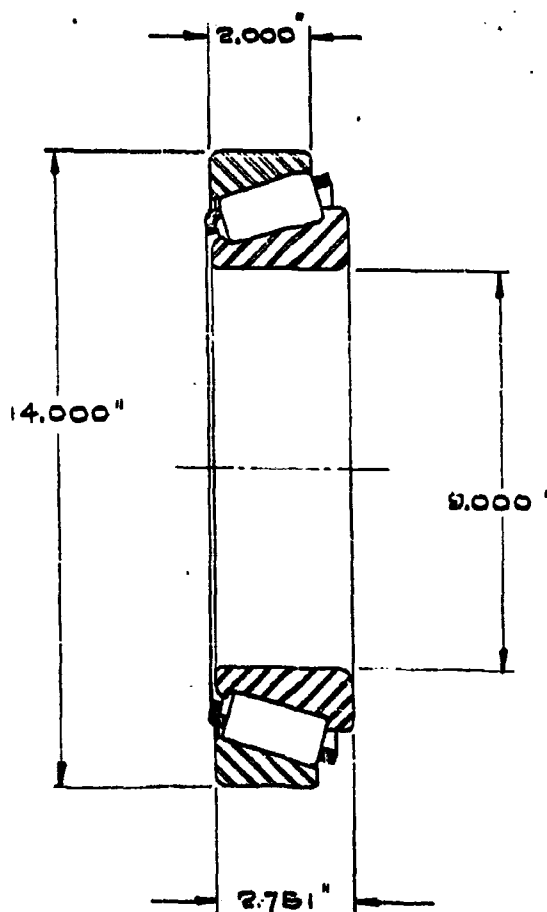


ENCLOSURE 60

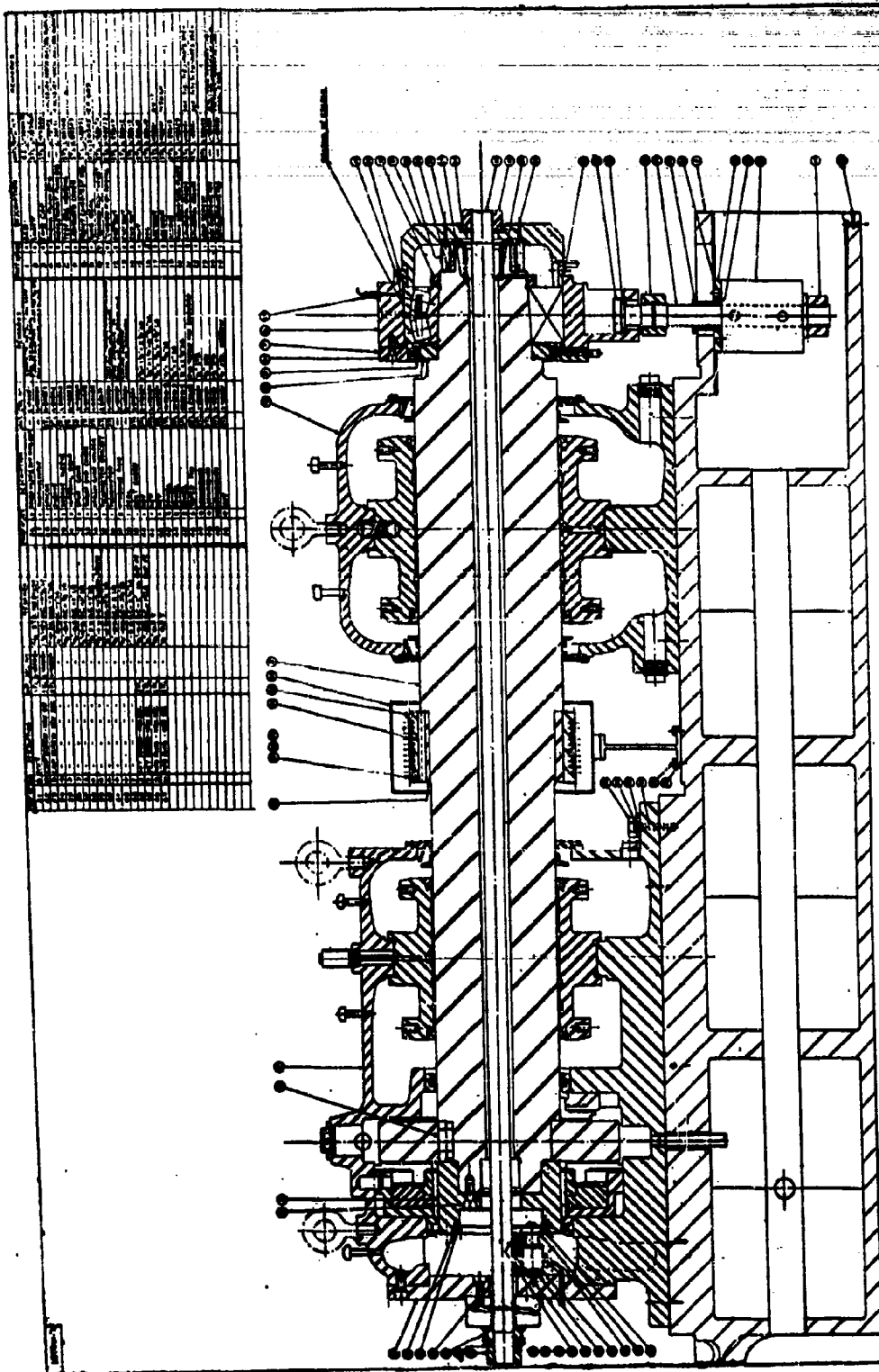
96900/96140 TAPERED ROLLER BEARING

AL63L023

ROLLER PITCH DIAMETER	11.15"	CONE ANGLE	= 17.5°
ROLLER MEAN DIAMETER	2.97"	BASIC DYNAMIC LOAD RATING	
ROLLER LENGTH	1.968"		
NO. OF ROLLERS	28	C	= 150,000 LBS

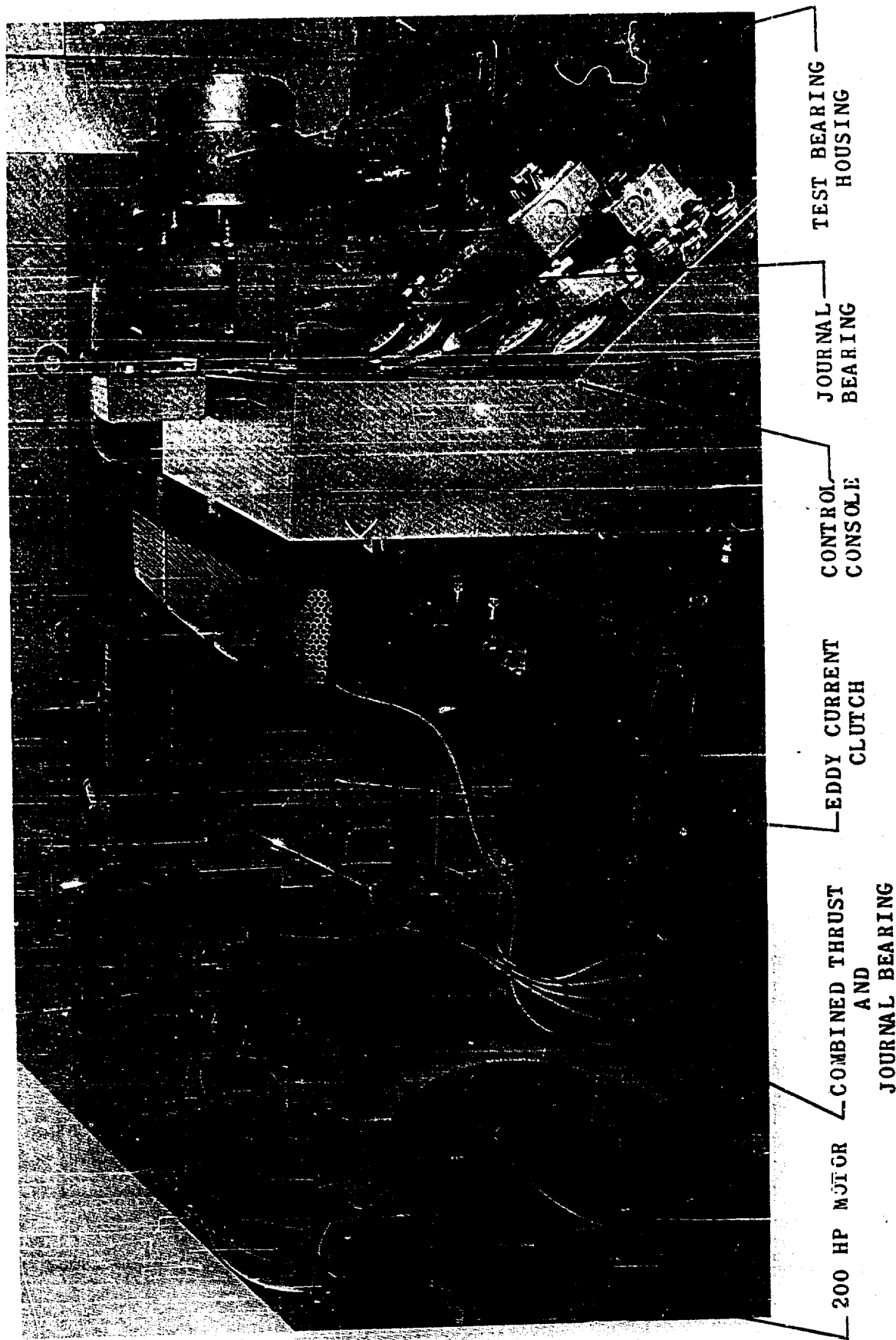


ENCLOSURE 61 HM746646/HM746610 TAPERED ROLLER BEARING

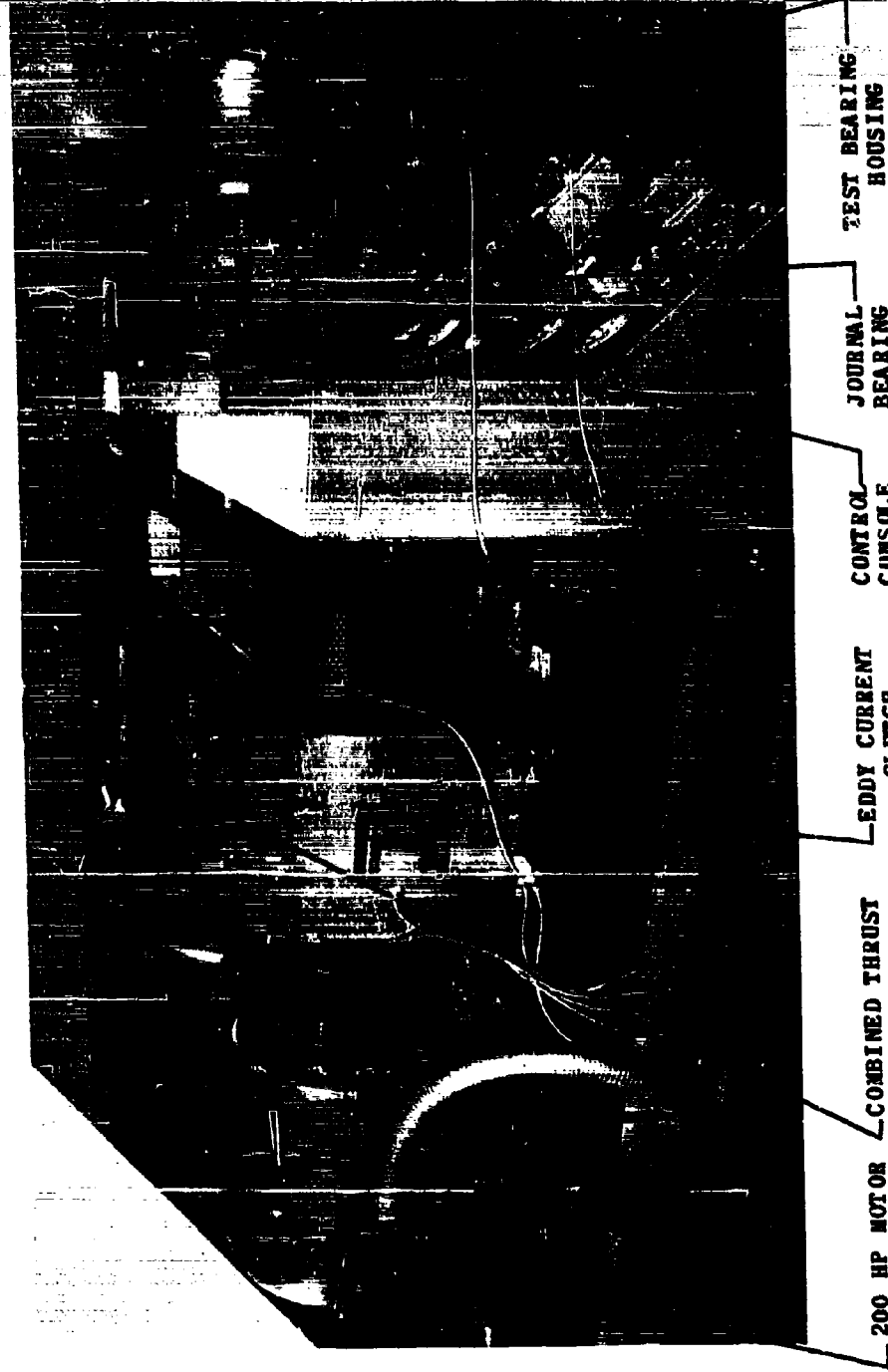


ENCLOSURE 62

CROSS-SECTION THROUGH LARGE VIBRATION TESTER



ENCLOSURE 63 GENERAL VIEW OF LARGE BEARING VIBRATION TESTER



200 HP MOTOR
COMBINED THRUST
AND
JOURNAL BEARING

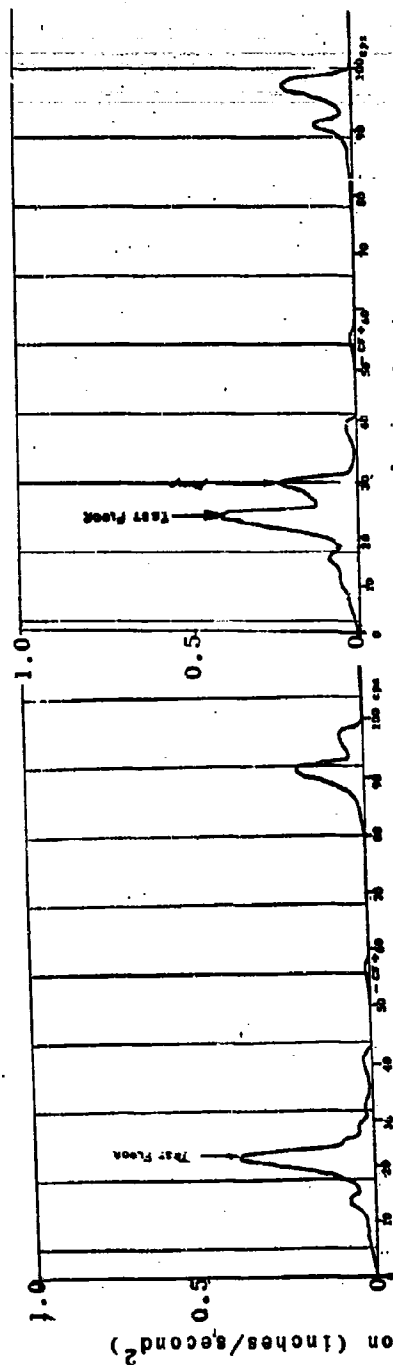
EDDY CURRENT
CLUTCH

CONTROL
CONSOLE

JOURNAL
BEARING

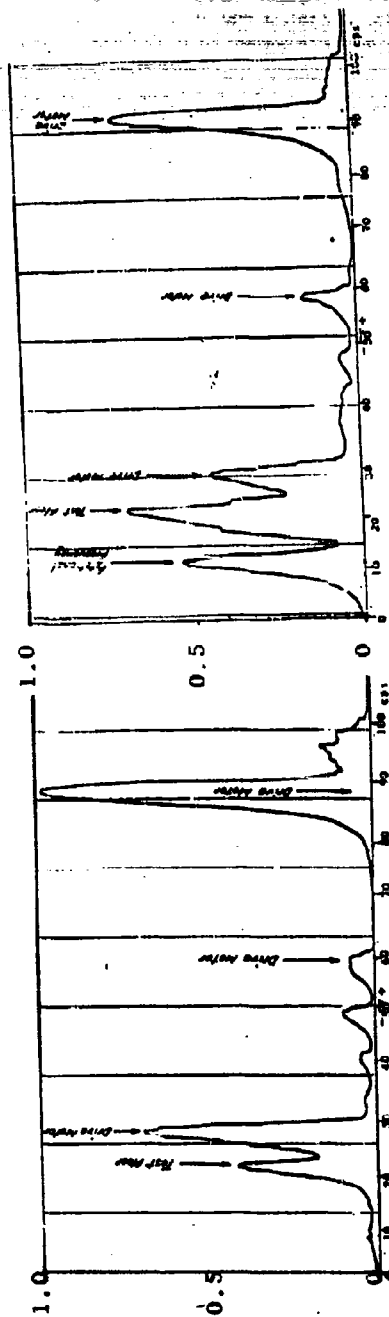
TEST BEARING
HOUSING

ENCLOSURE 63 GENERAL VIEW OF LARGE BEARING VIBRATION TESTER



Test Floor Ambient

Auxiliary Equipment "ON"



Drive Motor and Auxiliary Equipment "ON"

Test Shaft Operating at 800 RPM

ENCLOSURE 64 AMBIENT VIBRATION SPECTRA OF THE FRONT JOURNAL BEARING IN THE HORIZONTAL DIRECTION

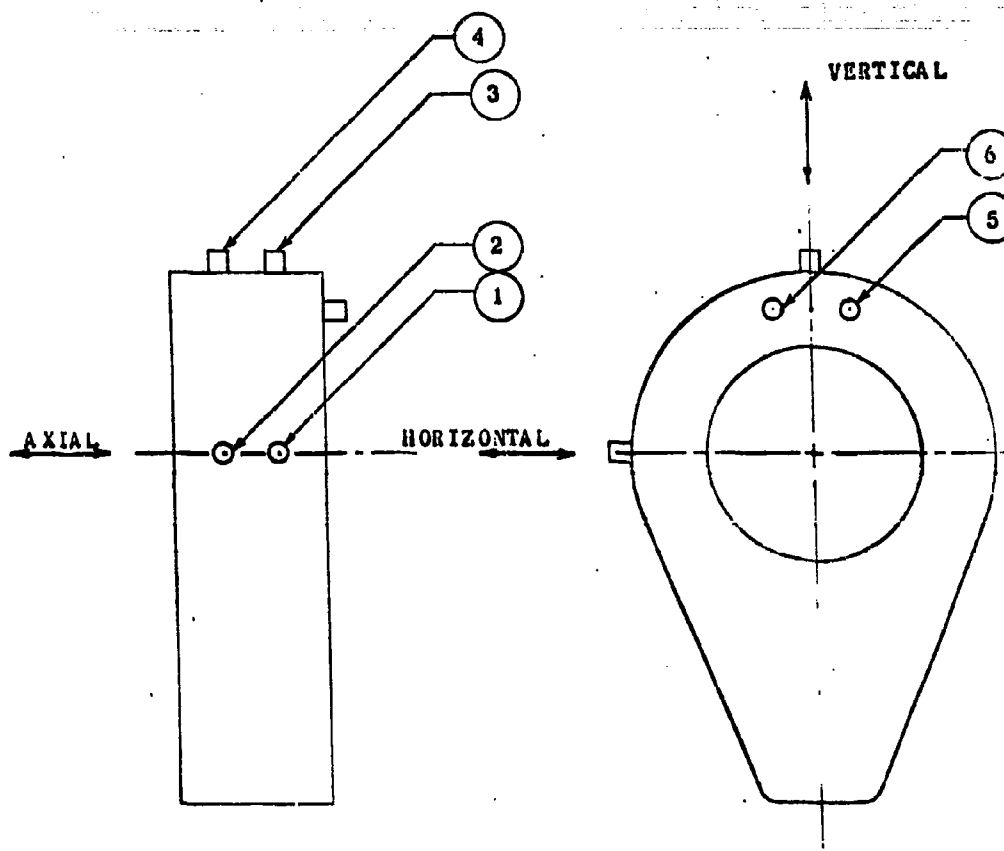
AL63L023

ENCLOSURE 65

OCTAVE BAND VIBRATIONAL ACCELERATIONS IN IN/SEC² MEASURED AT
THE TEST BEARING (23256-K) HOUSING WITH THE TEST BEARING
UNDER 20,000 LBS. RADIAL AND 10,000 AXIAL LOAD

Direction of Measure- ment	Test Shaft Rotational Speed RPM	FREQUENCY BANDS (CPS)											
		3.125 6.25	6.25 12.5	12.5 25	25- 50	50- 100	100- 200	200- 400	400- 800	800- 1600	1600- 3200	3200- 6400	6400- 12800
Vertical	0	0.17	0.07	0.53	1.15	0.46	0.97	0.53	0.47	0.22	0.55	0.28	0.44
	100	0.66	0.35	0.72	1.53	1.15	1.74	1.45	2.00	2.55	0.55	4.1	16
	300	0.21	0.93	1.5	1.62	0.74	2.3	5.4	22.0	20.9	47	22	63
	800	0.35	0.81	1.72	3.45	1.84	3.2	6.1	.43	64	210	170	540
Horizontal	0	0.34	0.34	0.32	0.4	1.15	1.5	0.74	0.80	0.53	0.39	0.16	0.28
	100	0.9	0.34	0.36	0.43	0.90	1.44	1.72	2.65	2.9	2.3	1.6	3.5
	300	0.8	0.8	0.92	0.52	0.78	2.3	6.3	13.8	22	19	10	4
	800	0.69	1.15	2.07	2.3	1.72	2.65	5.8	53	60	66	76	105
Axial	0	0.55	0.8	0.69	1.55	0.81	2.55	0.84	0.54	0.31	0.38	0.24	0.55
	100	1.7	0.69	1.25	2.9	3.2	6.2	2.6	2.2	3.2	2.4	2.1	10.0
	300	2.3	1.8	2.3	4.6	3.9	5.5	7.1	14.5	23	19	12.5	38
	800	.9	1.7	2.6	5.7	5.7	10.4	7.5	40	55	110	87	250

AL63L023

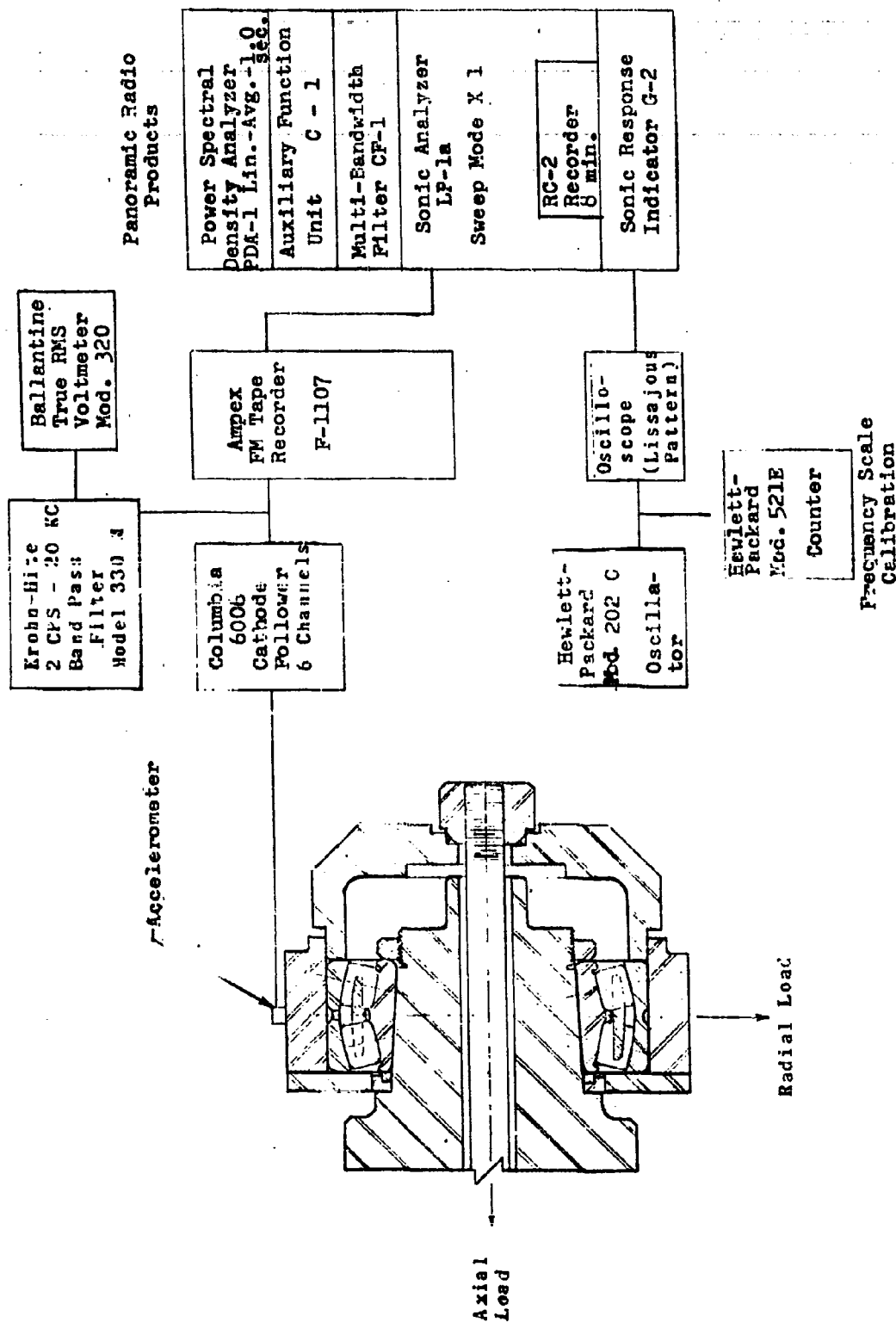


Location	Accelerometer	Model	Sensitivity mV/g
1	Endevco	CA 84*	355
2	Columbia-Research-Laboratory	CRL504-135	25
3	Endevco	CA 77*	365
4	Columbia-Research-Laboratory	CRL504-136	25
5	Columbia-Research-Laboratory	CRL410-100*	440
6	Columbia-Research-Laboratory	CRL504-130	25

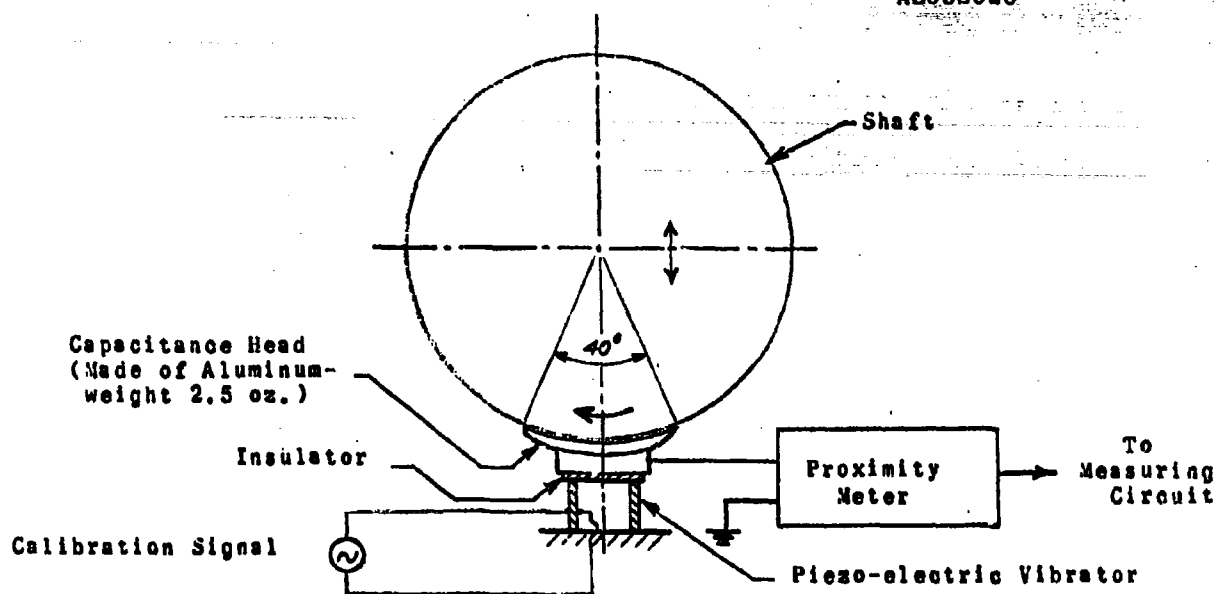
ENCLOSURE 66 ACCELEROMETER LOCATIONS ON TEST BEARING HOUSING

* These are high sensitivity accelerometers and are not used in the high frequency range, above 2,000 cps.

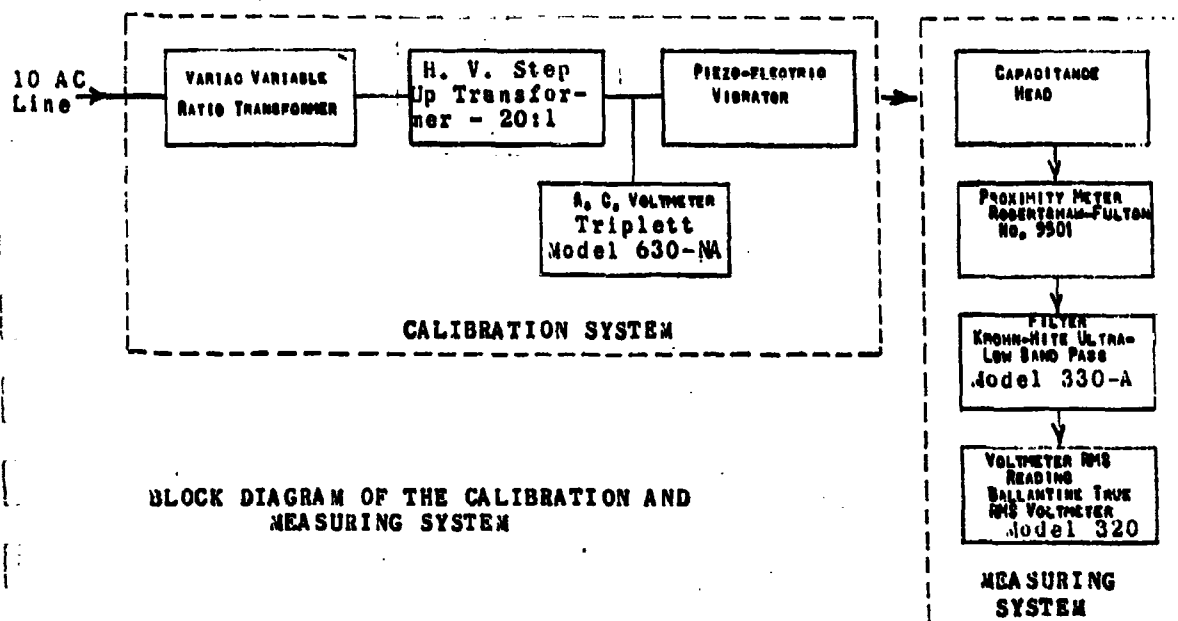
AL63L023



AL63L023



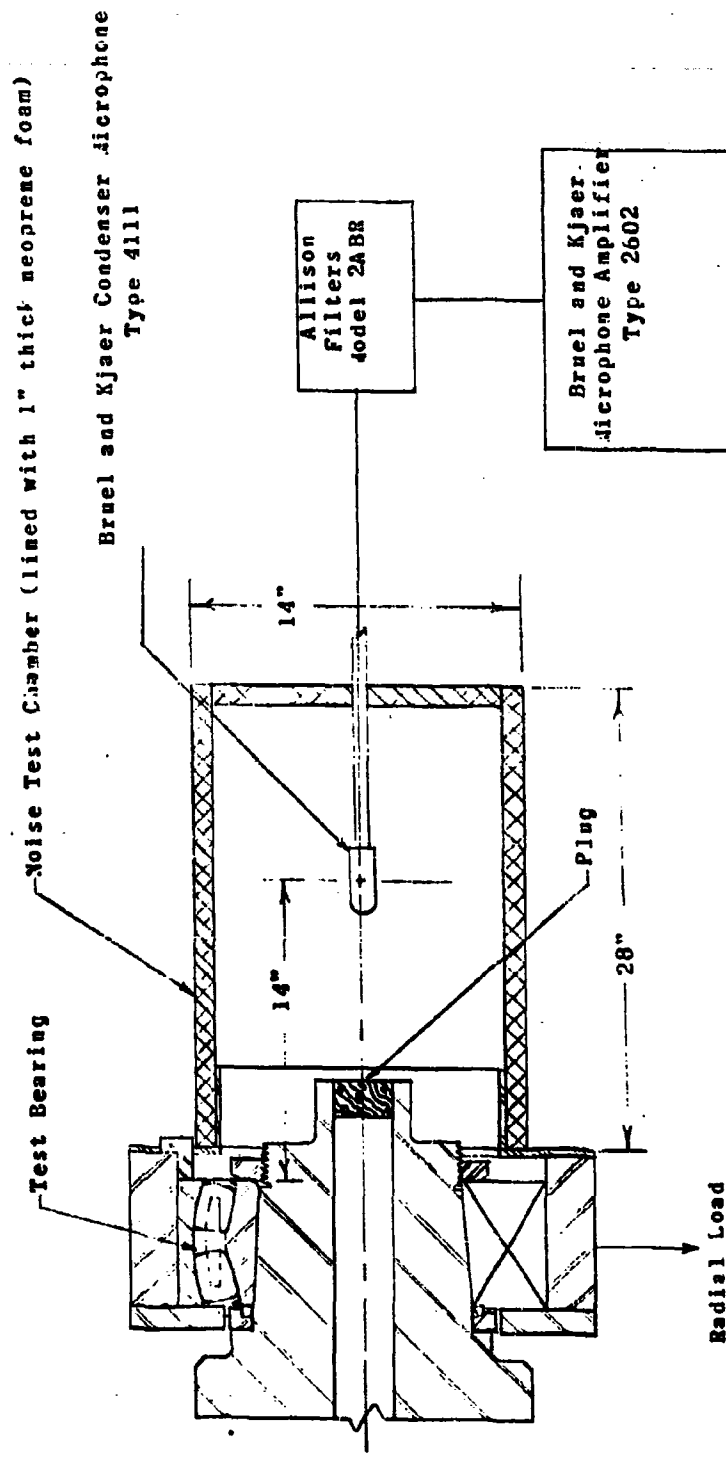
ARRANGEMENT OF THE CAPACITANCE HEAD



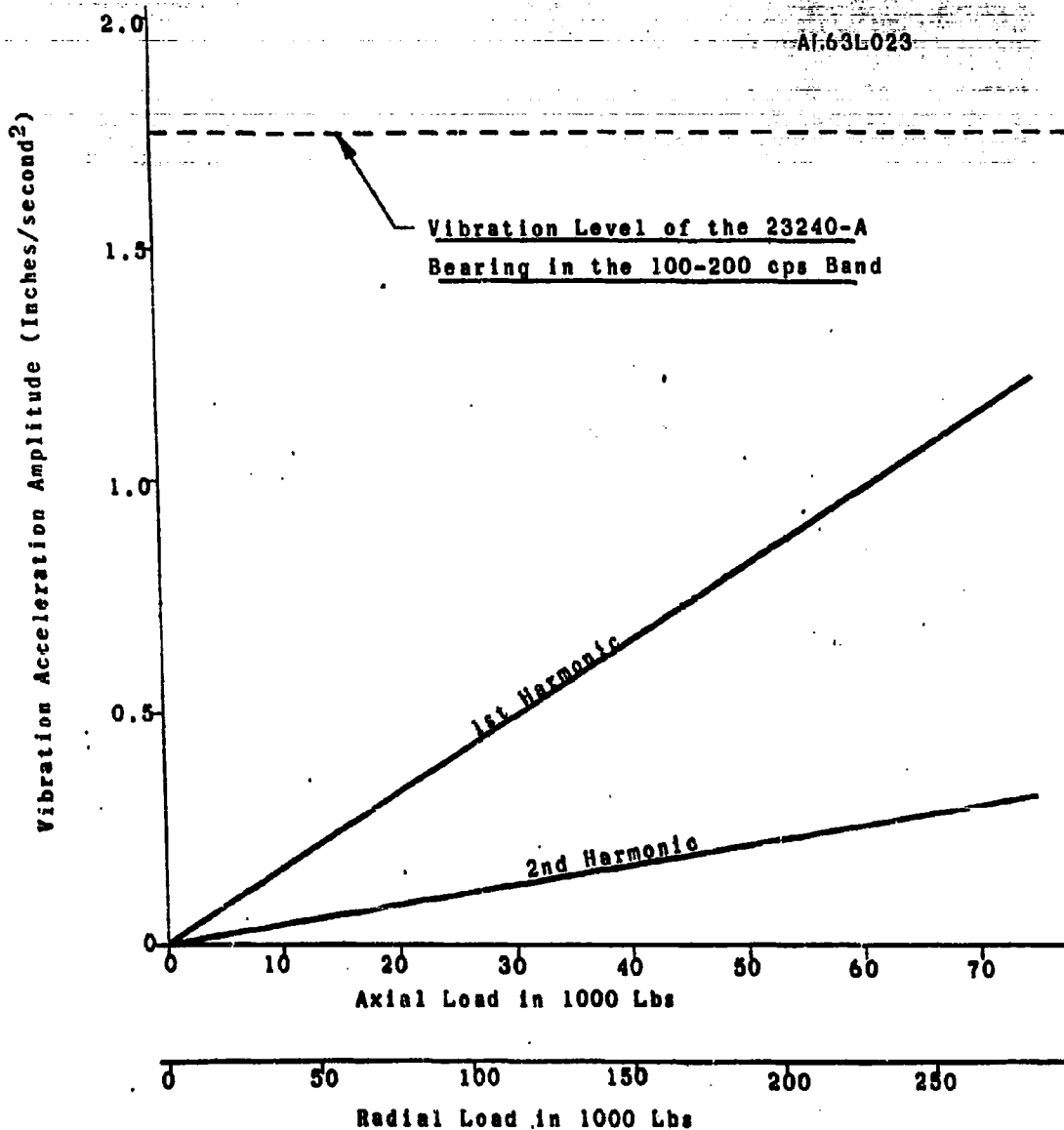
BLOCK DIAGRAM OF THE CALIBRATION AND MEASURING SYSTEM

ENCLOSURE 68 GENERAL ARRANGEMENT OF INSTRUMENTATION OF CAPACITANCE GAGE FOR SHAFT VIBRATION MEASUREMENT

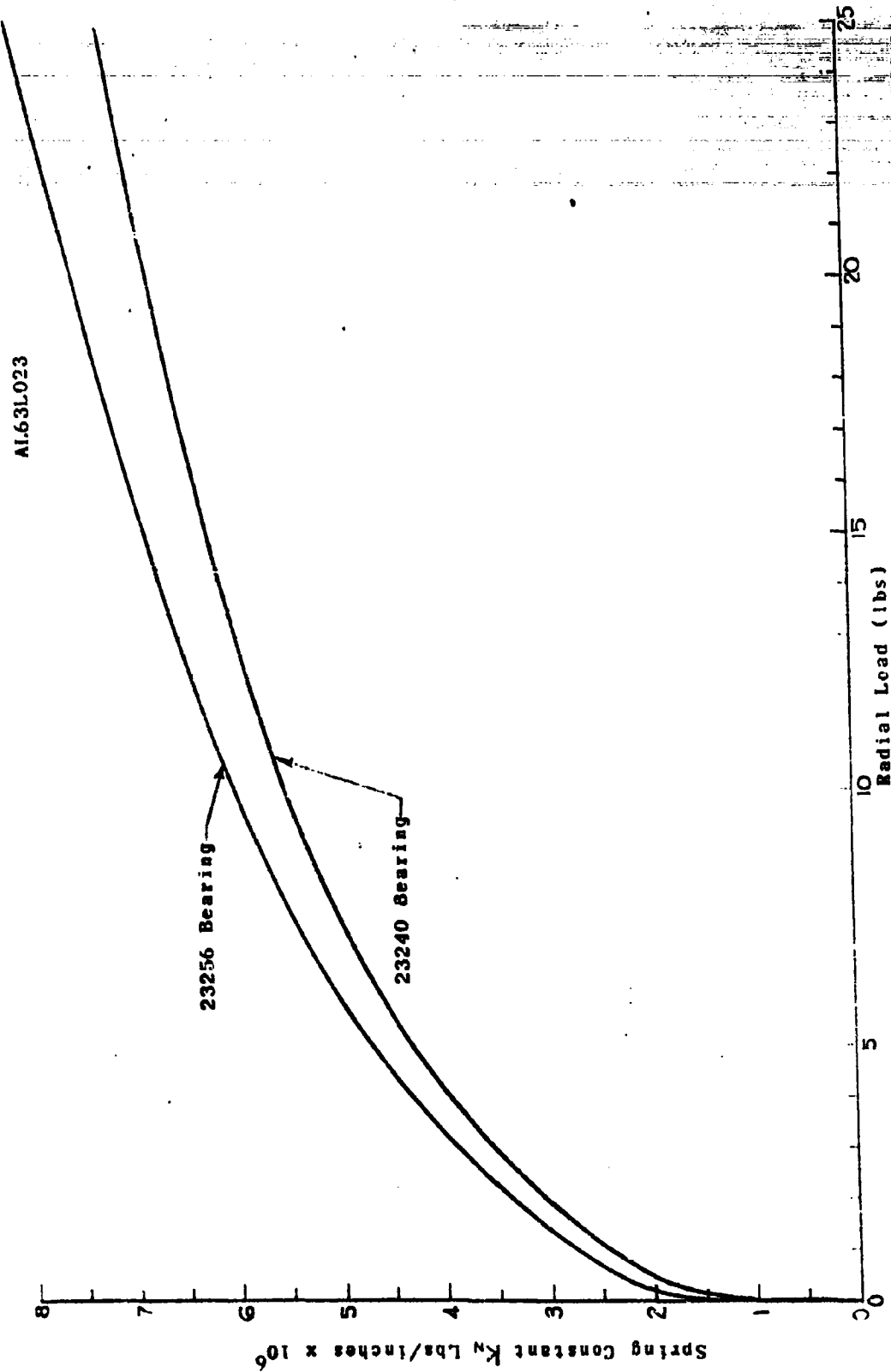
AL63L023



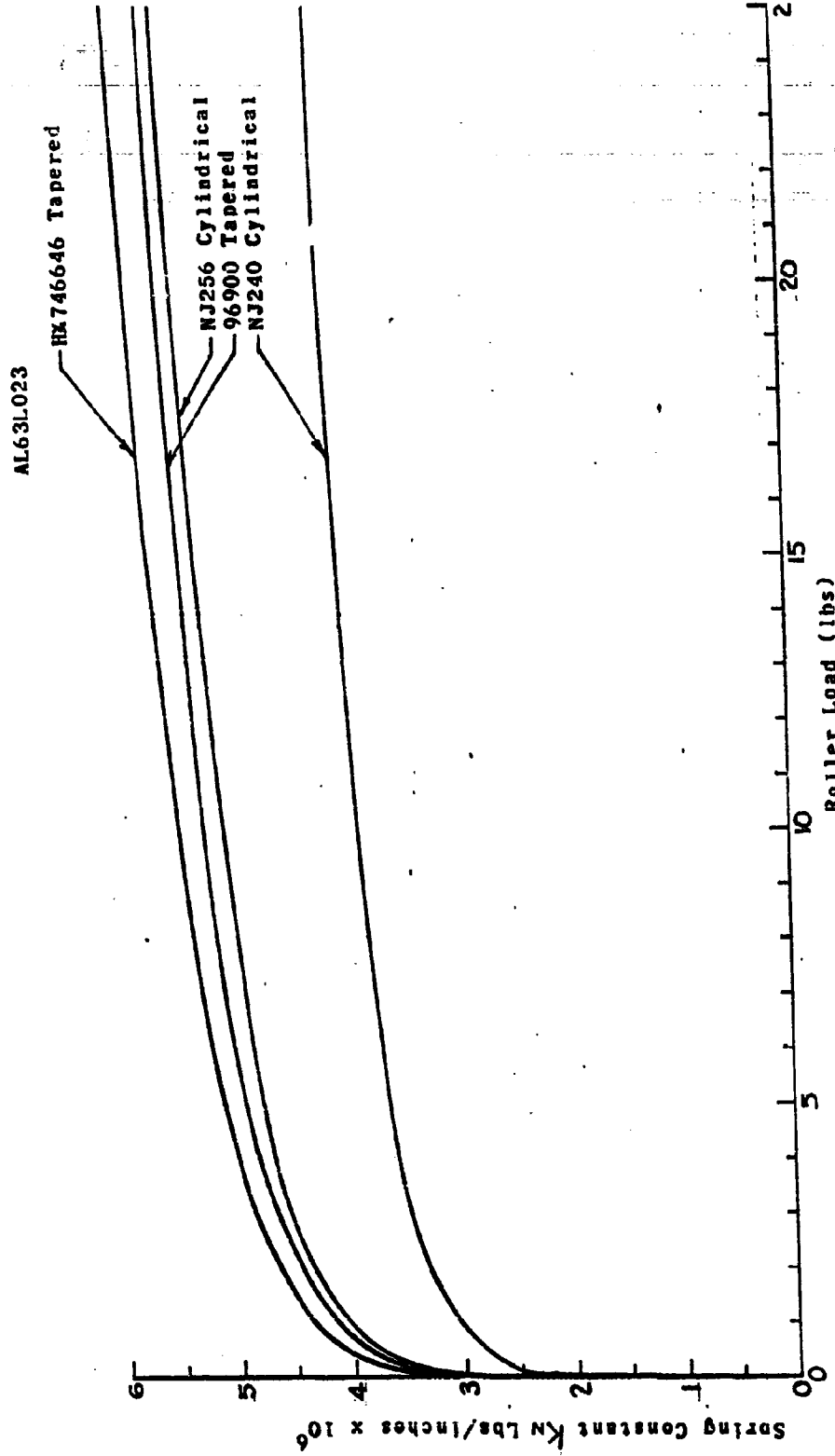
ENCLOSURE 69 NOISE TEST CHAMBER AND INSTRUMENTATION FOR
AIRBORNE NOISE MEASUREMENTS OF LARGE ROLLER
BEARINGS.



ENCLOSURE 70 VIBRATION INDUCED BY ROLLER LOADS IN A 23240
SPHERICAL ROLLER BEARING ROTATING AT 800 RPM

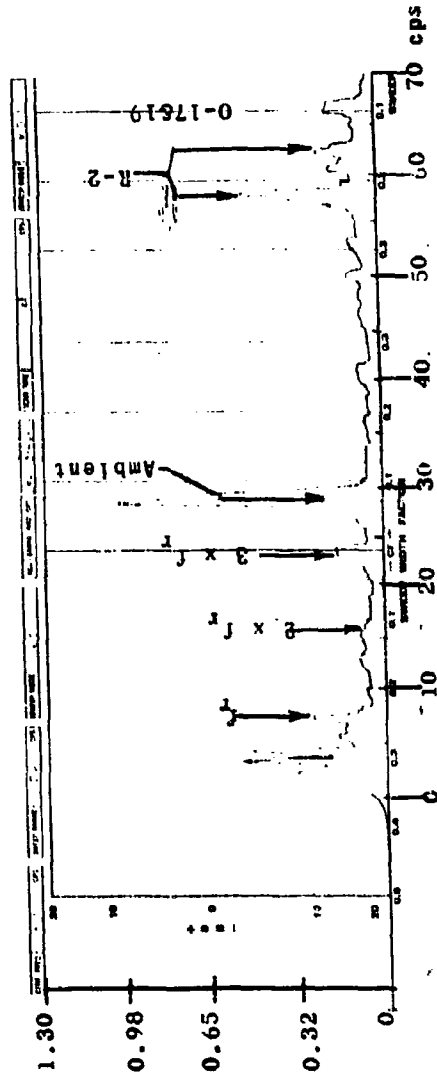


ENCLOSURE 71 ROLLER SPRING CONSTANT K_N AS A FUNCTION OF ROLLER LOAD FOR 23256 AND 23240 SPHERICAL ROLLER BEARINGS

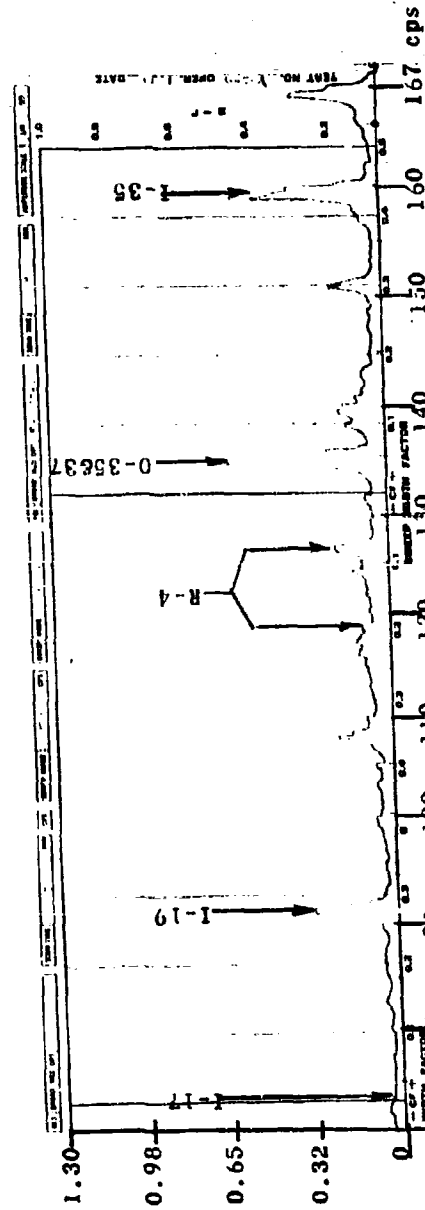


ENCLOSURE 72 ROLLER SPRING CONSTANT K_N AS A FUNCTION OF ROLLER LOAD FOR CYLINDRICAL AND TAPERED ROLLER BEARINGS

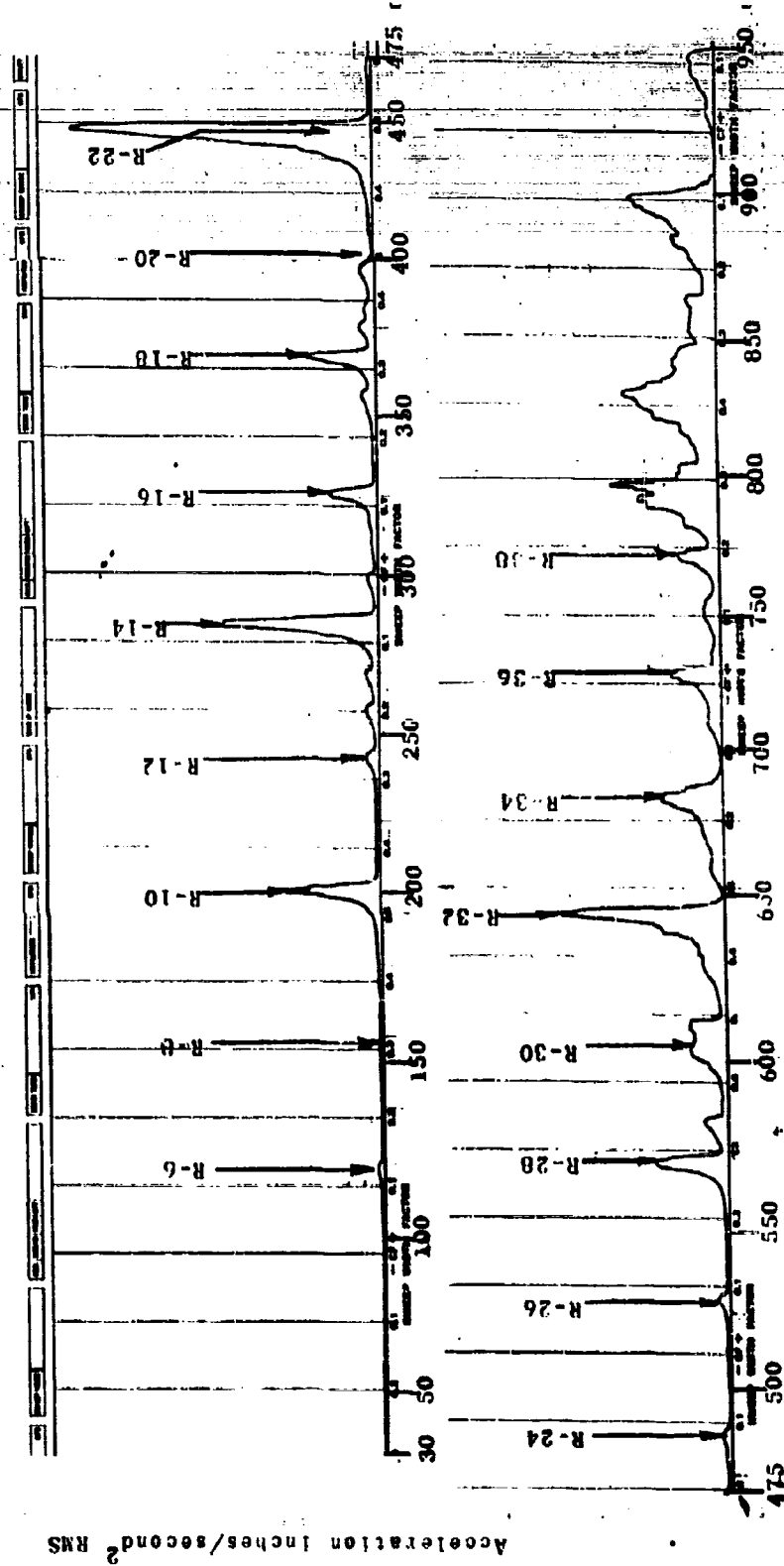
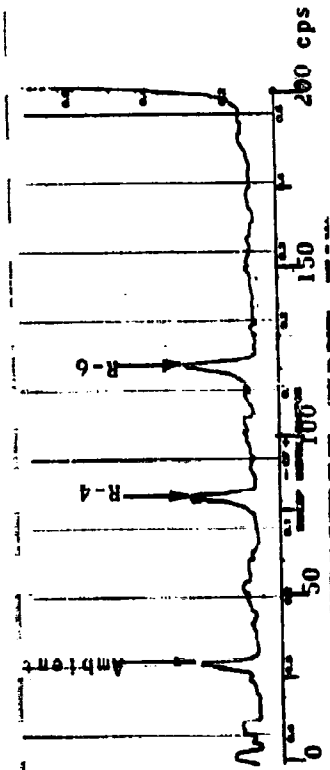
Acceleration in inches/second² RMS



Acceleration in inches/second² RMS



ENCLOSURE 73 LOW FREQUENCY VIBRATION SPECILA OF 23256-6 BEARING IN DIRECTION OF LOAD UNDER 20,000 LBS RADIAL LOAD AT 300 RPM (0-160 cps)



ENCLOSURE 74 VIBRATION SPECTRA OF NJ256-2 BEARING UNDER 20,000 LBS RADIAL LOAD IN THE 0-1000 CPS FREQUENCY RANGE

AL63L023

ENCLOSURE 75

**TABLE OF ORDERS OF WAVINESS CORRESPONDING TO VIBRATION
GENERATED AT OCTAVE BAND CUTOFF FREQUENCIES FOR
23240 SPHERICAL ROLLER BEARINGS**

Vibration Frequency									
	50	100	200	400	800	1600	3200	6400	12800 cps
<u>100 RPM</u>									
W _o	69	139	270	555	1110	2220	4440	8880	17800 wpc
W _i	53	105	210	421	842	1680	3320	6730	13500 wpc
W _b	8	17	33	67	134	269	538	1080	2150 wpc
<u>320 RPM</u>									
W _o	23	46	92	184	369	737	1470	2950	6000 wpc
W _i	18	35	70	141	282	563	1130	2250	4510 wpc
W _b	3	6	11	23	46	91	182	365	730 wpc
<u>1000 RPM</u>									
W _o	8	17	35	69	138	277	554	1108	2217 wpc
W _i	7	13	26	49	100	200	399	799	1598 wpc
W _b	1	2	4	9	17	34	68	137	274 wpc

 W_o = order of outer race waviness W_i = order of inner race waviness W_b = order of roller waviness

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ENCLOSURE 76

TABLE OF ORDERS OF WAVINESS CORRESPONDING TO VIBRATION
GENERATED AT OCTAVE BAND CUTOFF FREQUENCIES FOR
NJ240 CYLINDRICAL ROLLER BEARINGS

	Vibration Frequency								
	50	100	200	400	800	1600	3200	6400	12800 cps
<u>100 RPM</u>									
W _o	76	152	303	606	1212	2424	4848	9697	19394 wpc
W _i	61	119	238	476	952	1905	3810	7619	15238 wpc
W _b	8	16	32	65	129	258	516	1032	2065 wpc
<u>300 RPM</u>									
W _o	23	45	91	182	363	727	1454	2909	5818 wpc
W _i	18	36	71	143	286	571	1143	2286	4571 wpc
W _b	2.4	5	10	19	39	77	155	309	619 wpc
<u>600 RPM</u>									
W _o	9	17	34	68	137	274	547	1094	2188 wpc
W _i	7	13	27	54	107	215	430	859	1718 wpc
W _b	0.9	1.8	3.6	7.3	15	29	58	116	232 wpc

W_o = Order of Outer Race WavinessW_i = Order of Inner Race WavinessW_b = Order of Ball Waviness

AL63L023

ENCLOSURE 77

TABLE OF ORDERS OF WAVINESS CORRESPONDING TO VIBRATION
GENERATED AT OCTAVE BAND CUTOFF FREQUENCIES FOR
96900/96140 TAPERED ROLLER BEARINGS

		Vibration Frequency							
		50	100	200	400	800	1600	3200	6400 12800 cps
<u>100 RPM</u>									
W _o	65	131	263	526	1052	2105	4210	8420	16840 wpc
W _L	55	110	220	441	882	1765	3531	7060	14130 wpc
W _b	5	10	20	40	80	160	320	641	1280 wpc
<u>300 RPM</u>									
W _o	22	43	87	175	350	701	1400	2800	5600 wpc
W _L	18	36	73	147	294	586	1160	2360	4710 wpc
W _b	1.5	3	6	13	26	53	106	213	427 wpc
<u>900 RPM</u>									
W _o	8	16	33	66	133	266	532	1070	2130 wpc
W _L	7	14	27	55	110	221	442	885	1770 wpc
W _b	0.6	1.2	3	5	10	20	40	80	160 wpc

W_o = Order of Outer Race Waviness

W_L = Order of Inner Race Waviness

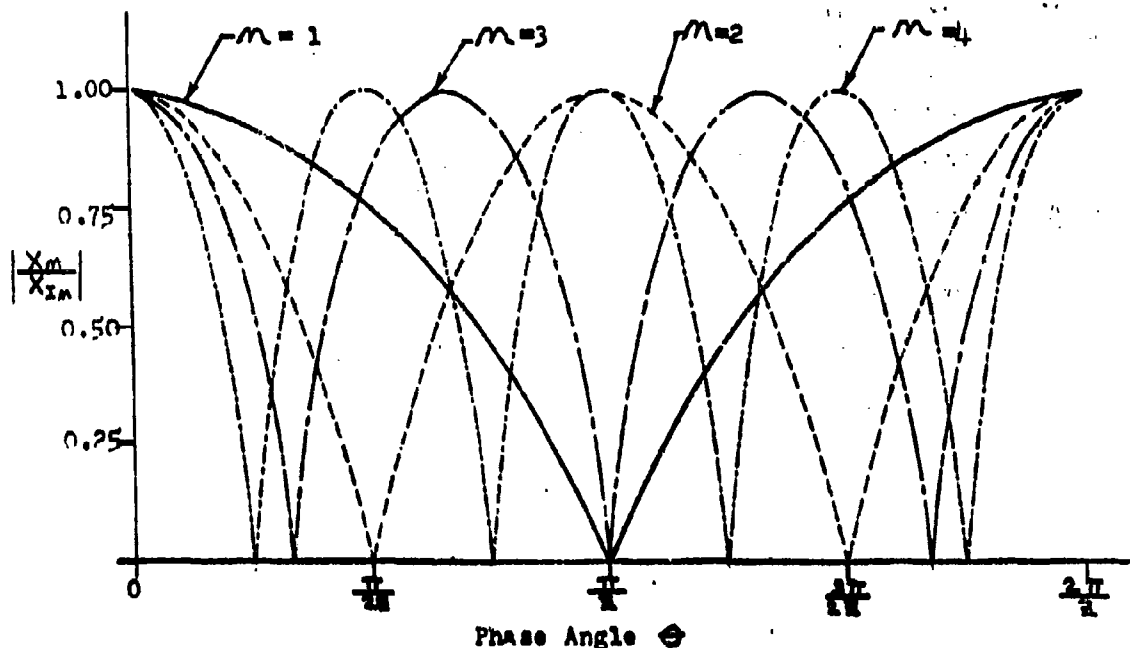
W_b = Order of Roller Waviness

X_{1m} = Vibration amplitude of single-row bearing with Z rolling elements.

X_m = Vibration amplitude of double row bearing with Z rolling elements per row.

θ = Phase angle between the rows of rolling elements.

$m = 1, 2, 3, 4, \dots$, corresponding to K was inner or outer ring waviness where $K = mZ \pm 1$.



ENCLOSURE 78 VIBRATION AMPLITUDES IN DOUBLE-ROW BEARINGS WITH IDENTICAL WAVINESS OF THE ROLLING PATH OF THE TWO ROWS, BUT VARYING "OFFSET ANGLE" θ OF THE ROWS.

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ENCLOSURE 79

VELOCITY BANDWIDTH AMPLIFICATION FACTORS FOR OUTER RING WAVINESS, U_o , INNER RING WAVINESS, U_i ,
AND ROLLER WAVINESS, U_b , FOR A BEARING ROTATING AT 100 RPM

BEARING SIZE	ROLLER DIAMETER	NO. OF ROLLERS PER ROW	PITCH CIRCLE DIAMETER	CONTACT ANGLE α	RADIAL VIBRATIONS			AXIAL VIBRATIONS		
					U_o	U_i	U_b	U_o	U_i	U_b
23256 CAX*	55mm	18	390mm	13°	0.049	0.065	0.138	0.240	0.288	0.615
23240CK*	39mm	19	282mm	13°	0.048	0.064	0.134	0.213	0.282	0.595
NJ256	50mm	19	390mm	0°	0.070	0.091	0.214	-	-	-
NJ240	36mm	19	280mm	0°	0.064	0.081	0.203	-	-	-
9600/ 96140-A	0.938"	32	11.31"	21.5°	0.052	0.068	0.234	0.142	0.186	0.638
96900/ 96140-B	0.938"	33	11.81"	21.5°		0.054	0.070	0.242	0.147	0.660
HM746646/ HM746610-C	1.217"	28	11.15"	17.5°		0.055	0.063	0.193	0.183	0.643

WAVINESS TESTER SPEED:

200 RPM FOR INNER AND OUTER RINGS

740 RPM FOR ROLLERS

* THESE BEARINGS ARE DOUBLE ROW BEARINGS, AND THE AMPLIFICATION FACTORS GIVEN FOR THEM ARE BASED ON
EQUAL LOAD SHARING BETWEEN THE TWO ROWS AND ON A CAGE WITH STAGGERED ROLLERS OR A TWO-PIECE CAGE.

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ENCLOSURE 60

TABULATION OF NATURAL FREQUENCIES OF THE BEARING OUTER RING FOR LARGE ROLLER BEARINGS

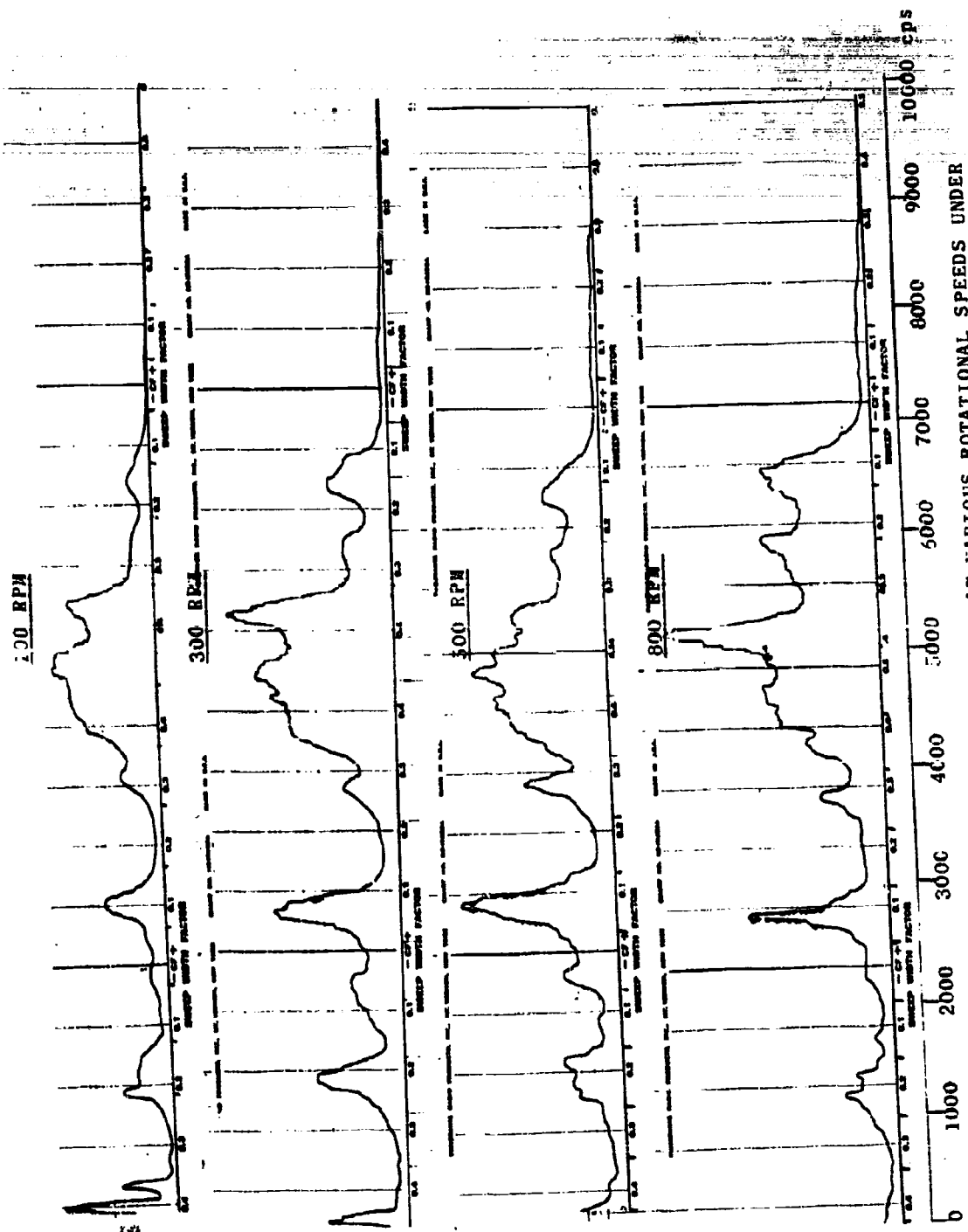
BEARING	NATURAL FREQUENCIES RADIAL VIBRATIONS								AXIAL VIBRATIONS		
	f_R	f_{F1}	f_{F2}	f_{F3}	f_{F4}	f_{F5}	f_{F6}	f_{F7}	f_{F8}	f_H	f_A
22256 SPHERICAL	2190	2780	3060	3410	4020	4800	6100	7560	-	8000	430
23240 SPHERICAL	2840	3640	4080	4760	5680	7080	9080	12500	15600	1210	235
63156 CYLINDRICAL	2590	3300	3690	4250	5320	6800	8720	11000	13700	678	-
NJ240 CYLINDRICAL	3540	4500	5050	5900	7400	9540	12200	15600	19400	1140	-
96900/56140 TAPERED	5930	7530	8120	8710	9640	11090	13100	17000	18600	1970	2590

f_R = THE NATURAL FREQUENCY OF THE RADIAL VIBRATIONS IN THE RIGID BODY MODE

$f_{F1}, f_{F2}, f_{F3}, f_{F4}, f_{F5}, f_{F6}, f_{F7}, f_{F8}$ = THE NATURAL FREQUENCIES OF THE LOWEST EIGHT HARMONICS OF FLEXURAL VIBRATIONS

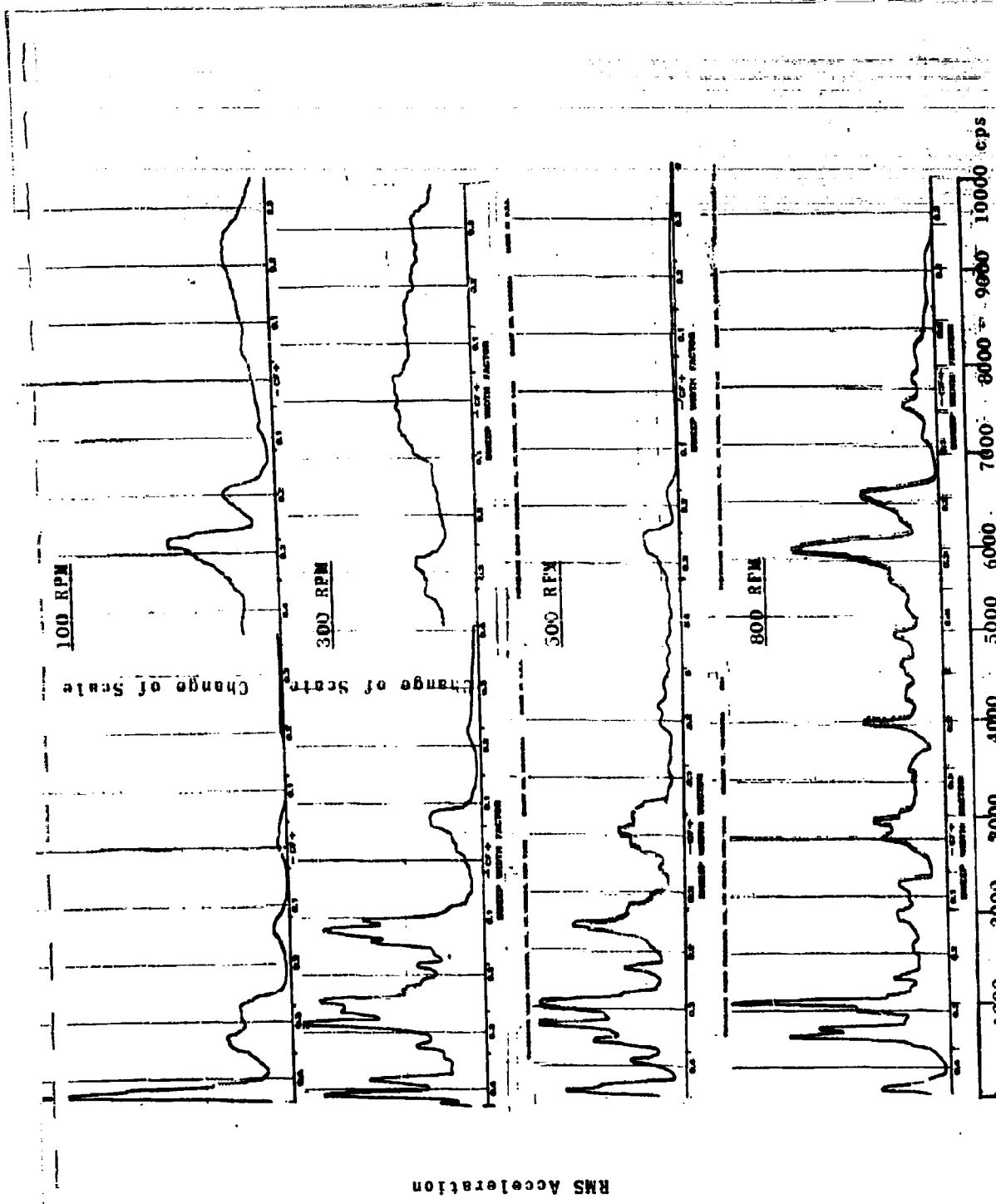
f_A = THE NATURAL FREQUENCY OF AXIAL VIBRATIONS IN THE RIGID BODY MODE

f_H = THE NATURAL FREQUENCY OF THE RADIAL VIBRATIONS OF A SYSTEM CONSISTING OF THE BEARING MOUNTED IN ITS HOUSING ON THE VIBRATION TESTER.



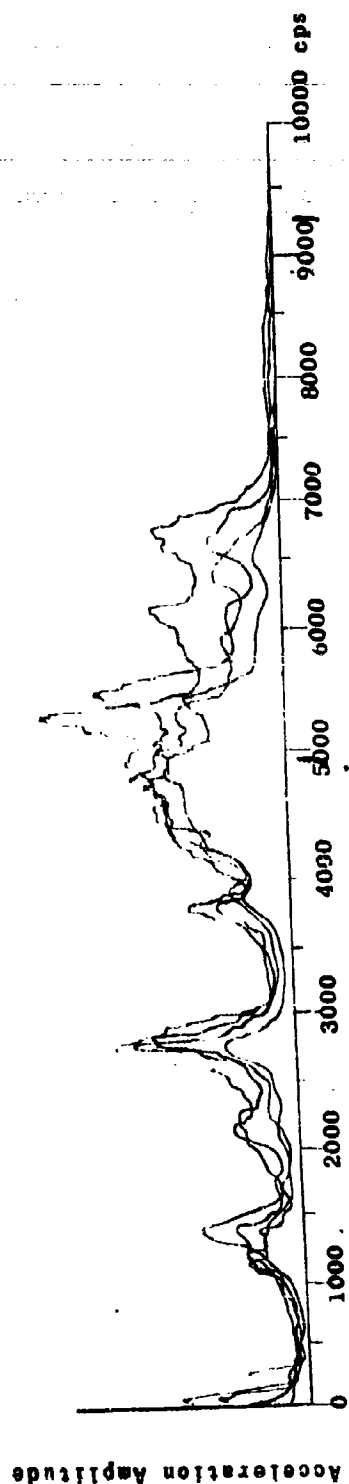
RMS Acceleration

ENCLOSURE 81 VIBRATION SPECTRA OF 23240 BEARING AT VARIOUS ROTATIONAL SPEEDS UNDER 10,000 LBS RADIAL LOAD.

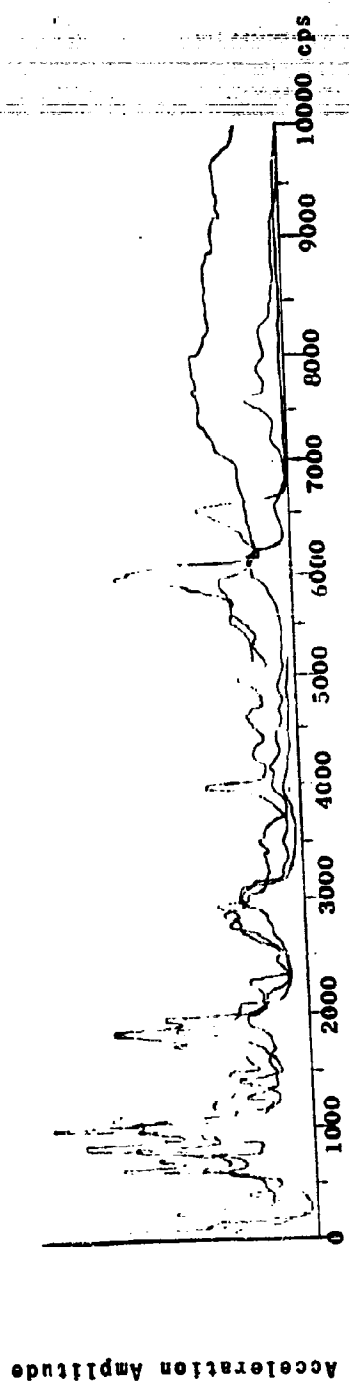


ENCLOSURE 82 VIBRATION SPECTRA OF 23256 BEARING AT VARIOUS ROTATIONAL SPEEDS UNDER 20,000 LBS RADIAL LOAD (0-10000 cps)

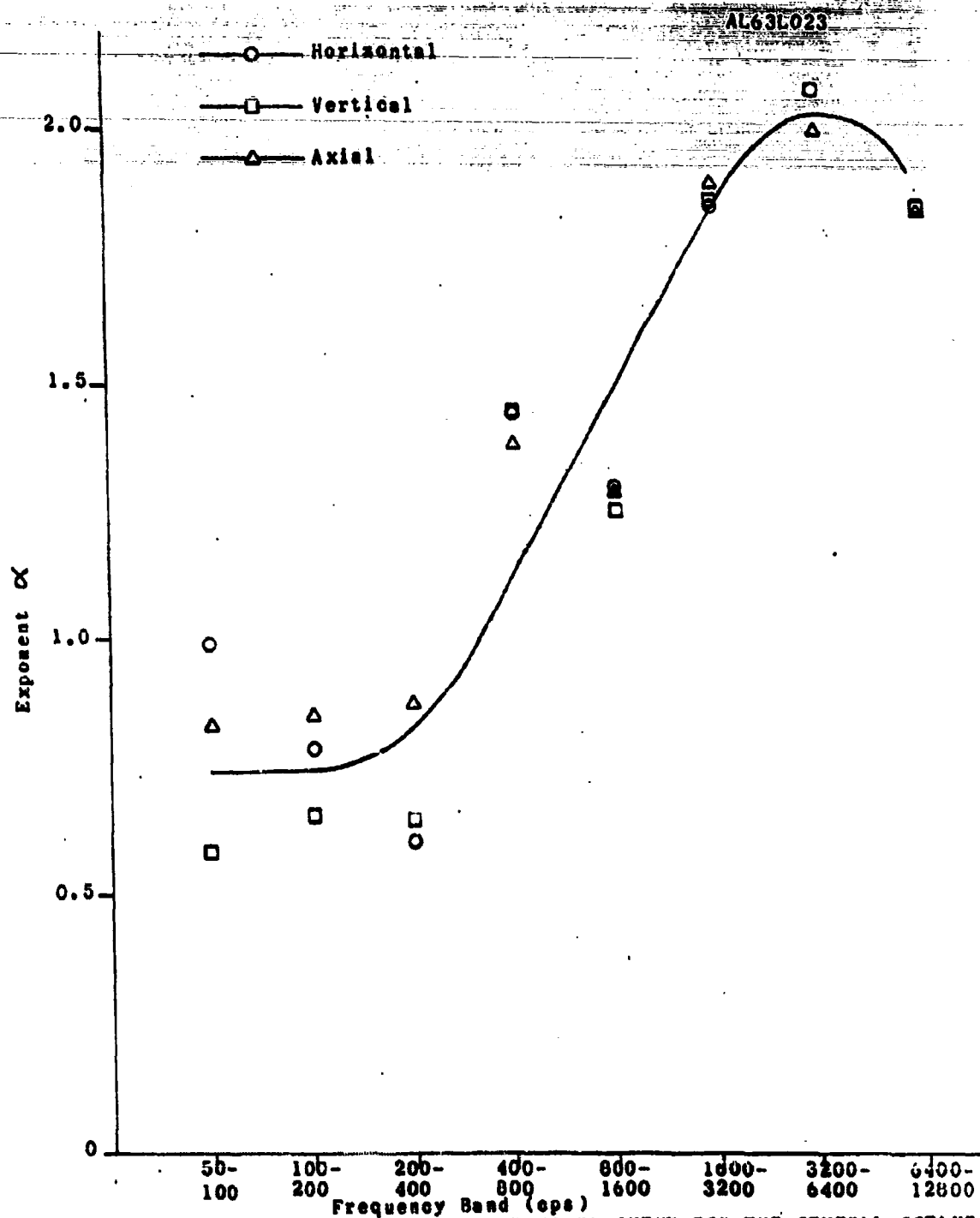
2324C Bearing



23256 Bearing

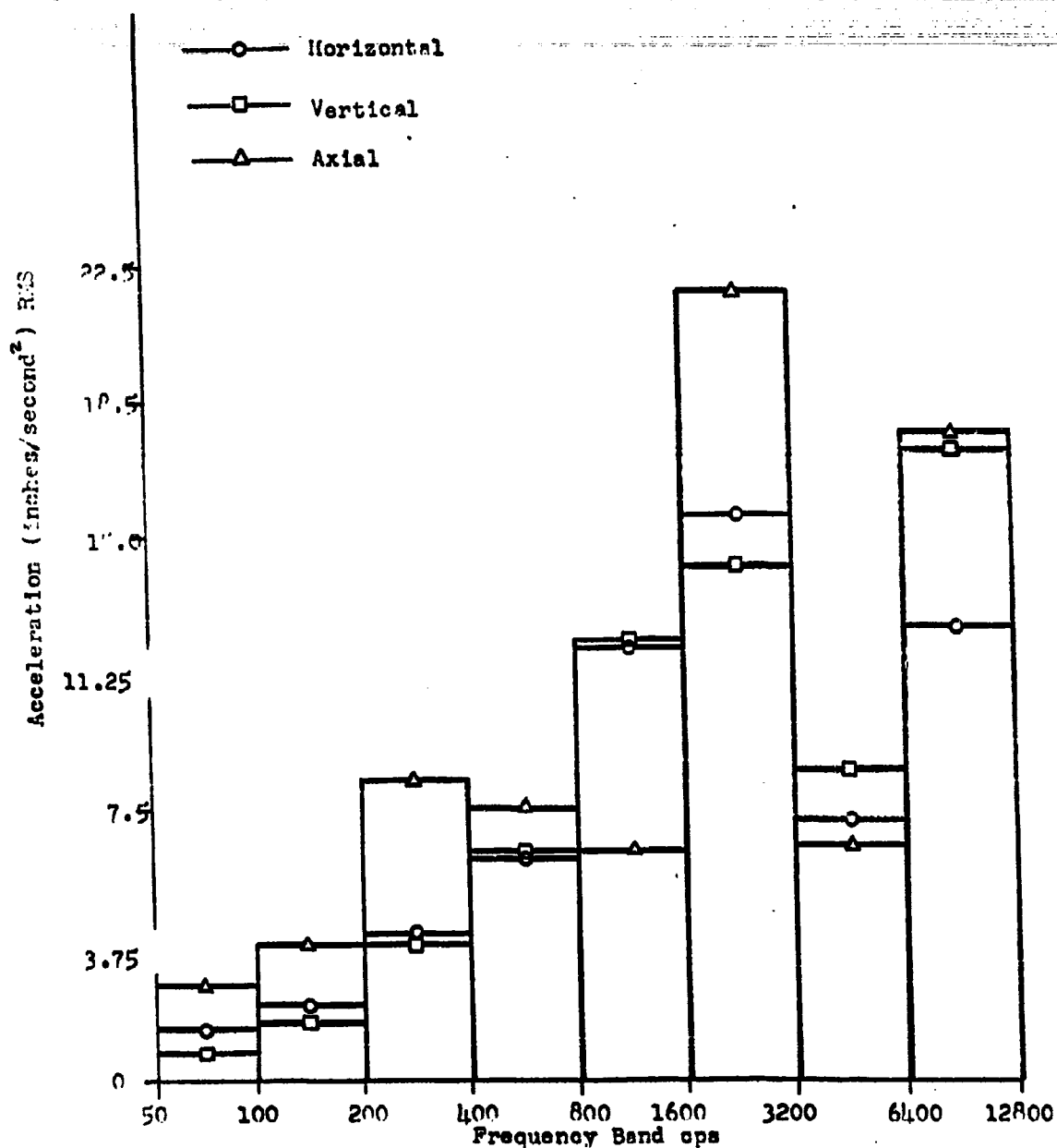


ENCLOSURE 83 SUPERIMPOSED VIBRATION SPECTRA OF A 23240 AND A 23256 BEARING OPERATING AT
ROTATIONAL SPEEDS OF 100, 300, 500 AND 800 RPM.



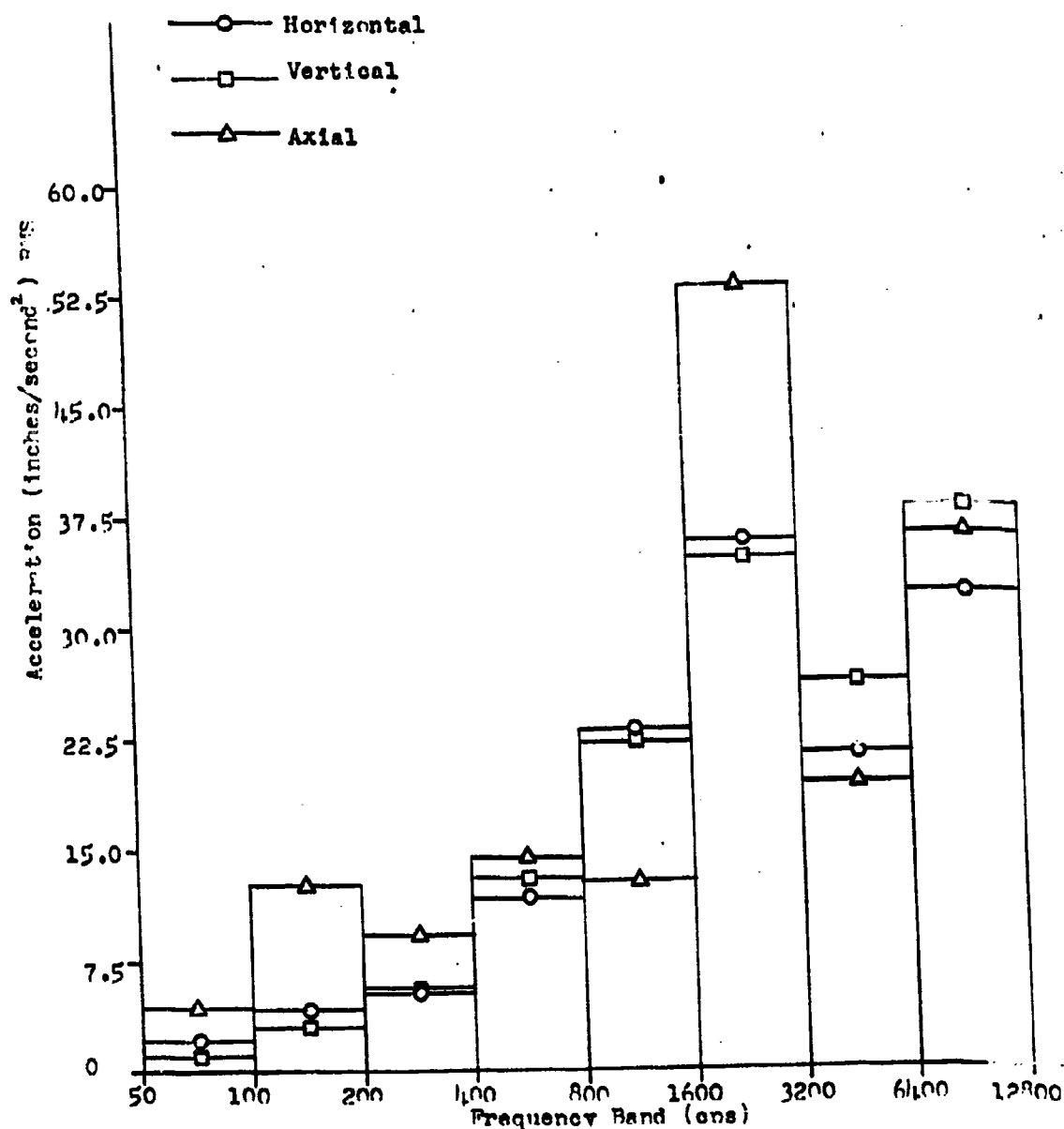
ENCLOSURE 64 ACCELERATION: - SPEED EXPONENT FOR THE SEVERAL OCTAVE BANDS AT 60,000 LBS FOR 23256 SPHERICAL ROLLER BEARINGS.

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ENCLOSURE 85 OCTAVE BAND VIBRATION SPECTRUM OF 23256 SPHERICAL ROLLER BEARING UNDER 30,000 LBS RADIAL LOAD AT 300 RPM.

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ENCLOSURE 86 OCTAVE BAND VIBRATION SPECTRUM OF 23256 SPHERICAL ROLLER BEARINGS UNDER 30,000 LBS RADIAL LOAD AND 500 RPM.

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ENCLOSURE 87

WAVINESS OF 23240CK SPHERICAL ROLLER BEARING COMPONENTS
(23240-A, 23240-B, AND 23240-C BEARINGS)

FREQUENCY BANDS (WPC)													
BEARINGS COMPONENTS	3- 6	6- 12	12- 24	24- 48	48- 96	15- 30	30- 60	60- 120	120- 240	240- 480	480- 960	960- 1920	1920- 3840

WAVINESS VELOCITY (MICROINCHES/SEC.) AT 200 RPM ROTATIONAL SPEED

INNER RACE

23240-A	1050	1740	2050	6350	5300	3750	9000	5250	14000	20000	28000	43500	-
23240-B	400	240	310	1050	475	450	1000	550	725	2000	3950	8250	12000
23240-C	250	200	380	970	450	225	970	525	1150	1800	3850	5700	1000

OUTER RACE

23240-A	775	1230	1150	4500	1850	2000	4750	2700	18500	17500	19000	60500	-
23240-B	550	450	675	1750	1340	1025	1500	900	1750	4850	16000	110000	70000
23240-C	650	425	675	1400	825	750	1250	825	1250	2300	4050	14500	5250

4- 8	8- 16	16- 32	32- 64	64- 128
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WAVINESS VELOCITY (MICROINCHES/SEC.) AT 740 RPM ROTATIONAL SPEED

ROLLERS (AVERAGE OF 30)

23240-A	2700	2950	5100	8200	16200
23240-B	950	1170	1650	2300	3400
23240-C	326	304	445	732	1510

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ENCLOSURE 68

WEARINESS OF NJ240K CYLINDRICAL ROLLER BEARING COMPONENTS (NJ240-A AND NJ240-B)

BEARING COMPONENTS	3-6	6-12	12-24	24-48	48-96	96-192	192-384	384-768	768-1536	1536-3072	3072-6144	6144-12288	12288-24576
--------------------	-----	------	-------	-------	-------	--------	---------	---------	----------	-----------	-----------	------------	-------------

MAXIMUM VELOCITY (MILES PER HOUR) AT 200 RPM ROTATIONAL SPEED

INNER RINGS

NJ240-A	325	560	650	2200	6000	9000	39000	95000	200000	209000	205000	167000	90000
NJ240-B	300	350	400	1200	1900	600	1400	1600	3000	7000	15000	28000	24000

OUTER RINGS

NJ240-A	1800	450	1000	1650	4500	1350	2100	7500	15500	31000	34000	48500	10500
NJ240-B	1800	1000	900	1800	1600	1700	1600	1200	9200	6000	8000	21000	1000

4-8	8-16	16-32	32-64	64-128
-----	------	-------	-------	--------

MAXIMUM VELOCITY (MILES PER HOUR) AT 750 RPM ROTATIONAL SPEED

ROLLERS AVERAGE OF 19

NJ240-A	2750	4350	6750	8850	11000
NJ240-B	520	245	390	670	940

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ENCLOSURE 82

WEARINESS OF TAPERED ROLLER BEARING COMPONENTS (26200/26100-A, 26200/26100-B AND M746666/M746610-C)

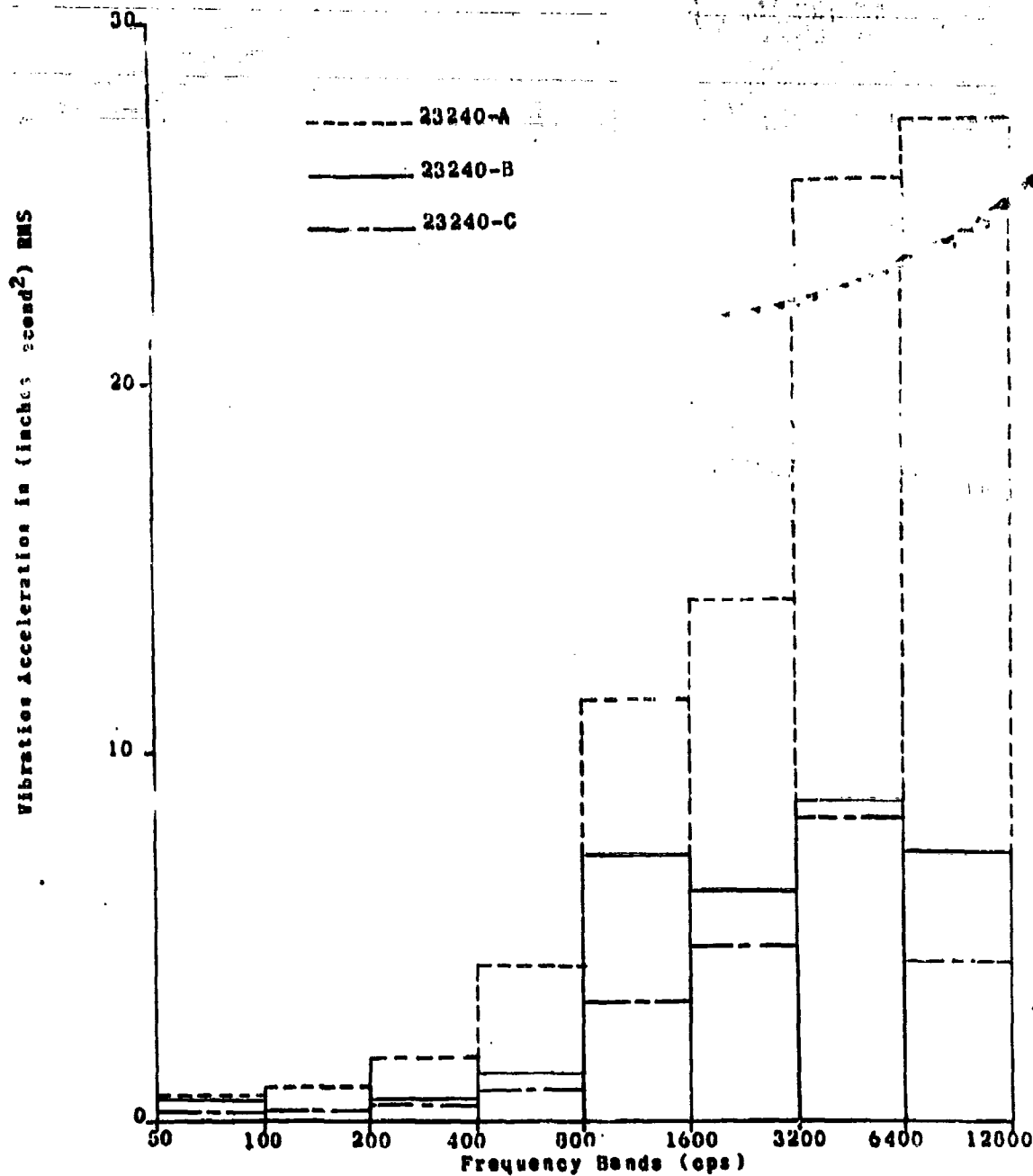
Bearing Component	3-6	6-12	12-24	24-48	48-96	96-192	192-384	384-768	768-1536	1536-3072	3072-6144	6144-12288	12288-24576
WEARINESS VELOCITY (MILS PER MINUTE) AT 200 ROTATIONAL SPEED													
SOBS													
26200-A	1000	700	1000	2000	5500	1600	2100	7000	14000	24000	80000	55000	7000
26100-B	700	950	800	2200	1200	2100	2600	3000	6000	26000	60000	7000	7000
M746666-C	500	350	800	5000	1600	5000	6000	16000	20000	28000	55000	120000	75000
SMR													
26200-A	500	1000	8000	1000	6000	2000	15000	10000	15000	25000	55000	90000	130000
26100-B	1200	800	1000	1800	1600	1800	2000	2500	2500	4800	12000	100000	130000
M746610-C	1000	700	900	1600	2000	1800	2000	3000	3000	6000	10000	65000	160000
CONV FLANGE MIXTURE													
26200-A	500	1000	1000	2000	5000	2000	1000	1000	500	6000	54000	44000	23000
26100-B	700	1000	1000	2000	2000	2000	2000	1000	1000	8000	45000	75000	
M746666-C	500	1000	1000	2000	2000	2000	2000	1000	1000	4000	10000		

WEARINESS VELOCITY (MILS PER MINUTE) AT 200 ROTATIONAL SPEED

TOLERANCE N. M. WEARINESS

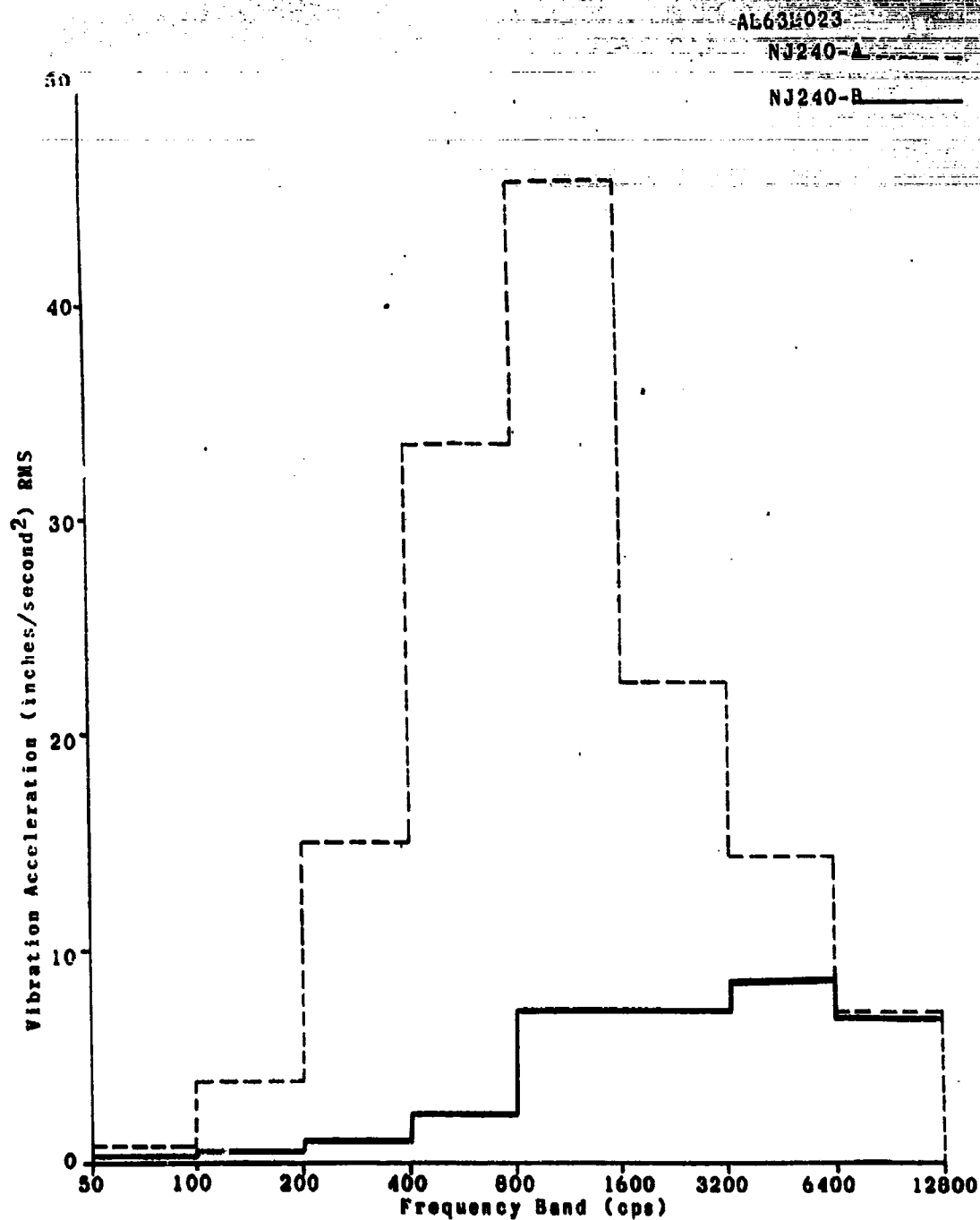
	3-6	6-12	12-24	24-48	48-96	96-192	192-384	384-768	768-1536	1536-3072	3072-6144	6144-12288	12288-24576
26200-A													
26100-B													
M746666-C													
26200-A	2000	2800	5000										
26100-B				1200	2200								
M746666-C				1000	2000								
26200-A													
26100-B													
M746666-C													

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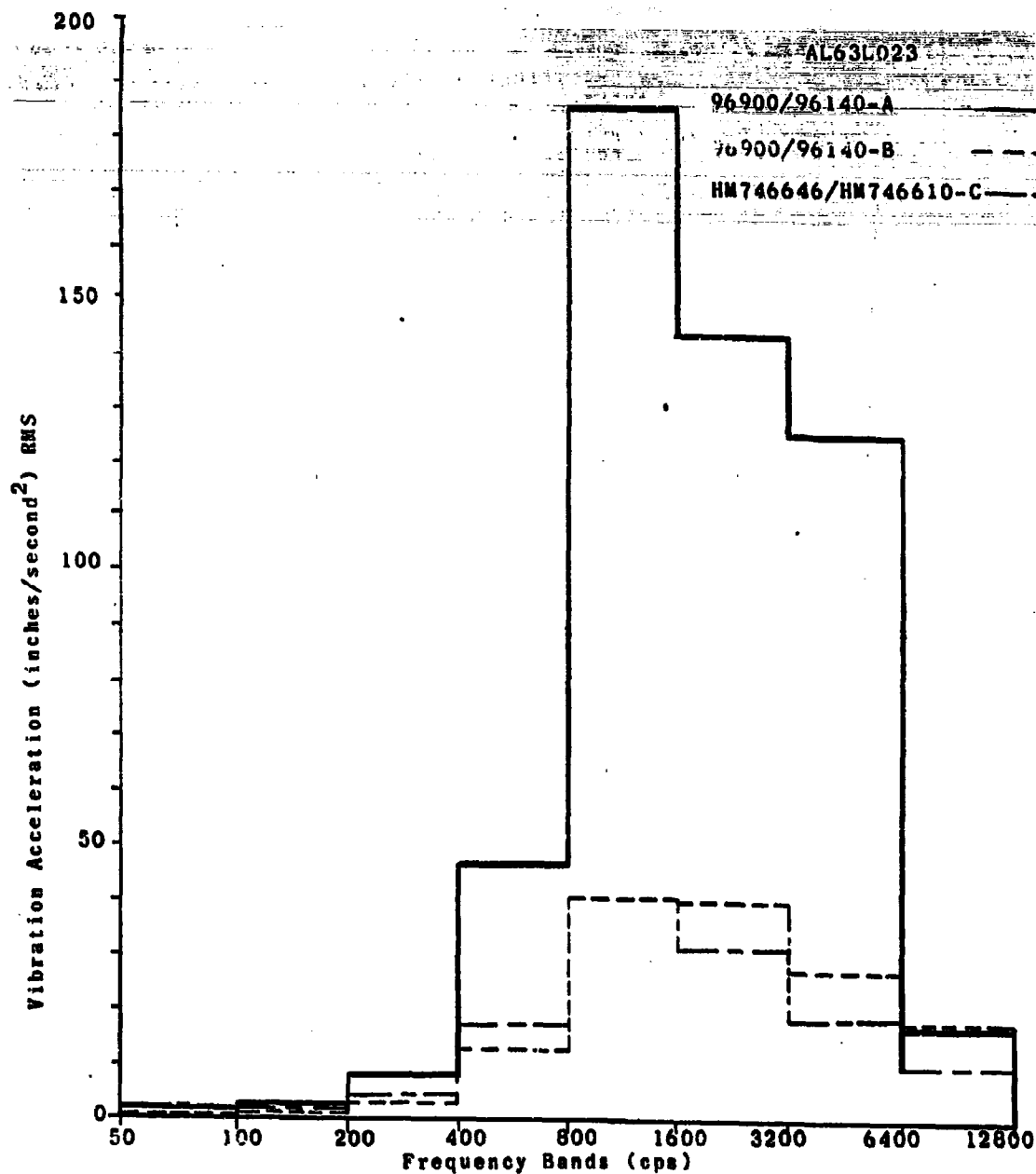


ENCLOSURE 90

OCTAVE BAND VIBRATION SPECTRUM OF THE STANDARD AND IMPROVED 23240 SPHERICAL BEARINGS IN THE DIRECTION OF LOAD AT 300 RPM.

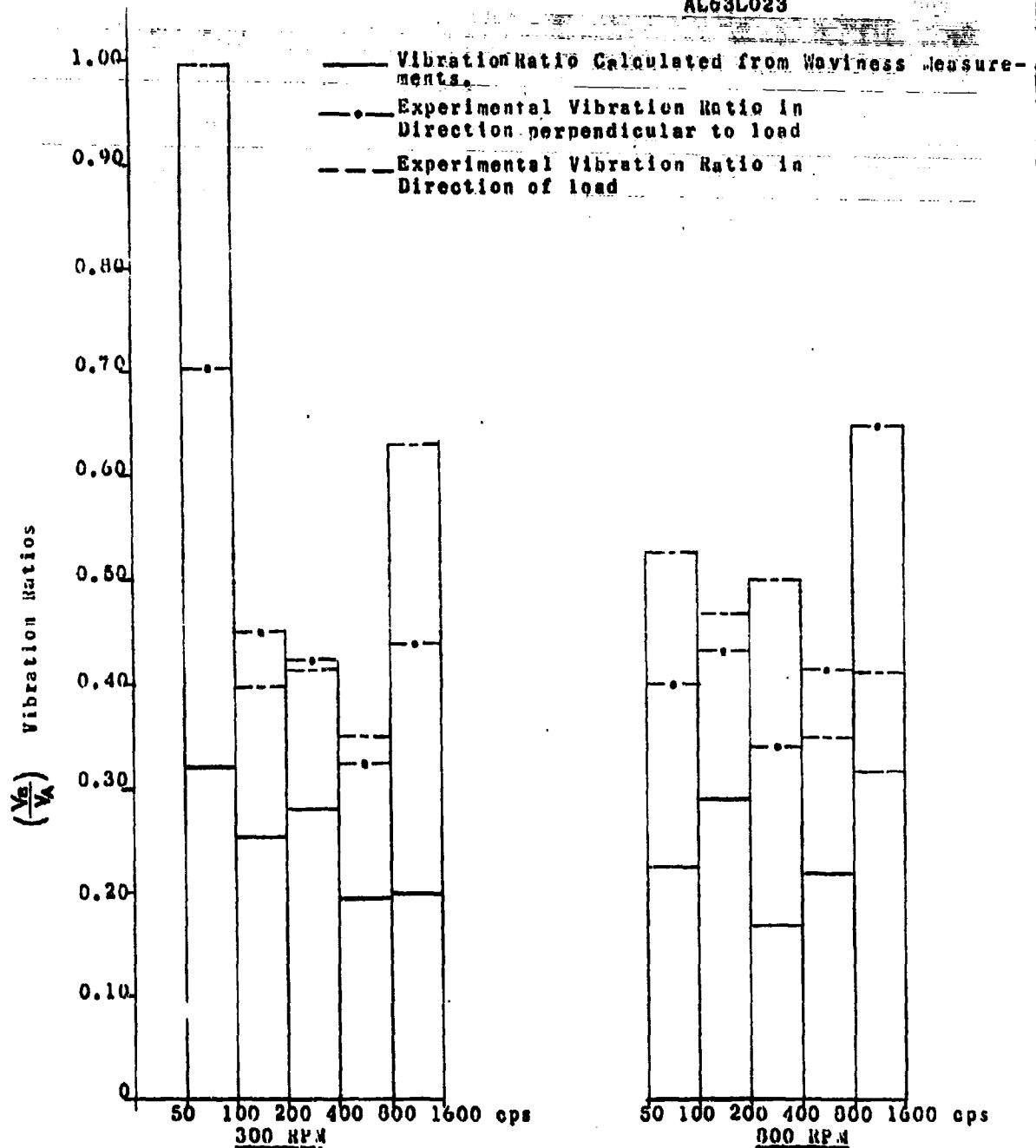


ENCLOSURE 91 OCTAVE BAND VIBRATION SPECTRUM OF STANDARD AND IMPROVED NJ240 CYLINDRICAL BEARINGS IN THE DIRECTION OF LOAD AT 300 RPM

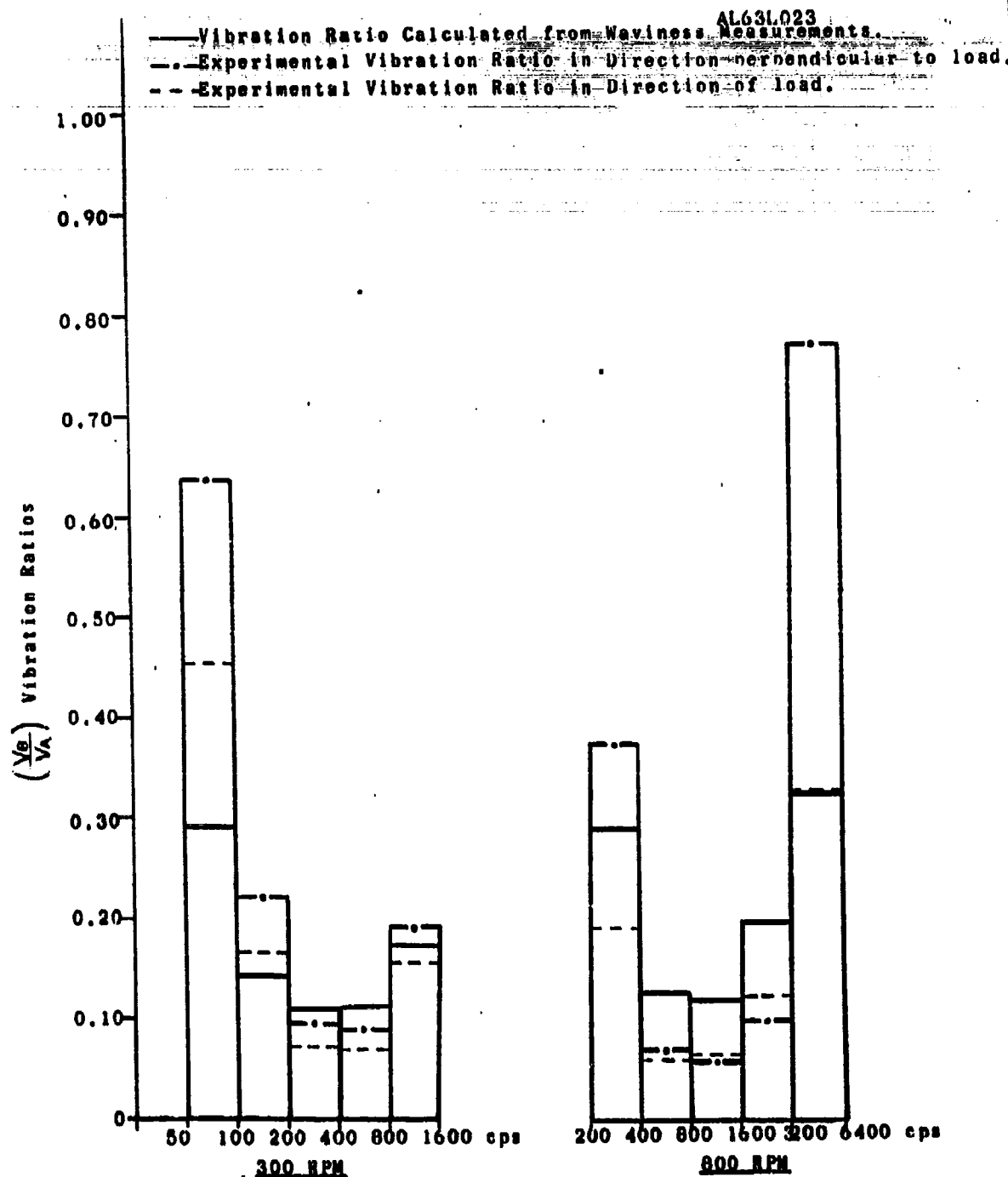


ENCLOSURE 92 OCTAVE BAND VIBRATION SPECTRUM OF THE STANDARD AND IMPROVED TAPERED ROLLER BEARINGS IN THE DIRECTION OF LOAD AT 300 RPM

AL63L023

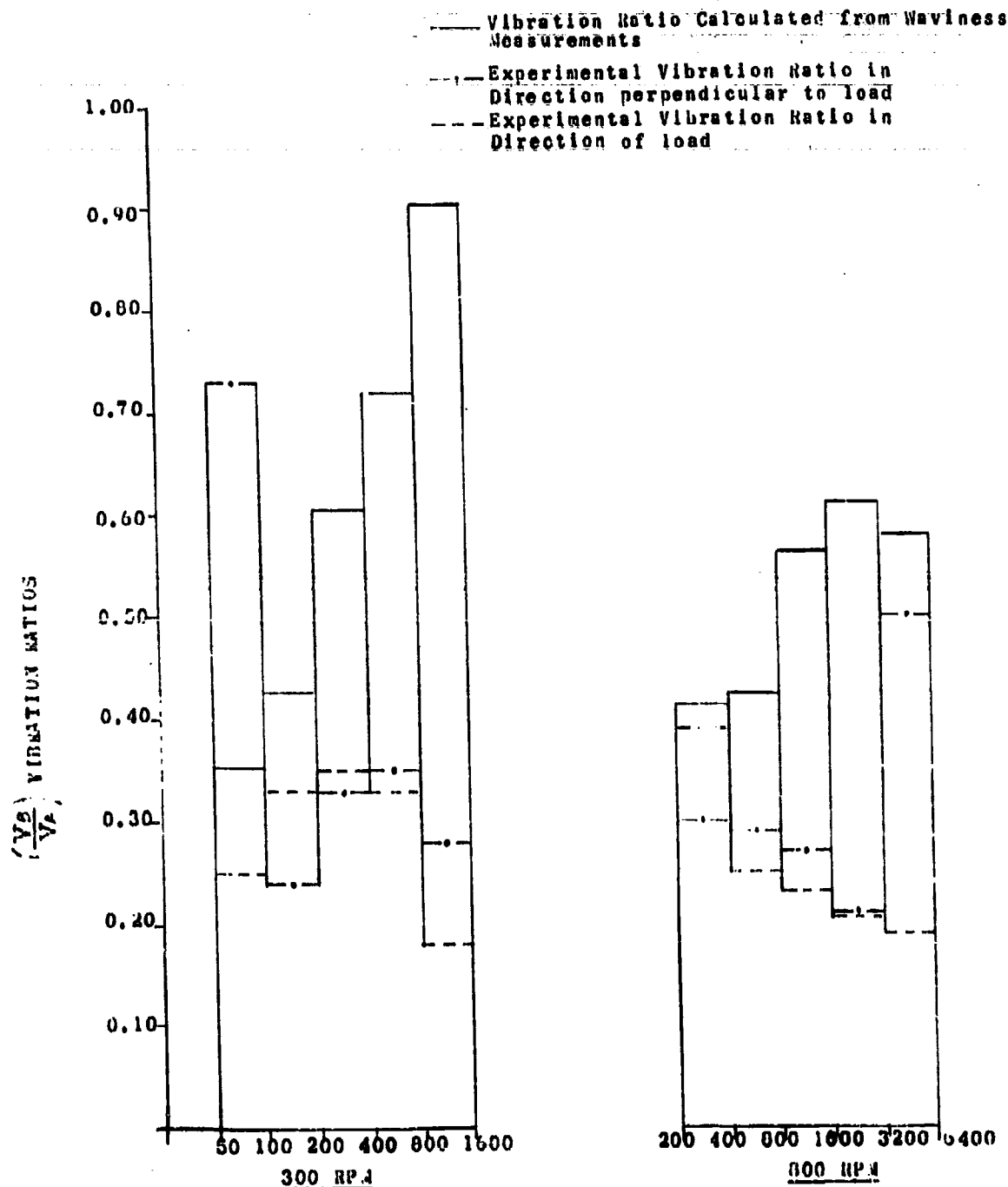


ENCLOSURE 93 THEORETICAL AND EXPERIMENTAL (V_a/V_a) RATIOS
OF 23240 SPHERICAL ROLLER BEARINGS IN SEVERAL OCTAVE BANDS
AT 300 AND 800 RPM.



ENCLOSURE 94 THE THEORETICAL AND EXPERIMENTAL (V_b/V_a) RATIOS FOR THE NJ240 CYLINDRICAL ROLLER BEARINGS IN SEVERAL OCTAVE BANDS AT 300 AND 800 RPM

AL531023



ENCLOSURE 95

THEORETICAL AND EXPERIMENTAL (V_b/V_a) RATIOS OF THE 96900/96140 TAPERED ROLLER BEARINGS IN SEVERAL OCTAVE BANDS AT 300 and 800 RPM.

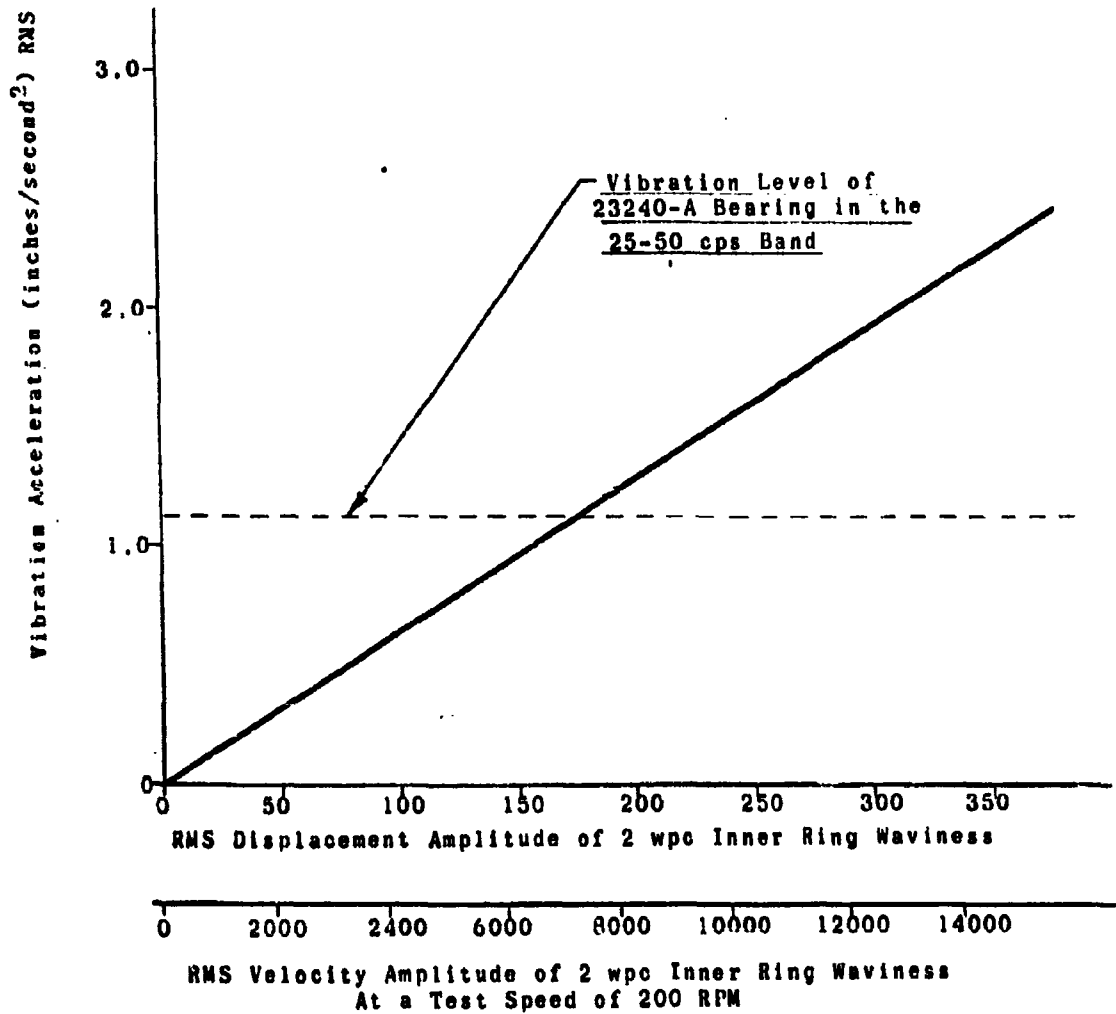
AL63L023

ENCLOSURE 96

SQUARED AMPLIFICATION FACTORS WHICH SHOW THE RELATIVE INFLUENCE
OF OUTER RING, INNER RING AND ROLLER WAVINESS ON THE
VIBRATION LEVEL OF THE ROLLER BEARINGS TESTED WHEN ROTATING AT 300 RPM

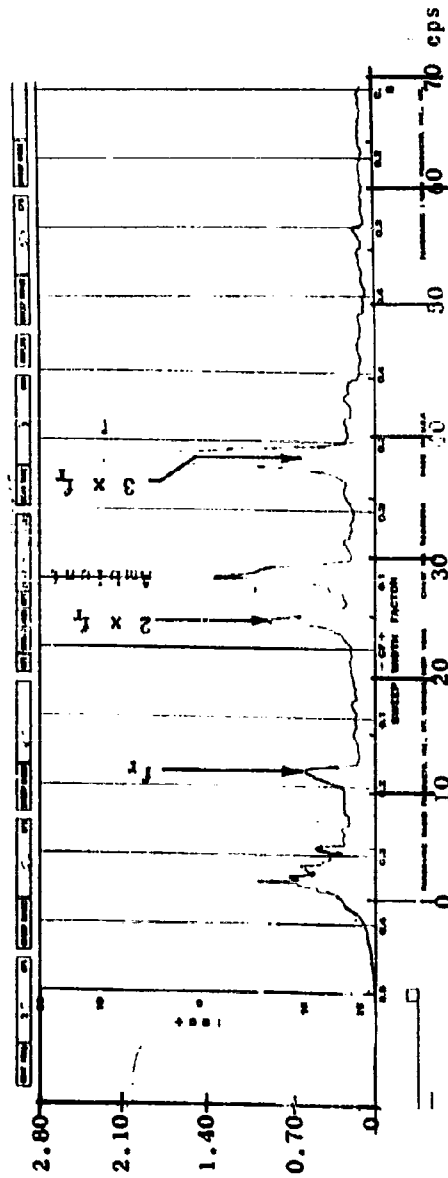
Bearing	Frequency Bands (cps)				
	50- 100	100- 200	200- 400	400- 800	800- 1600
23240-A Spherical					
Z_o^2	0.20	0.02	0.03	0.29	0.10
Z_i^2	0.24	0.66	0.24	0.28	0.23
Z_b^2	0.56	0.32	0.73	0.43	0.67
NJ240-A Cylindrical					
Z_o^2	0.04	0.09	0.10	0.21	0.29
Z_i^2	0.02	0.09	0.25	0.35	0.20
Z_b^2	0.94	0.83	0.67	0.44	0.51
96900/76140-A Tapered					
Z_o^2	0.02	0.07	0.02	0.02	0.13
Z_i^2	0.01	0.02	0.04	0.10	0.17
Z_b^2	0.97	0.91	0.94	0.88	0.70

AL63L023



ENCLOSURE 97 FLEXURAL VIBRATION INDUCED BY INNER RING 2 WPC
IN A 23240 SPHERICAL ROLLER BEARING ROTATING
AT 800 RPM

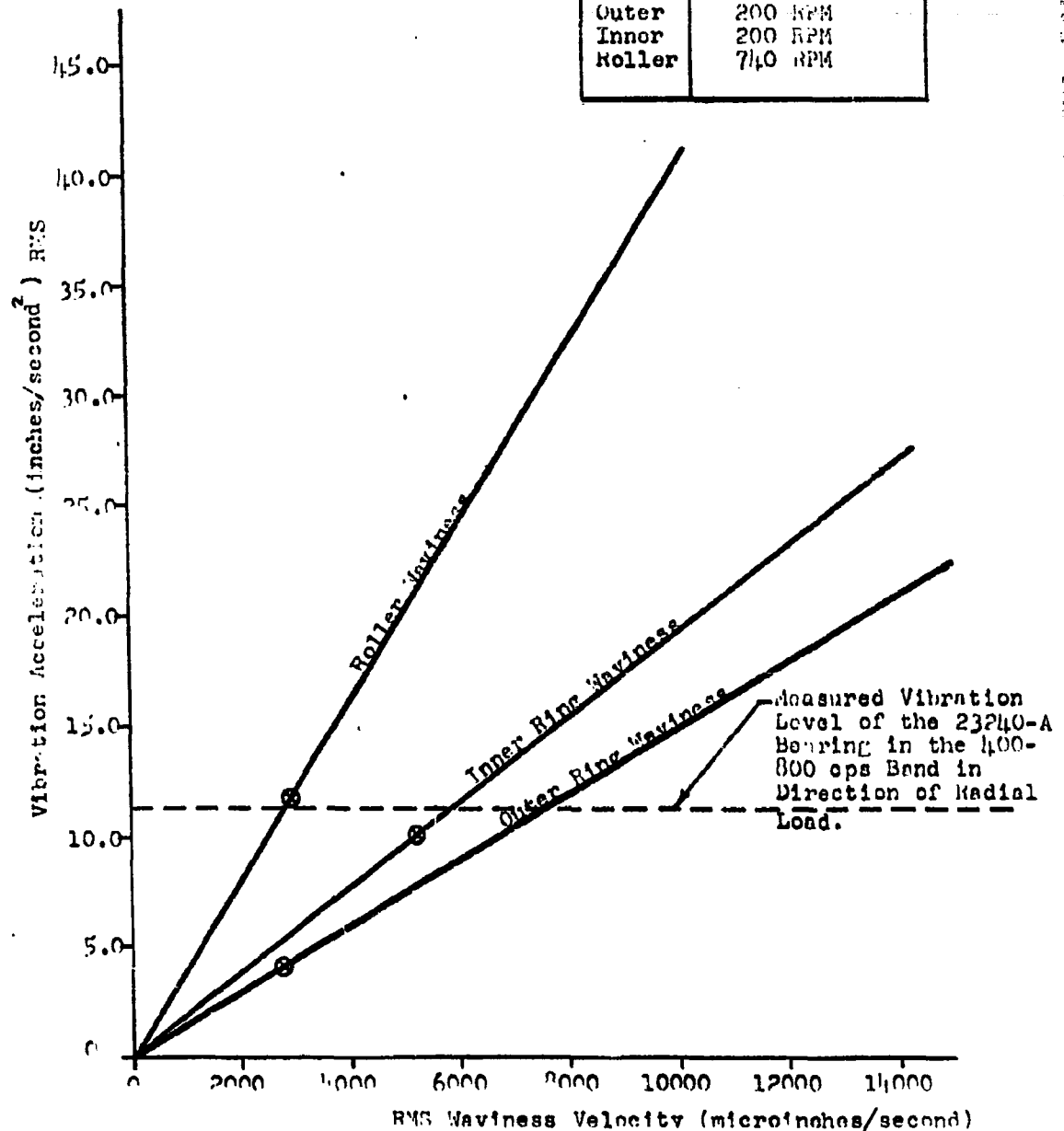
Acceleration in inches/second² RMS



ENCLOSURE 98 LOW FREQUENCY VIBRATION SPECTRUM OF 23256 BEARING IN
DIRECTION OF LOAD UNDER 2000 LBS RADIAL LOAD (0-70 CPS)

AL63L023

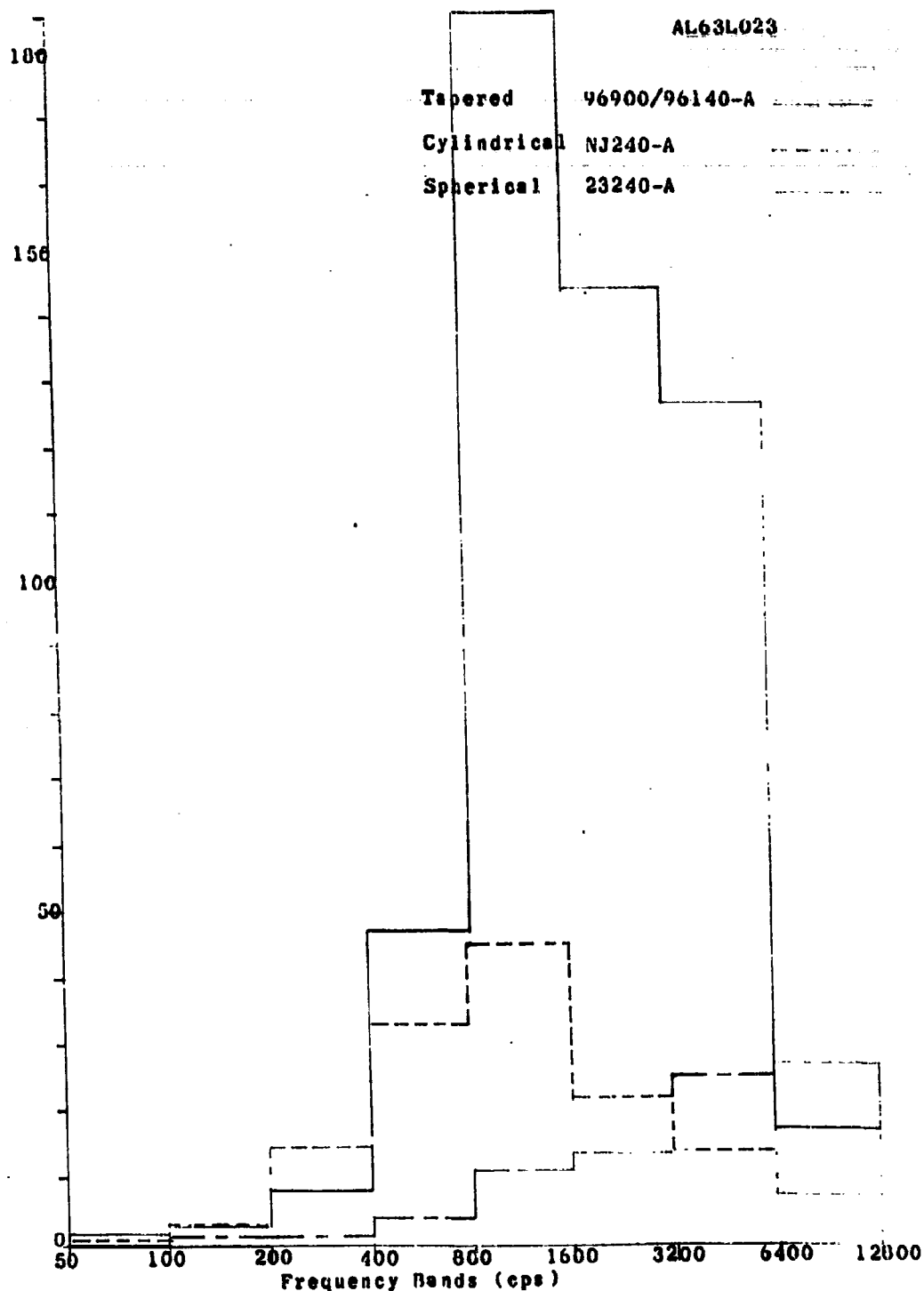
Bearing Part	Waviness Test Spindle Speed
Outer	200 RPM
Innor	200 RPM
Roller	740 RPM



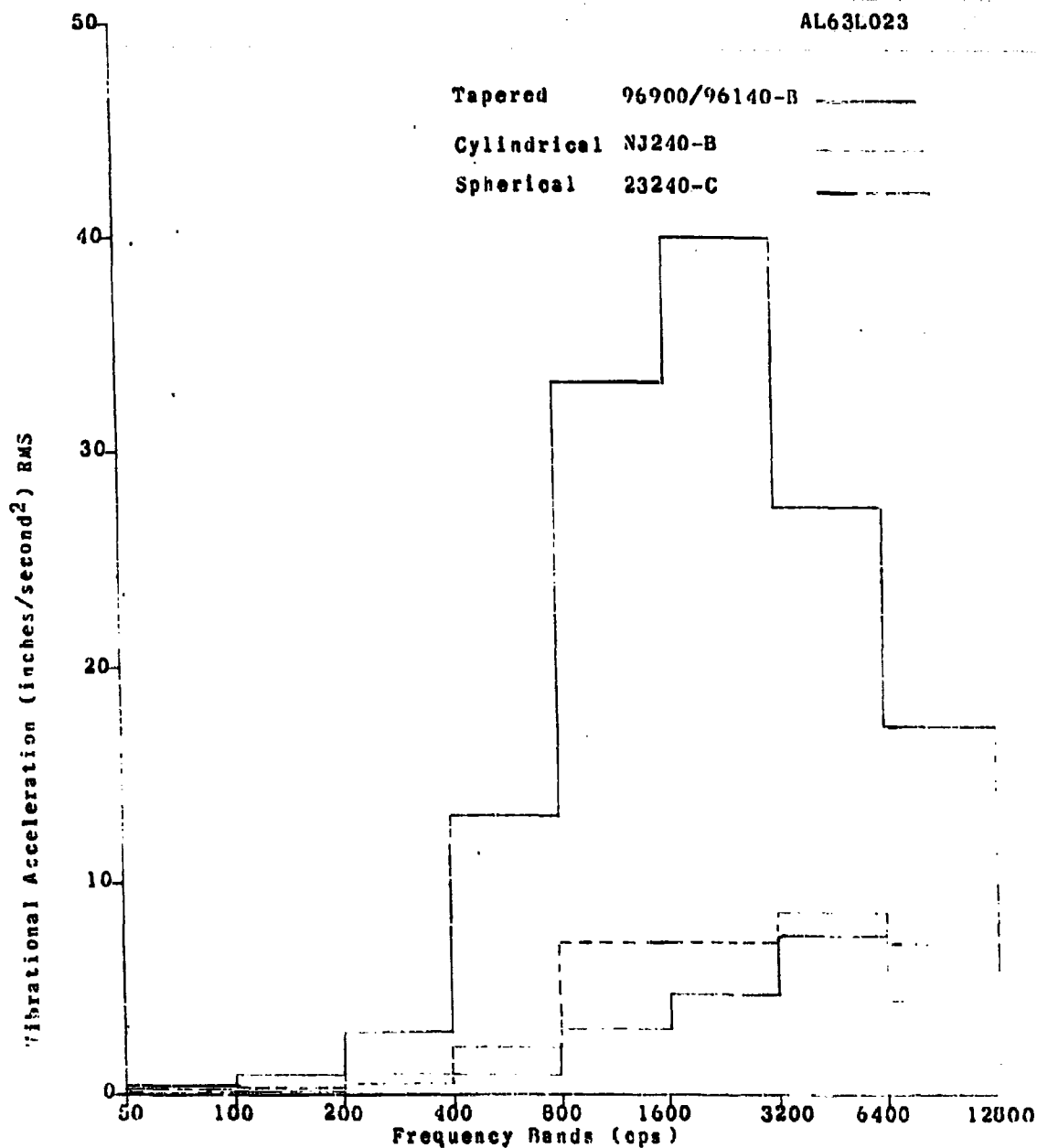
ENCLOSURE 99

Rigid Body Vibrations in the 400-800 cps Band for a 23240 Spherical Roller Bearing rotating at 800 RPM.

Vibrational Acceleration (inches/second²) R.M.S



ENCLOSURE 100 OCTAVE BAND SPECTRA OF STANDARD QUALITY ROLLER BEARINGS (300 RPM).



ENCLOSURE 101 OCTAVE BAND SPECTRA OF IMPROVED ROLLER BEARINGS
(300 RPM).

A63L023

ENCLOSURE 102

THEORETICAL AND EXPERIMENTAL VIBRATION RATIOS BETWEEN CYLINDRICAL AND
SPHERICAL ROLLER BEARINGS AND BETWEEN TAPERED AND SPHERICAL ROLLER BEARINGS
OF DIFFERENT QUALITY LEVELS

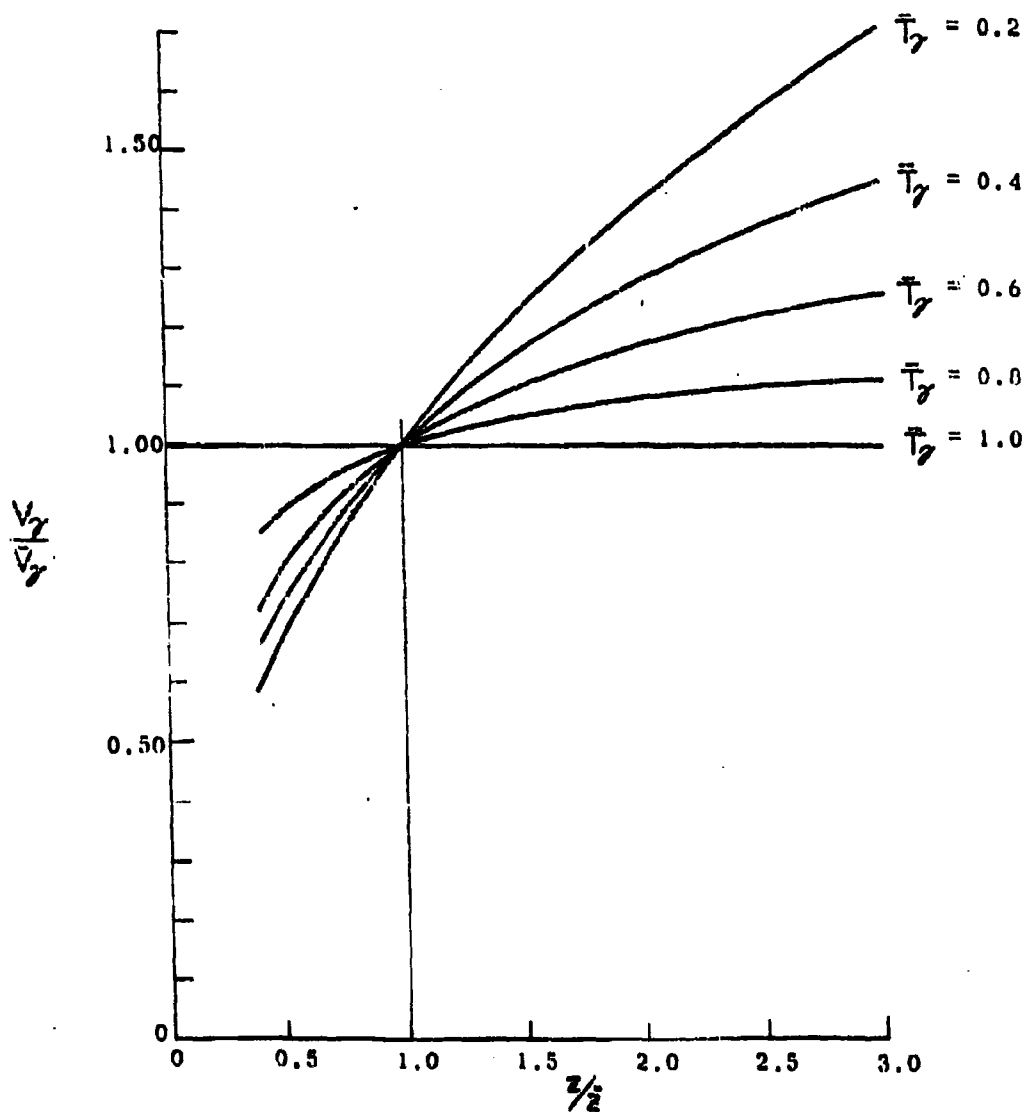
Vibration Ratio		Frequency Bands (cps)				
		50- 100	100- 200	200- 400	400- 800	800- 1600
$\left(\frac{V_{c-A}}{V_{s-A}}\right)$	Th.	1.28	1.50	2.18	1.64	1.52
	Ex	0.94	3.32	8.30	8.20	4.02
Ratio $\frac{Th}{Ex}$		1.36	0.45	0.26	0.20	0.38
$\left(\frac{V_{t-A}}{V_{s-A}}\right)$	Th	2.66	1.00	1.80	1.42	1.38
	Ex	1.85	2.54	4.45	11.50	16.30
Ratio $\frac{Th}{Ex}$		1.44	0.75	0.41	0.12	0.08
$\left(\frac{V_{c-B}}{V_{s-B}}\right)$	Th	2.12	1.60	2.34	2.44	2.78
	Ex	1.00	1.51	1.63	2.25	2.23
Ratio $\frac{Th}{Ex}$		2.12	1.06	1.43	1.08	1.25
$\left(\frac{V_{t-B}}{V_{s-B}}\right)$	Th	5.86	6.08	7.72	9.88	7.70
	Ex	1.03	2.25	4.25	13.00	10.40
Ratio $\frac{Th}{Ex}$		5.68	2.70	1.82	0.76	0.74

THE SUBSCRIPT A REFERS TO BEARINGS OF STANDARD PRODUCTION QUALITY,
THE SUBSCRIPTS B AND C TO IMPROVED BEARINGS.

Th = Theoretical
Ex = Experimental

Subscript c = cylindrical
s = spherical
t = tapered

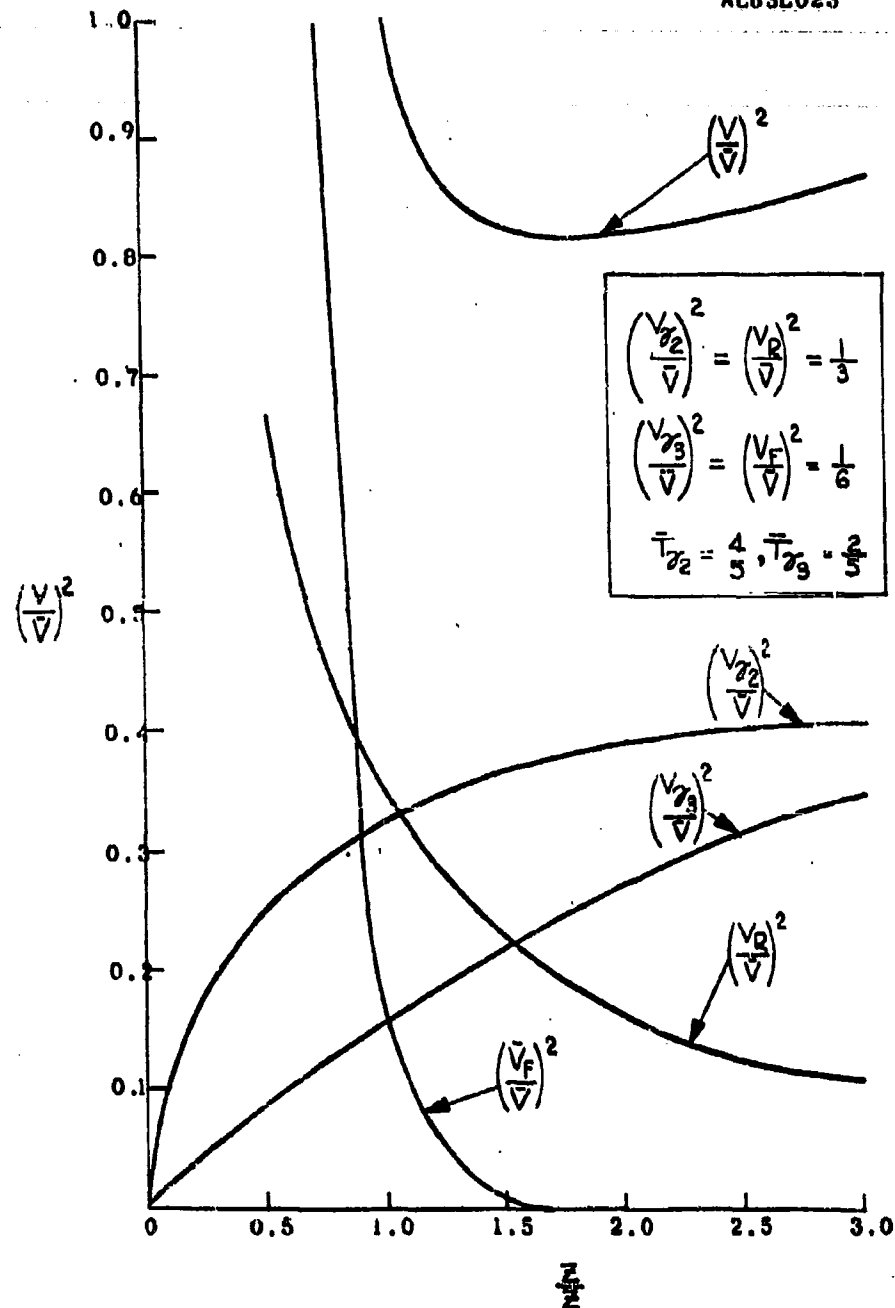
AL63L023



ENCLOSURE 103

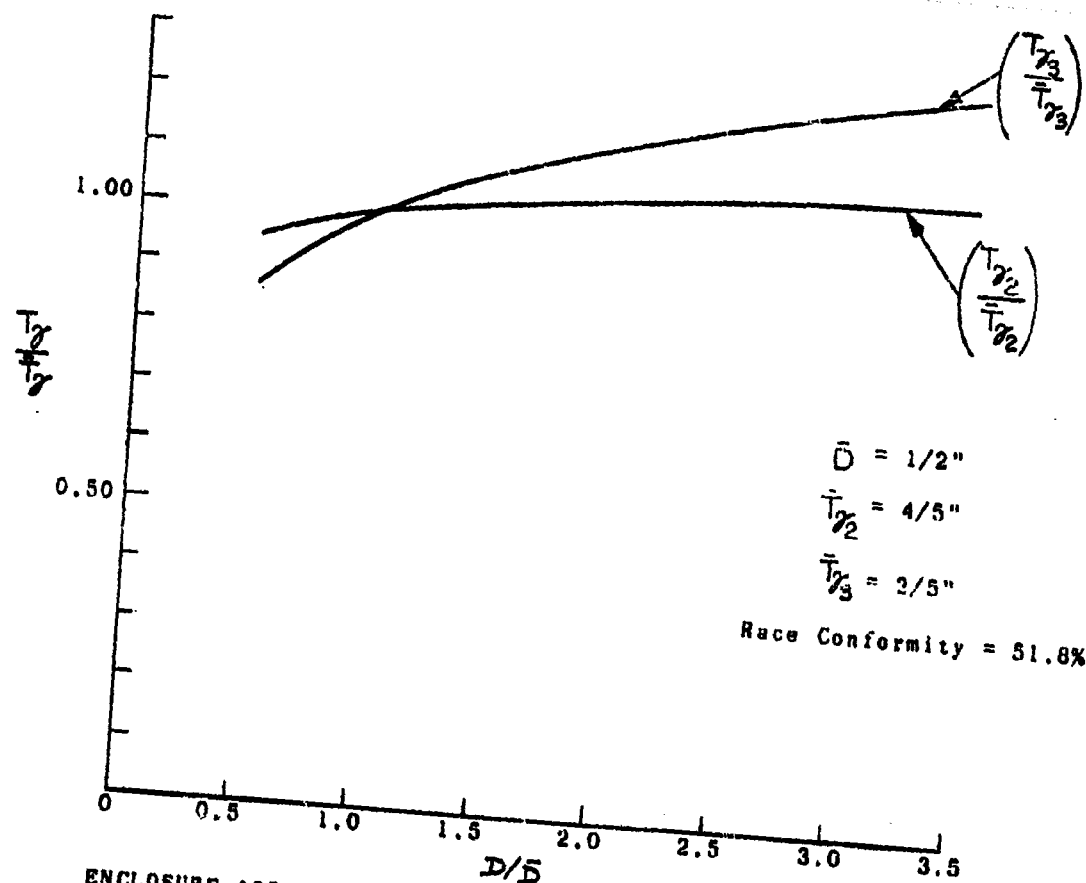
EFFECT OF NUMBER OF ROLLING BODIES ON
THE VIBRATION LEVEL OF FLEXURAL VIBRATIONS
DUE TO WAVINESS.

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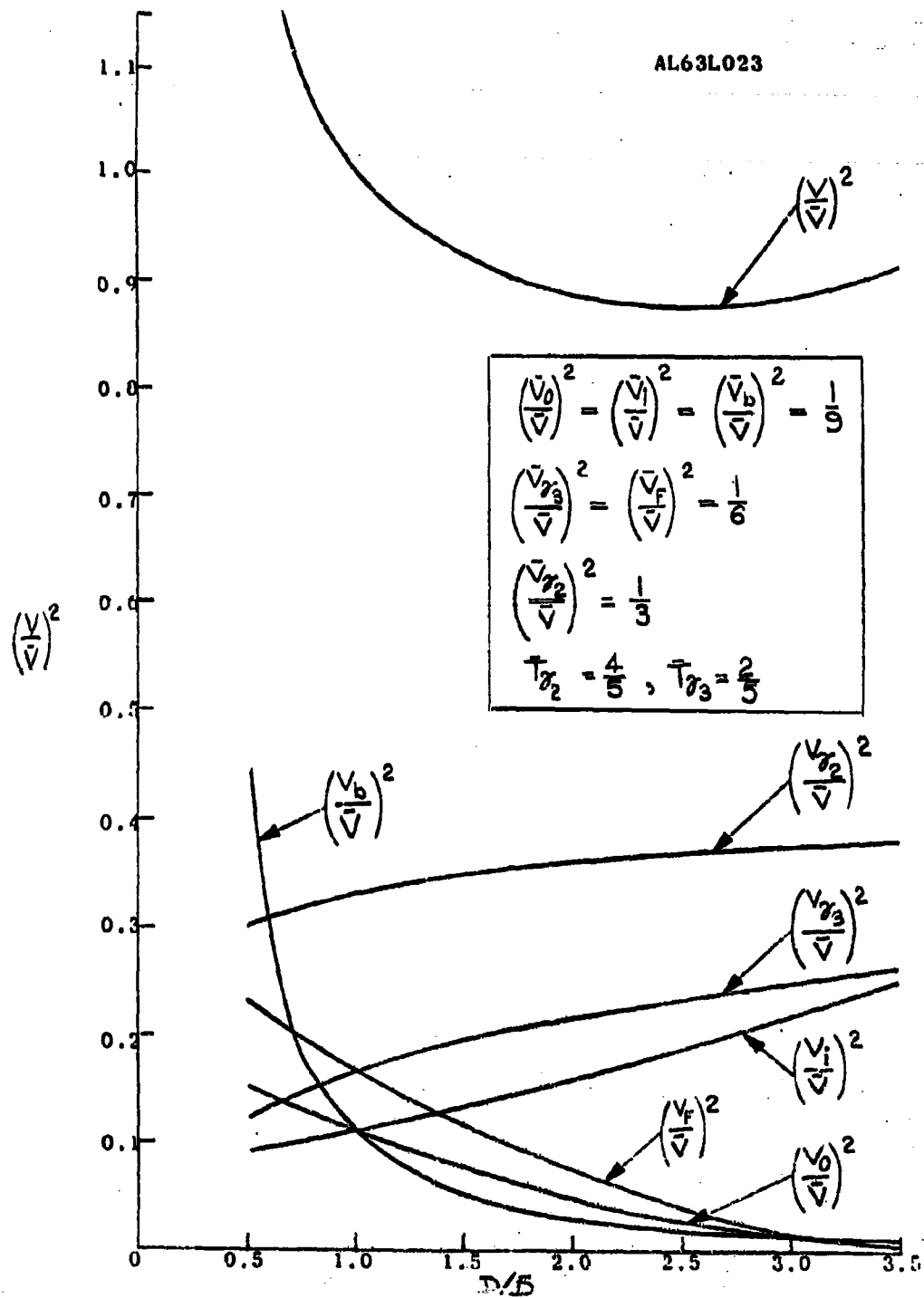
ENCLOSURE 104 VIBRATION LEVEL OF A BEARING AS A FUNCTION OF THE NUMBER OF ROLLING BODIES.

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ENCLOSURE 105

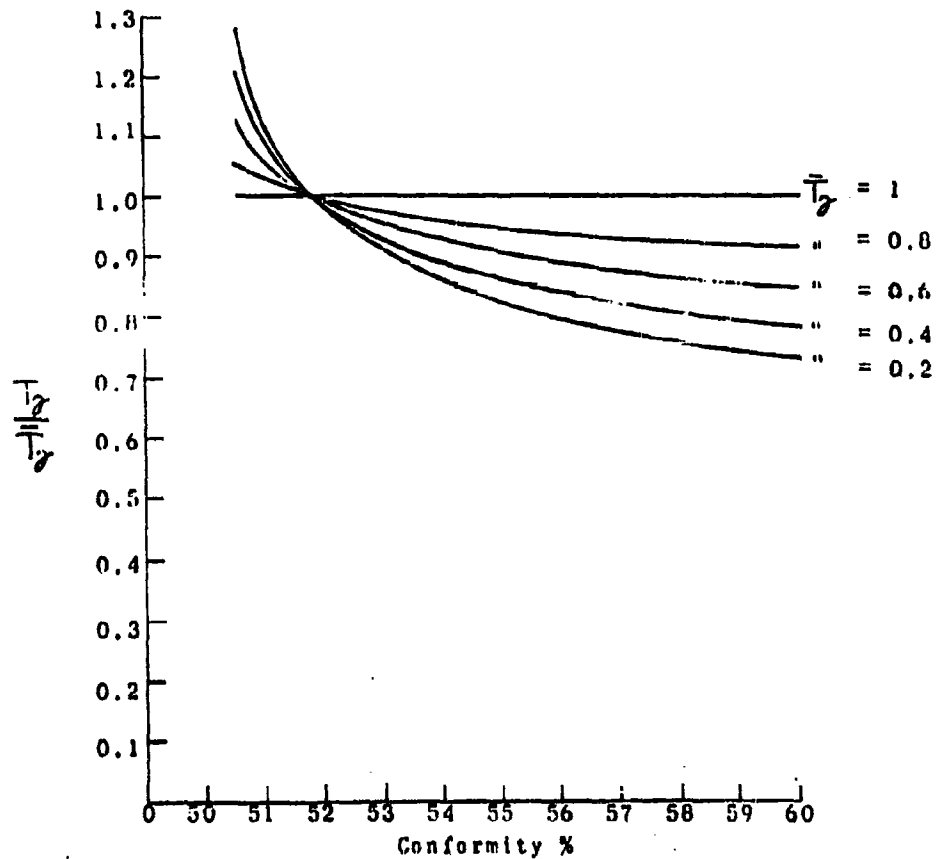
AMPLIFICATION FACTOR FOR FLEXURAL VIBRATIONS
 INDUCED BY INNER RING WAVINESS, AS A FUNCTION
 OF BALL DIAMETER



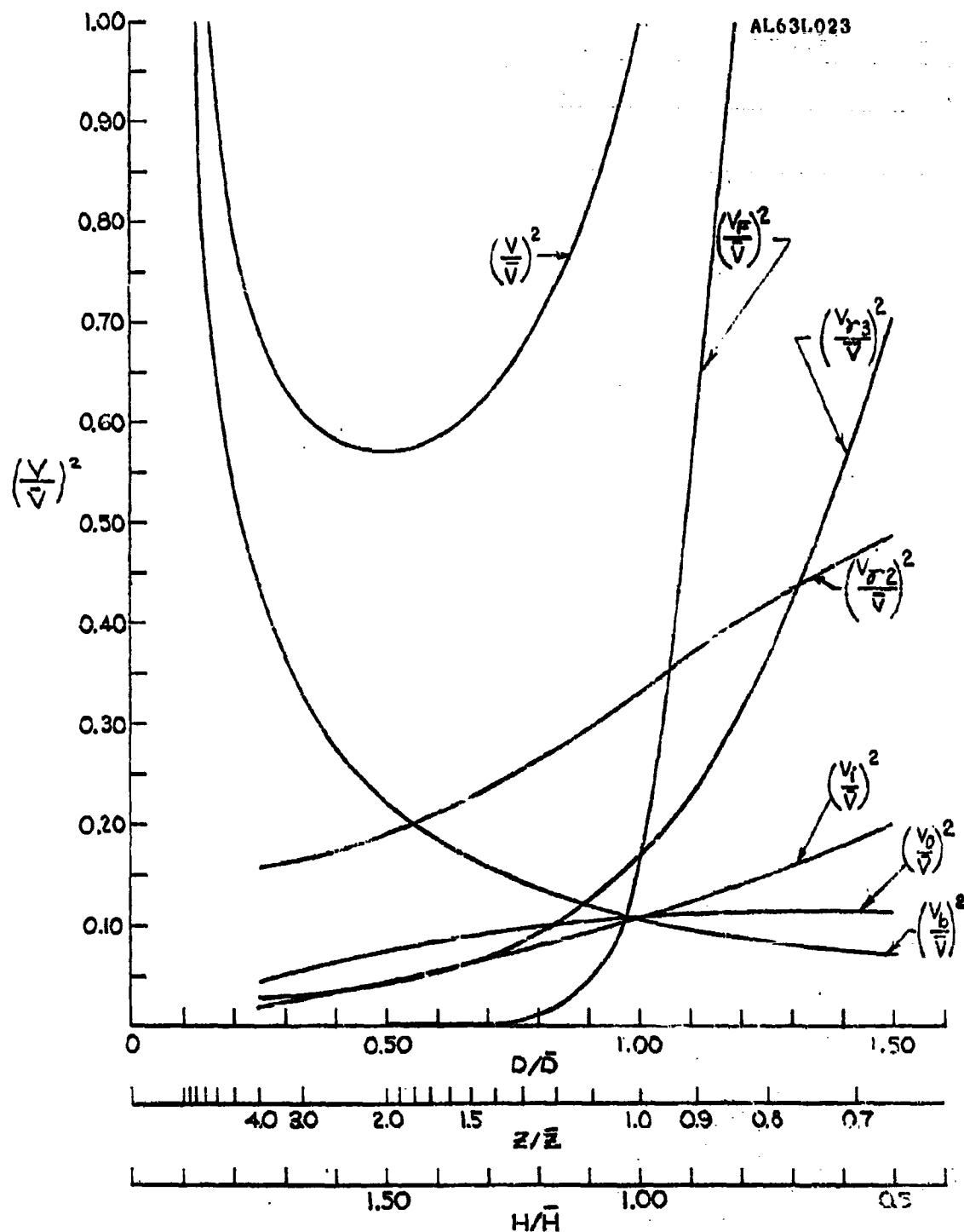
ENCLOSURE 106 VIBRATION LEVEL OF A BEARING AS A FUNCTION OF ROLLING BODY DIAMETER

AL63L023

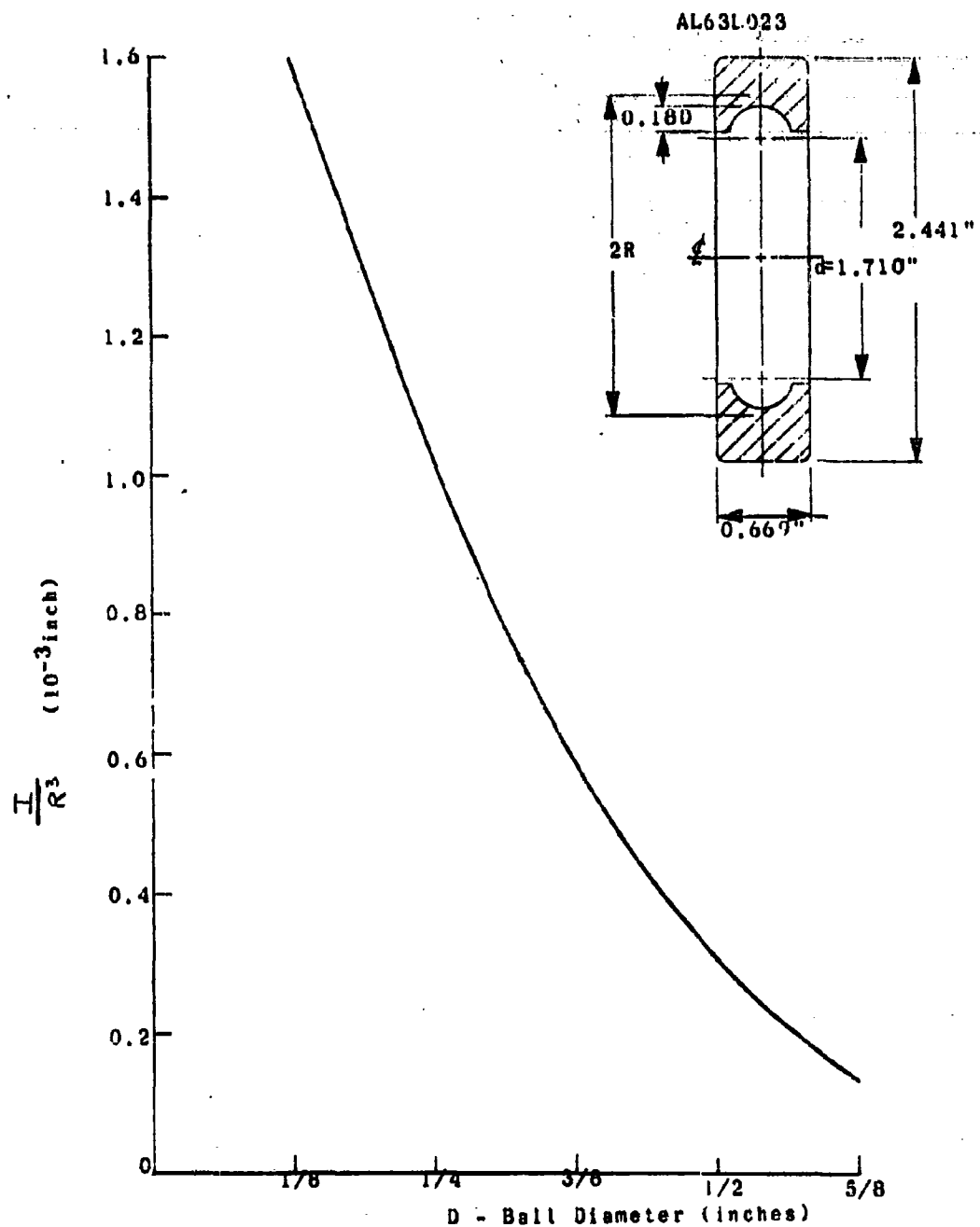
T_g = Amplification factor for bearing with 51.8% conformity and 1/2" balls.



ENCLOSURE 107 EFFECT OF CONFORMITY ON FLEXURAL VIBRATIONS INDUCED BY LOW ORDER INNER RING WAVINESS

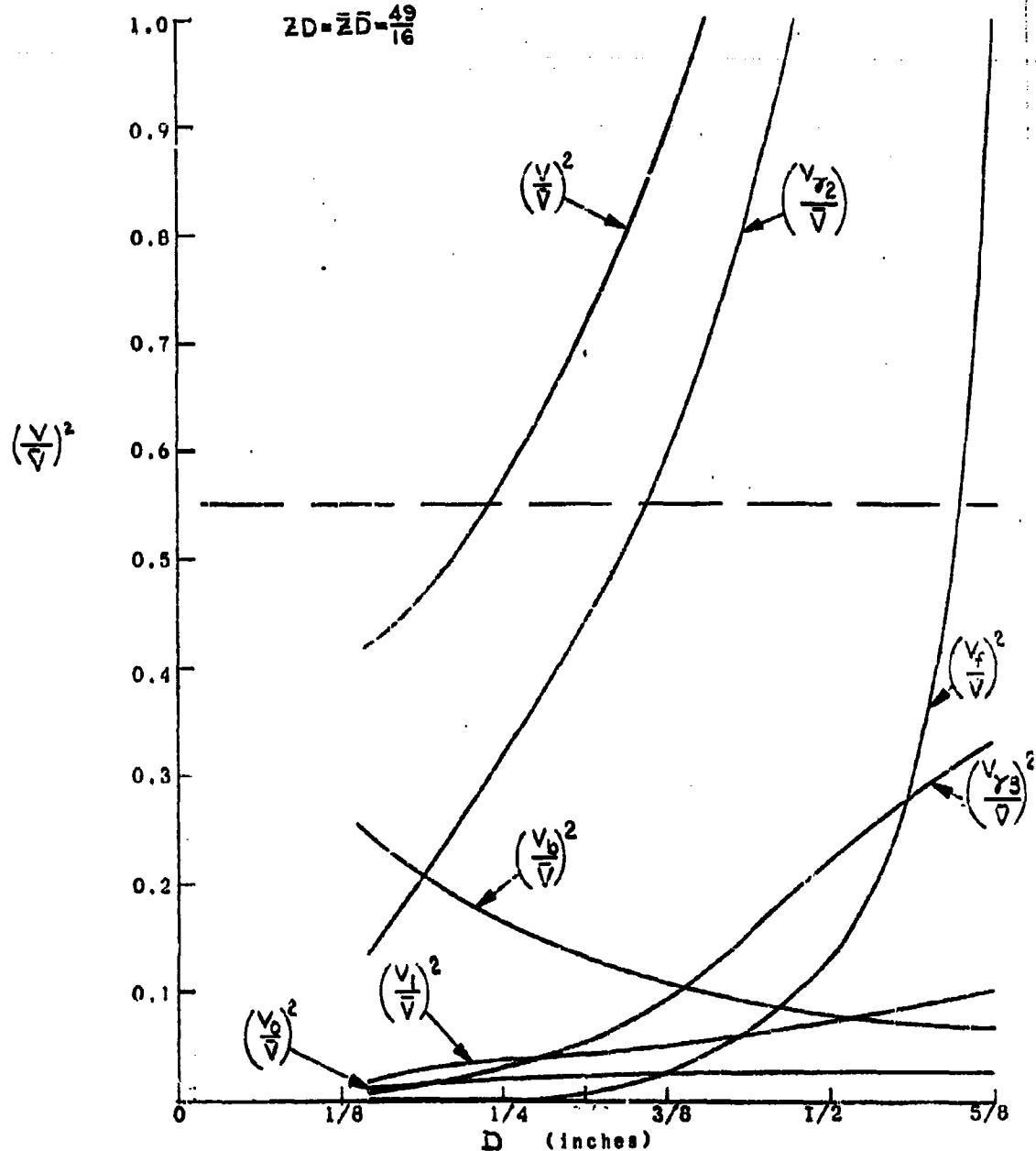


ENCLOSURE 108 VIBRATION OF A BEARING AS A FUNCTION OF ROLLING BODY DIAMETER D , WHERE Z AND H ARE ASSUMED TO DEPEND ON D .



ENCLOSURE 102 FLEXURAL RIGIDITY I/R^3 OF OUTER RING AS A FUNCTION OF BALL DIAMETER D .

AL63L023

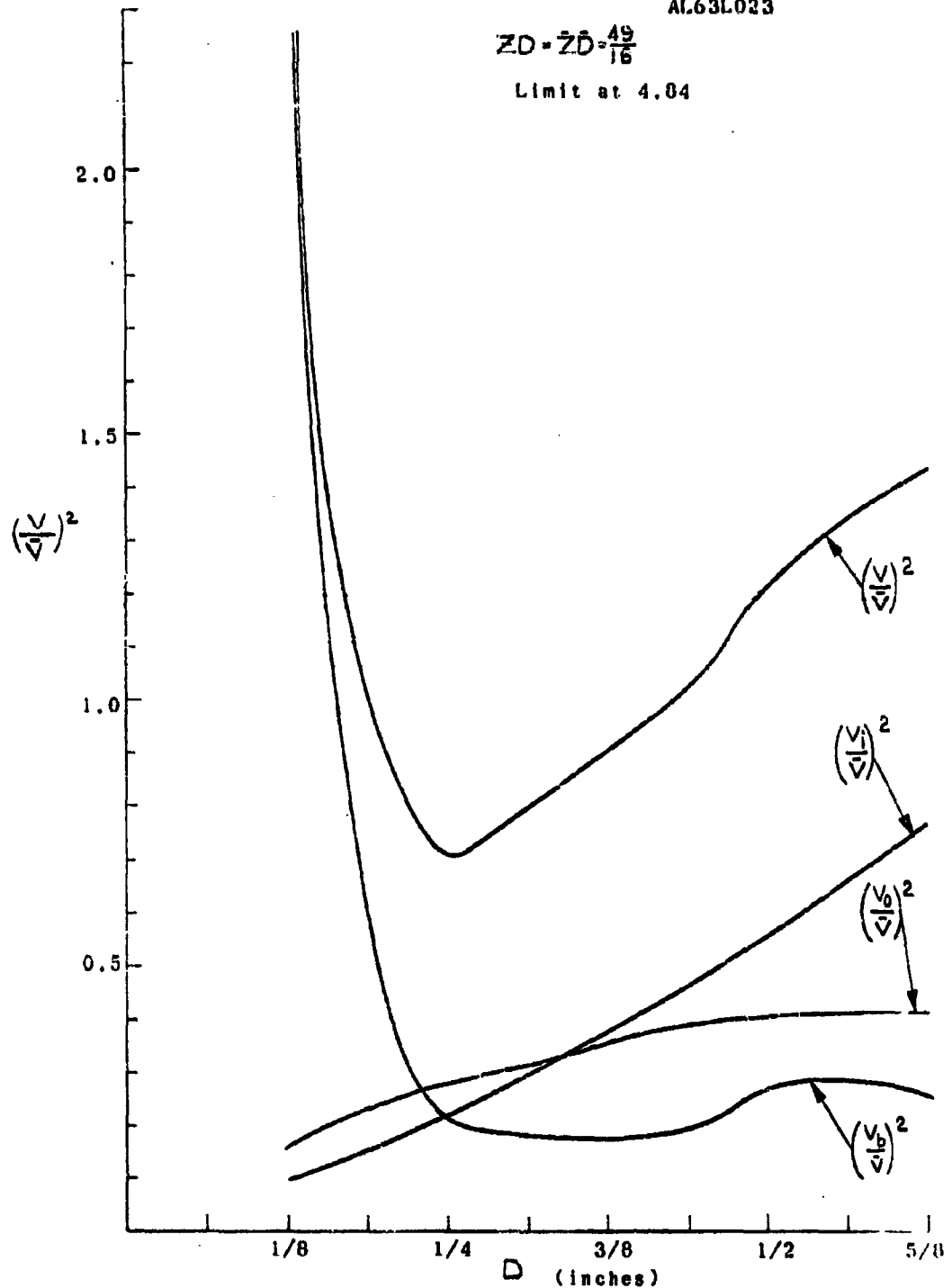
$$ZD = \bar{Z}\bar{D} = \frac{49}{16}$$


ENCLOSURE 110 SQUARED VIBRATION LEVEL RATIOS IN THE LOW (50-300 CPS) BAND AS A FUNCTION OF D.

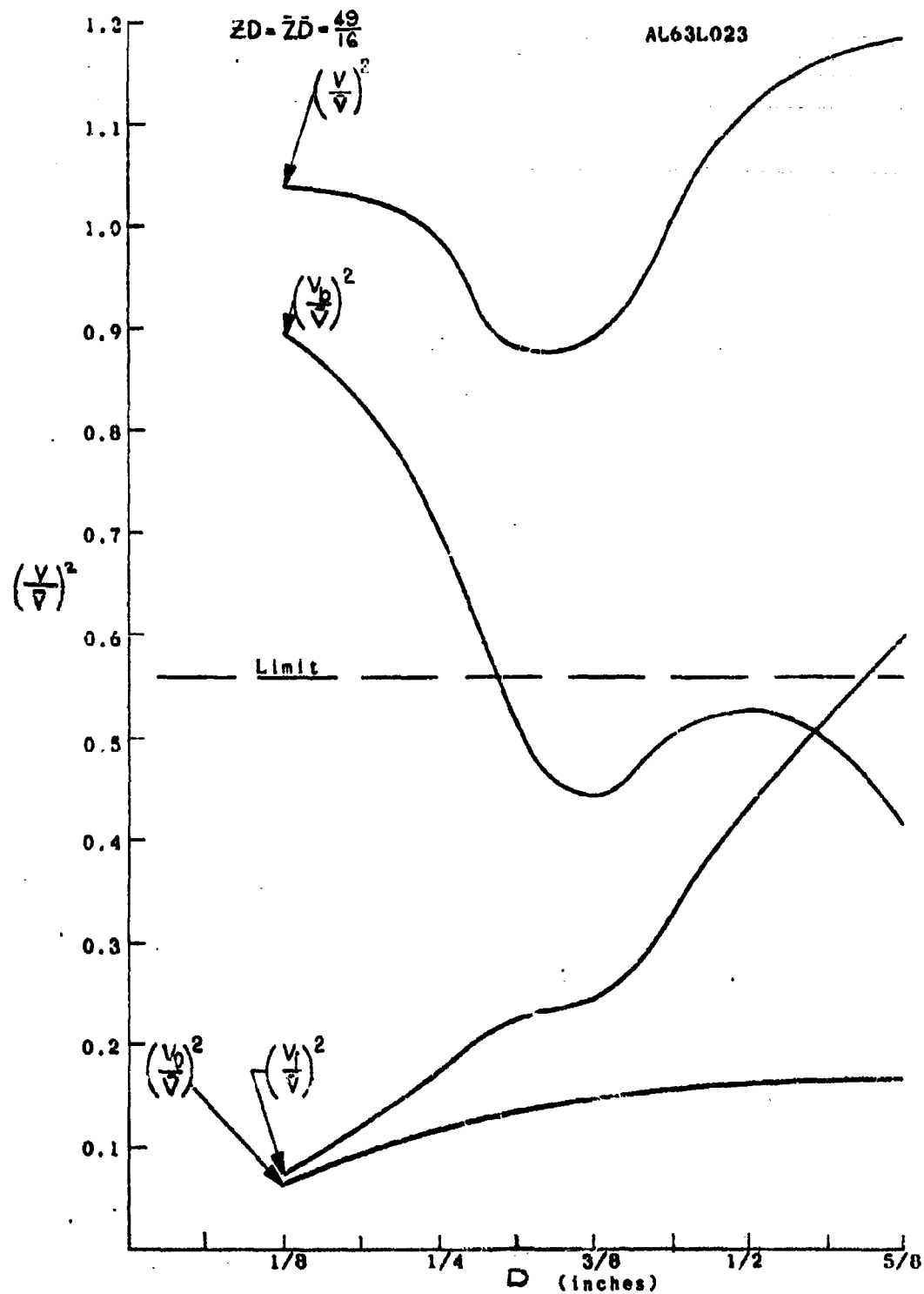
AL63L023

$$ZD = \bar{Z}\bar{D} = \frac{49}{16}$$

Limit at 4.04

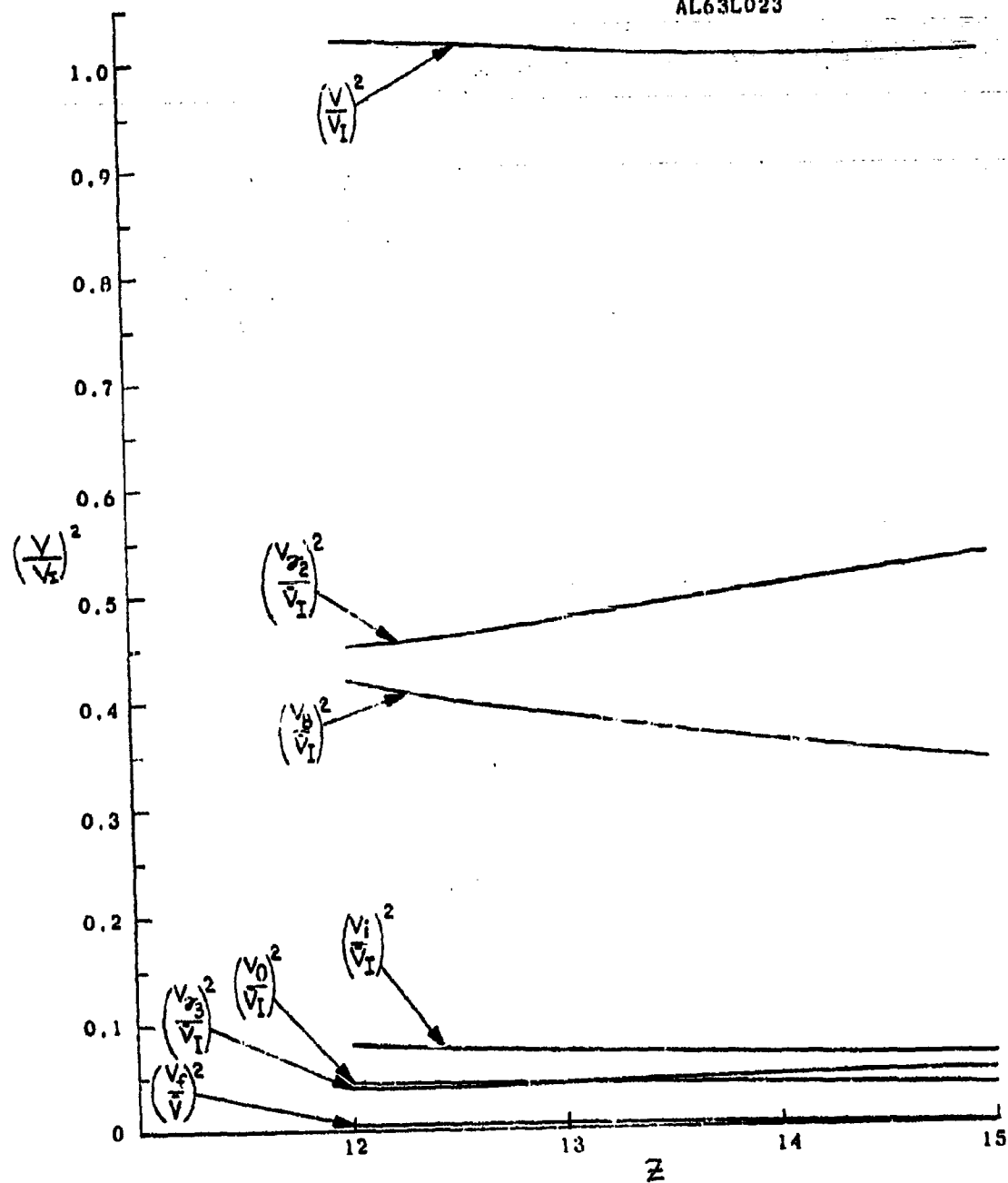


ENCLOSURE 111 SQUARED VIBRATION LEVEL RATIOS IN THE MEDIUM (300-1800 CPS) BAND AS A FUNCTION OF D

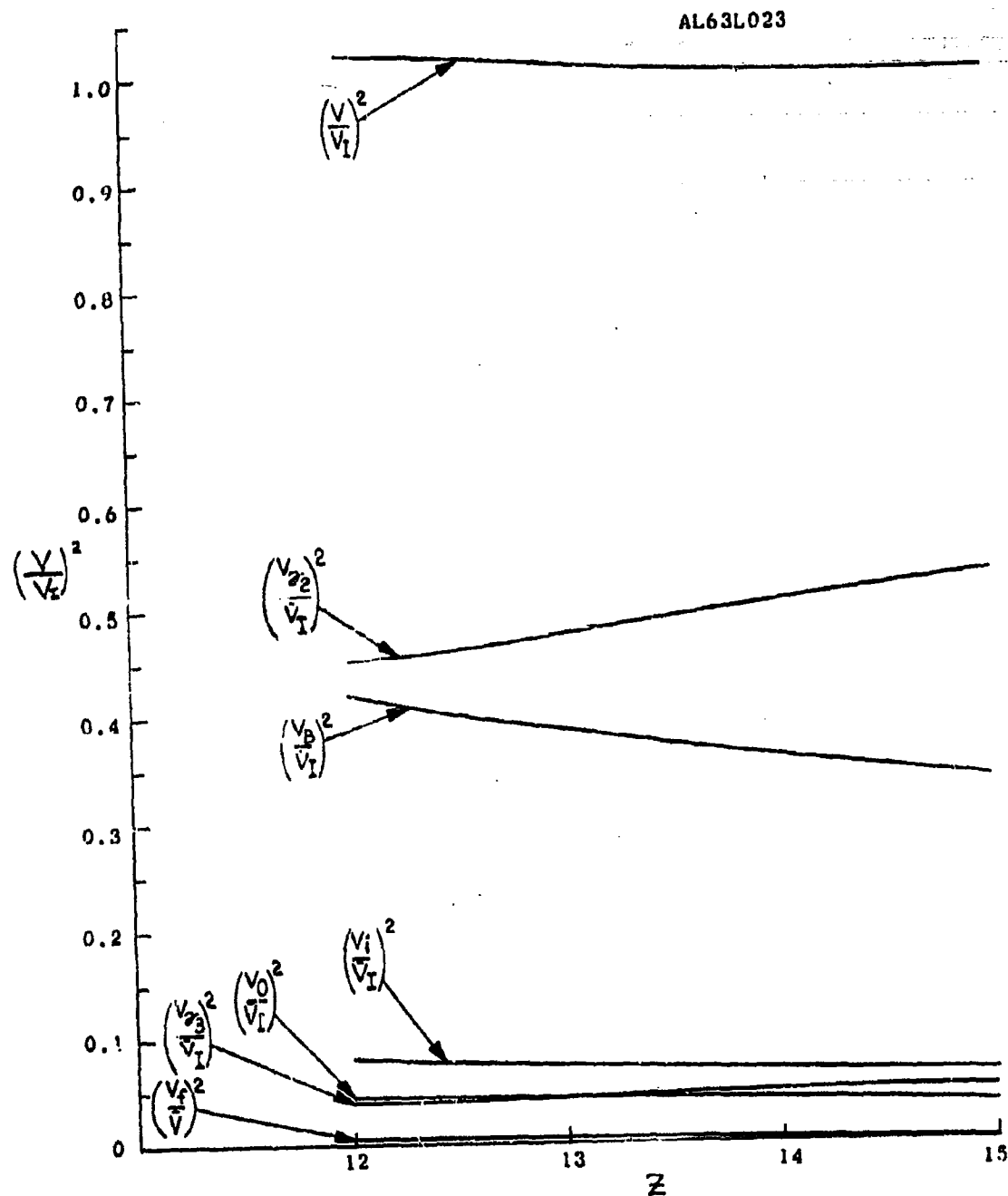


ENCLOSURE 112 SQUARED VIBRATION LEVEL RATIOS IN THE HIGH (1800-10000 CPS) BAND AS A FUNCTION OF D .

AL63L023

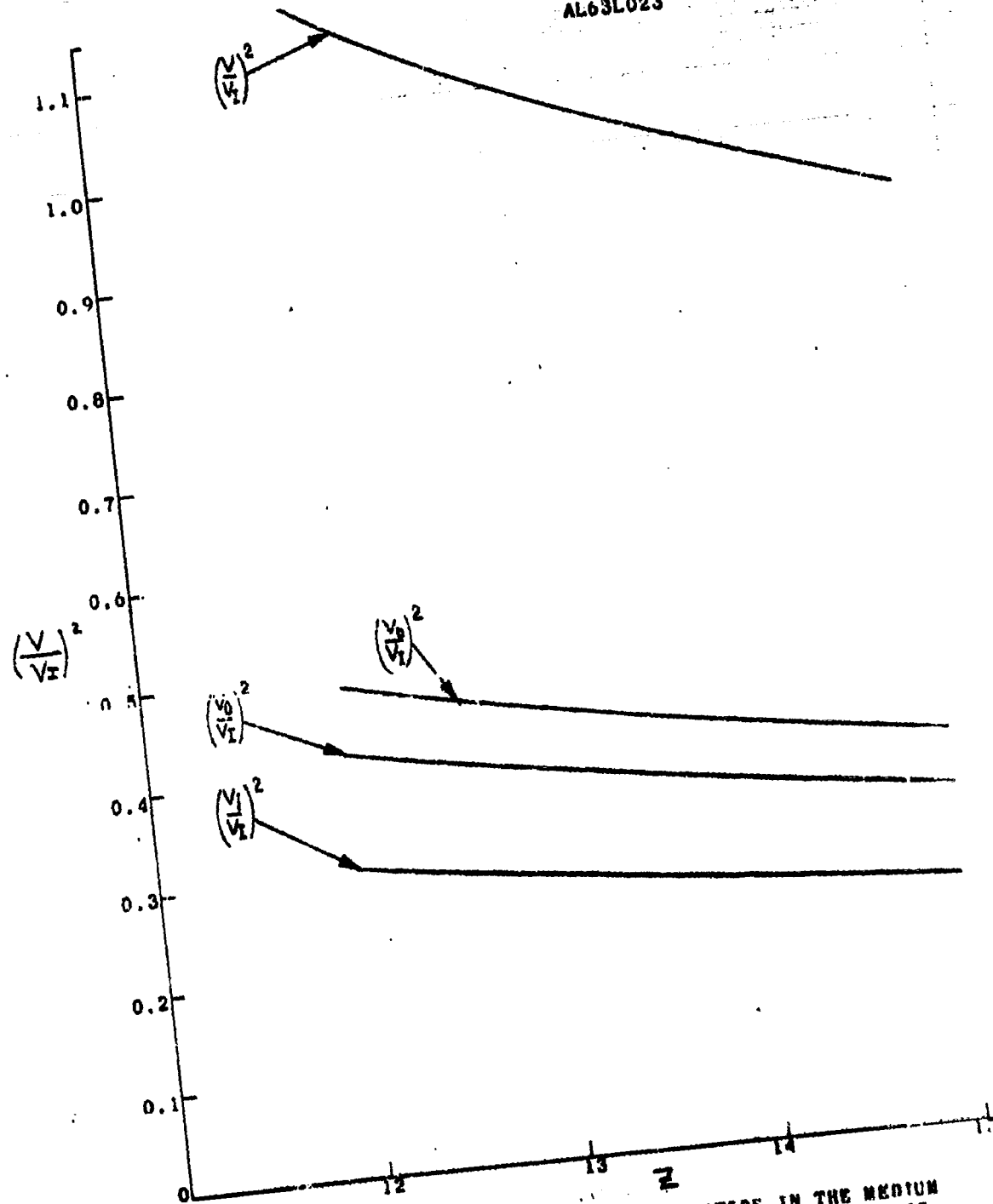


ENCLOSURE 113 SQUARED VIBRATION LEVEL RATIOS IN THE LOW (50-300 CPS) BAND AS A FUNCTION OF Z FOR BALL DIAMETER, $D_s \approx 7/32"$

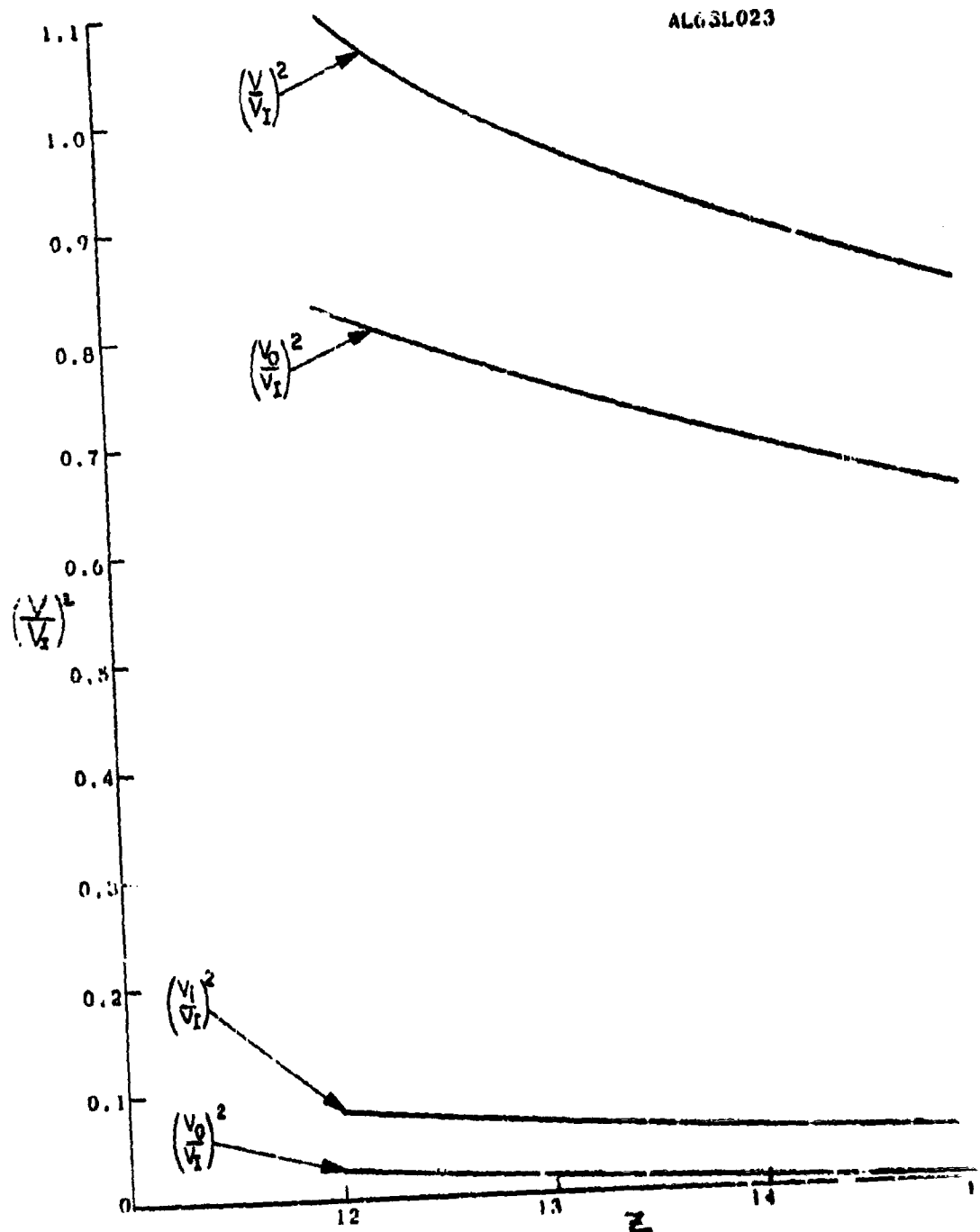


ENCLOSURE 113 SQUARED VIBRATION LEVEL RATIOS IN THE LOW (50-300 CPS) BAND AS A FUNCTION OF Z FOR BALL DIAMETER, $D_s = 7/32"$

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ENCLOSURE 114 SQUARED VIBRATION LEVEL RATIOS IN THE MEDIUM (300-1800 CPS) BAND AS A FUNCTION OF z FOR BALL DIAMETER, $D_1 = 7/32"$.



ENCLOSURE 115 SQUARED VIBRATION LEVEL RATIOS IN THE HIGH (1800-10000 CPS) BAND AS A FUNCTION OF Z FOR BALL DIAMETER, $D_2 = 7/32"$